

Calculus books have become full of clutter, distracting margin notes, and unneeded features. This calculus book clears out that clutter so that students can focus on the important ideas of calculus. The goal was to create a clean, streamlined calculus book that is accessible and readable for students while still upholding the standards required in science, mathematics, and engineering programs, and that is flexible enough to accommodate different teaching and learning styles.

This particular book is an alternative edition of Taalman/Kohn *Calculus* that covers Calculus I and part of what is usually taught as Calculus II, together with precalculus and algebra material. Students who need to strengthen their precalculus and algebra backgrounds while taking calculus can start with this book for an extended Calculus I course and then proceed to a Calculus II course using any other calculus text (although this book is structured to work particularly well with a continuation of the course using Taalman/Kohn *Calculus*).

Calculus before Calculations

The primary way that this book differs from Taalman/Kohn *Calculus* and other calculus books is that it begins by using relatively simple functions such as polynomials and simple algebraic functions to develop all of the main ideas of differential calculus. After students have gained a firm grounding in the ideas of calculus, that grounding is used to explore more complicated algebraic functions as well as exponential, logarithmic, and trigonometric functions. Students who struggle with precalculus and algebra benefit from this approach by being able to understand calculus first and then build their calculational skills afterwards.

A Full Calculus Course

Everything in Chapters 0 through 6 of Taalman/Kohn *Calculus* is included in this textbook, although the material appears in a different order and the book is supplemented with additional algebra and precalculus material. This means that students who learn from *Calculus I with Integrated Precalculus* are fully prepared to join their peers in Calculus II, and it is easy to run parallel sections of a course using *Calculus I with Integrated Precalculus* alongside sections of an existing Calculus course, while being confident that students leave with the same background and understanding of the material.

Linear Flow with Clean Margins

One thing that is distinctive about this calculus book is that it follows a linear writing style. Figures and equations flow with the text as part of a clear, structured exposition instead of being scattered about in the margins. This approach greatly increases the clarity of the book and encourages focused reading.

Exposition before Calculation

Another distinctive feature of this book is that in each section the exposition and illustrative examples are separated from the longer, more complicated calculational examples. Including these longer examples separately from the exposition increases flexibility: Students who want to read and understand the development of the material can do so without being bogged down or distracted by large examples, while students who want to use the book as a reference for looking up examples that are similar to homework problems can also do that.

Examples to Learn From

Within the exposition of each section are short examples that quickly illustrate the concepts being developed. Following the exposition is a set of detailed, in-depth examples that explore both calculations and concepts. Many steps and illustrations are provided in each example in order to aid the student, including details about how to get started on a problem and choose an appropriate solution method. One of the elements of the book that I am most proud of is the “Checking the Answer” feature, which is included after selected examples to encourage students to learn how to check their own answers.

Building Mathematics

This book approaches mathematics as a discipline that is developed logically, theorem by theorem. Whenever possible, theorems are followed by proofs that are written to be understood by students. These proofs are included because they are part of the logical development of the material, but each proof is clearly labeled and indented to indicate that it can be covered or skipped, according to instructor preference. Each exercise set contains an optional subsection of proofs, many of which are accessible even to beginning students. In addition, the interconnections among topics are emphasized by “Thinking Back” and “Thinking Forward” exercises that appear in each section and “Capstone” problems at the end of each chapter.

Consistency and Reliability

Another improvement in this book is that it has a consistent and predictable structure. For example, instructors can rely on every section concluding with a “Test Your Understanding” feature which includes five questions that students can use to self-test and that instructors can choose to use as pre-class questions. The exercises are always consistently split into subsections of different types of problems: “Thinking Back,” “Concepts,” “Skills,” “Applications,” “Proofs,” and “Thinking Forward.” In addition, the “Concepts” subsection always begins with a summary exercise, eight true/false questions, and three example construction exercises. Instructors and students alike can rely on this consistent structure when assigning exercises and choosing a path of study.

