Appendix E

Notes

About the Geogebra® Lessons

Geogebra® is software that can be found at https://www.geogebra.org/home; it allows the drawing and manipulation of precise shapes. Prose can be added to a drawing, and both drawing and prose can be copied and pasted to word-processing software documents. The software can be a valuable resource for classroom demonstrations and the preparation of documents with geometry shapes.

We include a few optional online video lessons on how to use Geogebra, which serve as an introduction to this powerful software program and perhaps a motivation to learn more about it. The lessons, developed by Dr. Janet Bowers, allow explorations of various shapes and the properties of those shapes. They can be related to various sections of the chapters where these shapes and properties are learned. The lessons will not touch on all of Geogebra's features but should give you an idea of its capabilities. With replays of the lessons, you can learn how to make several kinds of drawings. You do not need the software to view the lessons, but you do need it to practice on your own. The lessons are ADA-compliant and can be found at http://crmse.sdsu.edu/nickerson.

1. Special Triangles
2. Special Quadrilaterals
3. Sum of Interior Angles
4. Transformation Geometry

Three of these units contain the following features:

- **a.** Interactive applets with reflection questions and answers
- **b.** Video tutorial showing you how to create the sketch using Geogebra

Note: Geogebra is a free, online tool that can be downloaded to a laptop or used online to create and share interactive applets. For more information, see https://www.geogebra.org/home.

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Appendix F

A Review of Some Rules

Courses for preservice elementary school teachers usually assume competence with whole-number, fraction, and decimal arithmetic (addition, subtraction, multiplication, and division) and a previous exposure to elementary algebra. The courses themselves often focus on *why* the particular calculational procedures work rather than how on to do them.

Experience shows that some students, however, are quite rusty with some of the *rules* (and there are a lot of them!). If you are such a student, these pages offer a quick review of a few topics, without any explanation of why the rules give correct answers, or even what addition, subtraction, multiplication, and division mean. You should not use a calculator for any calculation, to assure yourself that your basic facts and techniques with whole numbers are still in good working order. Here are the areas reviewed; sample from them as you need, or as your instructor suggests:

1. **Fractions (including mixed numbers)**
2. **Decimals**
3. **Fraction, decimal, and percent conversions**
4. **Solving a proportion**
5. **Whole-number and negative exponents**
6. **Properties of operations**
7. **Order of operations**
8. **Signed number arithmetic**

Final answers are given at the end.

F.1 Fractions (Including Mixed Numbers)

Equal (or Equivalent) Fractions

RULE: Multiply or divide the numerator and denominator by the same number (not zero) to get an equal fraction. $\left(\text{Terms: } \dfrac{\text{numerator}}{\text{denominator}}\right)$

EXAMPLES: $\dfrac{2}{3} = \dfrac{2 \cdot 7}{3 \cdot 7} = \dfrac{14}{21}$ $\quad \dfrac{60}{72} = \dfrac{60 \div 2}{72 \div 2} = \dfrac{30}{36} = \dfrac{30 \div 3}{36 \div 3} = \dfrac{10}{12} = \dfrac{5}{6}$

$\dfrac{4\frac{1}{2}}{7} = \dfrac{4\frac{1}{2} \times 2}{7 \times 2} = \dfrac{9}{14}$ $\quad \dfrac{17}{40} = \dfrac{2\frac{1}{2} \times 17}{2\frac{1}{2} \times 40} = \dfrac{42\frac{1}{2}}{100}$

PRACTICE:

1.1. Simplify as much as possible.

a. $\dfrac{126}{35}$ **b.** $\dfrac{96}{100}$ **c.** $\dfrac{42}{56}$ **d.** $\dfrac{196}{240}$ **e.** $\dfrac{168}{64}$ **f.** $\dfrac{72}{216}$ **g.** $\dfrac{7\frac{1}{3}}{10}$

h. $\dfrac{248}{120}$ **i.** $\dfrac{28}{36}$ **j.** $\dfrac{5.2}{10}$ **k.** $\dfrac{588}{1000}$ **l.** $\dfrac{\frac{3}{4}}{9}$ **m.** $\dfrac{7\frac{1}{2}}{100}$ **n.** $\dfrac{384}{512}$

1.2. Write a fraction equal to the given fraction, but with the designated numerator or denominator. (This skill is needed for adding or subtracting fractions.)

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EXAMPLE: Write a fraction equal to $\frac{5}{16}$, but with the denominator 96. To get a denominator of 96, one must multiply by 6 (from 96 ÷ 16, thinking "What times 16 will give 96?"). So multiply the numerator and denominator by 6: $\frac{5 \times 6}{16 \times 6} = \frac{30}{96}$.

a. $\frac{9}{10}$, with denominator 70 **b.** $\frac{2}{3}$, with denominator 36

c. $\frac{8}{15}$, with denominator 180 **d.** $\frac{9}{11}$, with numerator 99

e. $\frac{3}{8}$, with numerator 27 **f.** $\frac{84}{96}$, with denominator 56 (*Hint:* Simplify first.)

1.3. In (a) and (b), write ten fractions equal to the given fraction.

a. $\frac{3}{12}$ **b.** $\frac{8}{7}$

1.4. Do any of your fractions in 1.3 have a smaller or greater value than the original fraction in each case?

Rewriting Fractions Greater Than One as Mixed Numbers and Vice Versa

RULE: To change a fraction greater than 1 to a mixed (or whole) number, divide the numerator by the denominator. If the fraction can be simplified first, the division involves smaller numbers.