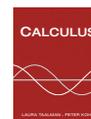
Flexibility

I recognize that instructors use calculus books in many different ways and that the real direction of a calculus course comes from the instructor, not any book. The streamlined, consistent structure of this book makes it easy to use with a wide variety of courses and pedagogical styles. In particular, instructors will find it easy to include or omit sections, proofs, examples, and exercises consistently according to their preferences and course requirements. Students can focus on mathematical development or on examples and calculations as they need to throughout the course. Later, they can use the book as a reliable reference.

It will be immediately clear to anyone opening either Taalman/Kohn *Calculus* or *Calculus I with Integrated Precalculus* that these books are substantially different from the other calculus books on the market today while still following the standard topics taught in most modern science, mathematics, and engineering calculus courses. My hope is that faculty who use either book will find it flexible for different pedagogical approaches and that students will be able to read it on different levels as they learn to understand the beauty of calculus. In addition, I hope that instructors find *Calculus I with Integrated Precalculus* to be a simple, streamlined way to address the needs of students who require additional help with algebra and precalculus in their first college calculus course.

Comparison of *Calculus I with Integrated Precalculus* with Taalman/Kohn *Calculus*

Calculus I with Integrated Precalculus includes all of the material in Chapters 0–6 of Taalman/Kohn *Calculus*, but in a different order and with supplementary precalculus and algebra material.



- ▶ Chapters 0–3 of *Calculus I with Integrated Precalculus* cover the same development of differential calculus topics as Taalman/Kohn *Calculus* but the more complicated calculational examples are deferred to later chapters.
- ▶ Chapters 4–6 of *Calculus I with Integrated Precalculus* revisit differential calculus through the lens of studying progressively more challenging types of functions. Any exercises or examples from Taalman/Kohn *Calculus* that were left out of Chapters 0–3 of *Calculus I with Integrated Precalculus* are included in Chapters 4–6. The requisite background precalculus and algebra material is built from the ground up.
- ▶ Chapters 7–9 of *Calculus I with Integrated Precalculus* are identical to Chapters 4–6 of Taalman/Kohn *Calculus* and cover all topics from integral calculus.

Instructors using *Calculus I with Integrated Precalculus* for a two-semester course in which they wish to cover all of integration might find it convenient to cover Chapters 0–4 in the first semester and Chapters 5–9 in the second semester. For *Calculus I with Integrated Precalculus* courses that do not need to cover as much integration, I suggest covering Chapters 0–3 in the first semester and Chapters 4–7 in the second semester. It is also possible to use *Calculus I with Integrated Precalculus* in an extended-hours one-semester course.

Students who learn Calculus I from *Calculus I with Integrated Precalculus* can continue with Calculus II using Taalman/Kohn *Calculus* or any other calculus textbook.

For an examination copy of Taalman/Kohn *Calculus*, please contact your local W. H. Freeman & Company representative.

For Instructors

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For Students

Student Solutions Manual

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Contains worked-out solutions to all odd-numbered exercises in the text.

Software Manuals

Maple™ and Mathematica® software manuals are available within CalcPortal. Printed versions of these manuals are available through custom publishing. They serve as basic introductions to popular mathematical software options and guides for their use with *Calculus I with Integrated Precalculus*.

Book Companion Web Site at www.whfreeman.com/integcalculus

For students, this site serves as a FREE 24–7 electronic study guide, and it includes such features as self-quizzes and interactive applets.

Online Homework Options

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W. H. Freeman offers approximately 2,500 algorithmically generated questions (with full solutions) through this free, open-source online homework system at the University of Rochester. Adopters also have access to a shared national library test bank with thousands of additional questions, including 1,500 problem sets matched to the book's table of contents.

Additional Media

SolutionMaster

This easy-to-use Web-based version of the Instructor's Solutions Manual allows instructors to generate a solution file for any set of homework exercises. Solutions can be downloaded in PDF format for convenient printing and posting.

[Interactive e-Book at ebooks.bfwpub.com/integralcalculus](http://ebooks.bfwpub.com/integralcalculus)

The Interactive e-Book integrates a complete and customizable online version of the text with its media resources. Students can quickly search the text, and they can personalize the e-Book just as they would the print version, with highlighting, bookmarking, and note-taking features. Instructors can add, hide, and reorder content, integrate their own material, and highlight key text.

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i-clicker

This two-way radio frequency classroom response solution was developed by educators for educators. University of Illinois physicists Tim Stelzer, Gary Gladding, Mats Selen, and Benny Brown created the i-clicker system after using competing classroom responses and discovering that they were neither appropriate for the classroom nor friendly to the student. Each step of i-clicker's development has been informed by teaching and learning. i-clicker is superior to other systems from both a pedagogical and a technical standpoint. To learn more about packaging i-clicker with this textbook, contact your local sales representative or visit www.iclicker.com.

Each section opens with a **list of the three main section topics**. The list provides a focus and highlights key concepts.

3.3 THE SECOND DERIVATIVE AND CURVE SKETCHING

- ▶ Using first and second derivatives to define and detect concavity
- ▶ The behavior of the first and second derivatives at inflection points
- ▶ Using the second-derivative test to determine whether critical points are maxima, minima, or neither

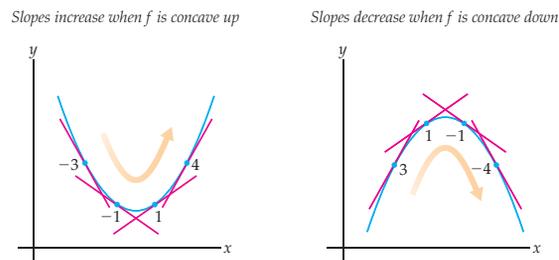
Definitions are clearly boxed, numbered, and labeled for easy reference. To reinforce their importance and meaning, definitions are followed by brief, often illustrated, examples.

DEFINITION 3.9 Formally Defining Concavity

Suppose f and f' are both differentiable on an interval I .

- (a) f is **concave up** on I if f' is increasing on I .
- (b) f is **concave down** on I if f' is decreasing on I .

How does this formal definition of concavity correspond with our intuitive notion of concavity? Consider the functions graphed next. On each graph four slopes are illustrated and estimated. Notice that when f is concave up, its slopes increase from left to right, and when f is concave down, its slopes decrease from left to right.



Theorems are developed intuitively before they are stated formally, and simple examples inform the discussion. **Proofs** follow most theorems, although they are optional, given instructor preference.

THEOREM 3.4 Rolle's Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b) , and if $f(a) = f(b) = 0$, then there exists at least one value $c \in (a, b)$ for which $f'(c) = 0$.

Actually, Rolle's Theorem also holds in the more general case where $f(a)$ and $f(b)$ are equal to each other (not necessarily both zero). For example, Rolle's Theorem is also true if $f(a) = f(b) = 5$, or if $f(a) = f(b) = -3$, and so on, because vertically shifting a function by adding a constant term does not change its derivative. However, the classic way to state Rolle's Theorem is with $f(a)$ and $f(b)$ both equal to zero.

Proof. Rolle's Theorem is an immediate consequence of the Extreme Value Theorem from Section 1.4 and the fact that every extremum is a critical point. Suppose f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , with $f(a) = f(b) = 0$. By the Extreme Value Theorem, we know that f attains both a maximum and a minimum value on $[a, b]$. If one of these extreme values occurs at a point $x = c$ in the interior (a, b) of the interval, then $x = c$ is a local extremum of f . By the previous theorem, this means that $x = c$ is a critical point of f . Since f is assumed to be differentiable at $x = c$, it follows that $f'(c) = 0$ and we are done.

It remains to consider the special case where all of the maximum and minimum values of f on $[a, b]$ occur at the endpoints of the interval (i.e., at $x = a$ or at $x = b$). In this case, since $f(a) = f(b) = 0$, the maximum and minimum values of $f(x)$ must both equal zero. For all x in $[a, b]$ we would have $0 \leq f(x) \leq 0$, which means that f would have to be the constant function $f(x) = 0$ on $[a, b]$. Since the derivative of a constant function is always zero, in this special case we have $f'(x) = 0$ for all values of c in (a, b) , and we are done. ■

Cautions are appropriately placed at points in the exposition where students typically have questions about the nuances of mathematical thinking, processes, and notation.

CAUTION It is important to note that although we use the notation x^{-1} to denote the reciprocal $\frac{1}{x}$, the notation f^{-1} does *not* stand for the reciprocal $\frac{1}{f}$ of f . The notation f^{-1} used in Definition 0.10 is pronounced “ f inverse.” We are now using the same notation for two very different things, but it should be clear from the context which one we mean.

Every section includes short, illustrative examples as part of the discussion and development of the material. Once the groundwork has been laid, more complex **examples** and calculations are provided. Students find this approach easier to handle because the difficult calculations do not interfere with the development of why things work. Example **solutions** are explained in detail and include all the steps necessary for student comprehension.

EXAMPLE 4 Using critical points and Rolle’s Theorem to find local extrema

The function $f(x) = x(x-1)(x-3)$ is a cubic polynomial with one local maximum and one local minimum. Use Rolle’s Theorem to identify intervals on which these extrema exist. Then use derivatives to find the exact locations of these extrema.

SOLUTION

The roots of $f(x) = x(x-1)(x-3)$ are $x = 0$, $x = 1$, and $x = 3$. Since f is a polynomial, it is continuous and differentiable everywhere. Therefore Rolle’s Theorem applies on the intervals $[0, 1]$ and $[1, 3]$, and it tells us that at least one critical point must exist inside each of these intervals.

The critical points of f are the possible locations of the local extrema that we seek. To find the critical points we must solve the equation $f'(x) = 0$. It is simpler to do some algebra before differentiating:

$$f'(x) = \frac{d}{dx}(x(x-1)(x-3)) = \frac{d}{dx}(x^3 - 4x^2 + 3x) = 3x^2 - 8x + 3.$$

By the quadratic formula, we have $f'(x) = 0$ at the points

$$x = \frac{-(-8) \pm \sqrt{8^2 - 4(3)(3)}}{2(3)} = \frac{8 \pm \sqrt{28}}{6} = \frac{4 \pm \sqrt{7}}{3}.$$

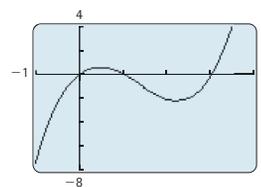
These x -values are approximately $x \approx 0.451$ and $x \approx 2.215$. If we look at the graph of f , then we can see that the smaller of these two x -values is the location of the local maximum and the larger is the location of the local minimum; see the figure that follows. \square

Following many example solutions, **Checking the Answer** encourages students to learn to check their work, using technology such as a graphing calculator when appropriate.

CHECKING THE ANSWER

The graph of $f(x) = x(x-1)(x-3)$ is shown next. Notice that the local extrema do seem to occur at the values we just found.

Extrema at $x \approx 0.451$ and $x \approx 2.215$



Each section closes with five **Test Your Understanding** questions that test students on the concepts and reading presented in the section. Because answers are not provided, instructors may choose to use these questions for discussion or assessment.

? TEST YOUR UNDERSTANDING

- ▶ Why could we not give a precise mathematical definition of concavity before this section?
- ▶ The domain points $x = c$ where $f''(c) = 0$ or where $f''(c)$ does not exist are the critical points of the function f' . Why?
- ▶ Why is it not clear to say a sentence such as “Because it is positive, it is concave up”? How could this information be conveyed more precisely?
- ▶ Why does it make sense that f' is increasing when f'' is positive?
- ▶ Suppose $x = c$ is a critical point with $f'(c) = 0$. Why does it make graphical sense that f has a local minimum at $x = c$ when f is concave up in a neighborhood around $x = c$?

Section Exercises are provided in a consistent format that offers the same types of exercises within each section. This approach allows instructors to tailor assignments to their course, goals, and student audience.

Thinking Back exercises ask students to review relevant concepts from previous sections and lessons.

Concepts exercises are consistently formatted to start with the following three problems:

- Problem 0 tests understanding.
- Problem 1 consists of eight true/false questions.
- Problem 2 asks the student to create examples based on their understanding of the reading.

Skills exercises offer ample practice, grouped into varying degrees of difficulty.

Applications exercises contain at least two in-depth real-world problems.

Proofs exercises can be completed by students in non-theoretical courses. Hints are often provided, and many exercises mimic work presented in the reading and examples. Often, these exercises are a continuation of a proof offered as a road map in the narrative.

Thinking Forward exercises plant seeds of concepts to come. In conjunction with the Thinking Back exercises, they tie together past and future topics, thereby providing a seamless flow of concepts.

Chapter Review, Self-Test, and Capstones, found at the end of each chapter, present the following categories:

Definitions exercises prompt students to recall definitions and give an illustrative example.

Theorems exercises ask students to complete fill-in-the-blank theorem statements.

Formulas, Notation, and/or Rules ask students to show a working understanding of important formulas, equations, notation, and rules.

Skill Certification exercises provide practice with basic computations from the chapter.

Capstone Problems pull together the essential ideas of the chapter in more challenging mathematical and application problems.

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Chris Brazfield, now at Carroll Community College, helped with the initial development of the text and the ideas behind it. Kevin Cooper of Washington State University contributed many interesting and challenging real-world applications. Laura Person provided valuable class-testing feedback that helped improve the text. I owe all of them great thanks for their expertise.

I also owe thanks to all of the people at W. H. Freeman who helped with the development of this text. Developmental editor Leslie Lahr has been with this project from the beginning. Even under pressure, Leslie always maintains a positive attitude and finds

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Finally, I would like to thank all of my students who showed determination and courage while tackling the monumental task of learning calculus, precalculus, and algebra all at the same time. I am so proud of all of you.

Learning something new can be both exciting and daunting. To gain a full understanding of the material in this text, you will have to read, you will have to think about the connections between the new topics and the topics that were previously presented, and you will have to work problems—many, many problems.

The structure of this text should help you understand the material. The material is laid out in a linear fashion that I think will facilitate your understanding. Each section is separated into two main parts: first, a presentation of new material and then second, a set of *Examples and Explorations*, where you will find problems that are carefully worked through. Working through these examples on your own, as you read the steps for guidance, will help prepare you for the exercises.

Reading a mathematics book isn't like reading a novel: You may have to read some parts more than once, and you may need to make notes or work things out on paper. Pay special attention to the "Checking Your Answer" features, so that you can learn how to check your own answers to many types of questions.

To succeed in calculus, you need to do homework exercises. The exercises in every section of this text are broken into six categories: "Thinking Back," "Concepts," "Skills," "Applications," "Proofs," and "Thinking Forward."

- As the title suggests, the *Thinking Back* problems are intended to tie the current material to material you've seen in previous sections or even previous courses.
- The *Concepts* problems are designed to help you understand the main ideas of the section, which are presented without a lot of calculation. Every group of *Concepts* exercises begins by asking you to summarize the section, continues with eight true–false questions, and then asks for three examples illustrating ideas from the section.
- The bulk of the exercises in each section consists of *Skills* problems that may require more calculation.
- The *Applications* exercises use the concepts from the section in "real-world" problems.
- The *Proofs* exercises ask you to prove some basic theory from the section.
- Finally, the *Thinking Forward* questions use current ideas to introduce topics that you will see in subsequent sections.

I hope this structure allows you to tie together the material as you work through the book. I have supplied the answers to the odd-numbered exercises, but don't restrict yourself to those problems. You can check answers to even-numbered questions by hand or by using a calculator or an online tool such as wolframalpha.com. After all, on a quiz or test you won't have the answers, so you'll have to know how to decide for yourself whether or not your answers are reasonable.

Some students may like to work through each section "backwards," starting by attempting the exercises, then checking back to the examples as needed when they get stuck, and, finally, using the exposition as a reference when they want to see the big picture. That is fine; although I recommend that you at least try reading through the sections in order to see how things work for you. Either way, I hope that the separation of examples from exposition and the division of homework problems into subsections will help make the process of learning this beautiful subject easier. I have written this text with you, the student, in mind. I hope you enjoy using it!