EXAMPLES: $\frac{13}{5} = 13 \div 5 = 2\frac{3}{5}$ $\frac{100}{12} = 100 \div 12 = 8\frac{1}{3}$ $\frac{368}{23} = 368 \div 23 = 16$

PRACTICE:

1.5. Write each as a mixed (or whole) number.

a. $\frac{493}{72}$ **b.** $\frac{15}{8}$ **c.** $\frac{52}{16}$ **d.** $\frac{1000}{15}$

e. $\frac{2400}{128}$ **f.** $\frac{96}{9}$ **g.** $\frac{360}{28}$ **h.** $\frac{1010}{12}$

RULE: To change a mixed number to a fraction, multiply the denominator by the whole number, add the product to the numerator, and write the sum over the denominator. A whole number can be written as a fraction in many ways.

EXAMPLES: $4\frac{2}{3} = \frac{4 \cdot 3 + 2}{3} = \frac{14}{3}$ $7\frac{5}{8} = \frac{7 \cdot 8 + 5}{8} = \frac{61}{8}$

$$9 = \frac{9}{1} = \frac{18}{2} = \frac{27}{3} = \frac{144}{16} = \ldots$$

PRACTICE:

1.6. Write each as a fraction.

a. $7\frac{1}{2}$ **b.** $19\frac{1}{3}$ **c.** $52\frac{7}{8}$ **d.** 17 **e.** $11\frac{9}{10}$

Adding or Subtracting Fractions and Mixed Numbers

RULE: If the fractions have the same denominator, then add/subtract the numerators, writing that answer over the original denominator. If the fractions do not have the same denominator, then replace them with equal fractions that do have a common denominator and proceed as in the first sentence. The common denominator need not be the *least* common denominator, but a least common denominator keeps the numbers smaller. It is customary to simplify the answer.

If a mixed number is involved, there are two ways to proceed. The first way is to change each mixed number to a fraction first. The second way is to deal with the fraction parts and

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the whole number parts separately, perhaps renaming the first mixed number if necessary to do the fraction subtraction.

EXAMPLES: $\frac{5}{14} + \frac{3}{14} - \frac{2}{14} = \frac{5+3-2}{14} = \frac{6}{14} = \frac{3}{7}$

$$\frac{2}{3} + \frac{5}{6} - \frac{7}{12} = \frac{48}{72} + \frac{60}{72} - \frac{42}{72} = \frac{48+60-42}{72} = \frac{66}{72} = \frac{11}{12}$$

One way, with mixed numbers:

$$12\tfrac{1}{4} - 6\tfrac{7}{8} = \frac{49}{4} - \frac{55}{8} = \frac{98}{8} - \frac{55}{8}$$
$$= \frac{98-55}{8} = \frac{43}{8} = 5\tfrac{3}{8}$$

A second way, with mixed numbers:

$$\begin{array}{r} 12\tfrac{1}{4} = 11\tfrac{5}{4} = 11\tfrac{10}{8} \\ -6\tfrac{7}{8} = -6\tfrac{7}{8} = -6\tfrac{7}{8} \\ \hline 5\tfrac{3}{8} \end{array}$$

PRACTICE:

1.7. Give the answers in simplest form (if the answer is a fraction greater than 1, give the mixed number).

a. $\frac{9}{16} - \frac{5}{12} + \frac{17}{24}$ **b.** $\frac{2}{3} + 6\tfrac{7}{8} + 2\tfrac{1}{2} - \frac{5}{4}$ **c.** $\frac{19}{36} + 2\tfrac{2}{3} - 1\tfrac{17}{18}$

d. $120\tfrac{1}{5} - 3\tfrac{7}{10} + 2\tfrac{1}{10}$ **e.** $600 - 60\tfrac{2}{7}$ **f.** $3\tfrac{1}{3} + 1\tfrac{5}{6} - \frac{3}{4}$

Multiplying Fractions (and Mixed Numbers)

RULE: To multiply two fractions, write the product (i.e., the result of multiplication) of the numerators over the product of the denominators. It is customary to simplify the answer. Mixed numbers should be changed to fractions before multiplying.

EXAMPLES: $\frac{2}{3} \times \frac{7}{8} = \frac{2 \times 7}{3 \times 8} = \frac{14}{24} = \frac{7}{12}$ $\quad 2\tfrac{1}{2} \times 3\tfrac{9}{10} = \frac{5}{2} \times \frac{39}{10} = \frac{195}{20} = \frac{39}{4} = 9\tfrac{3}{4}$

$$\frac{2}{3} \times 24 \times \frac{15}{16} = \left(\frac{2}{3} \times \frac{24}{1}\right) \times \frac{15}{16} = \frac{48}{3} \times \frac{15}{16} = \frac{16}{1} \times \frac{15}{16} = \frac{16 \times 15}{1 \times 16} = 15$$

PRACTICE:

1.8. Find the products (multiply).

a. $\frac{3}{4} \times \frac{7}{8}$ **b.** $\frac{5}{8} \times 120$ **c.** $\frac{2}{3} \times \frac{5}{2} \times \frac{9}{8}$

d. $3\tfrac{1}{3} \times 92$ **e.** $5\tfrac{1}{12} \times 7\tfrac{1}{5}$ **f.** $\frac{11}{12} \times \frac{2}{3} \times \frac{4}{5} \times \frac{10}{11}$

Dividing Fractions

RULE: To divide by a fraction, invert the fraction and multiply. If mixed numbers are involved, change them to fractions first.

EXAMPLES: $7 \div \frac{2}{3} = \frac{7}{1} \times \frac{3}{2} = \frac{21}{2} = 10\tfrac{1}{2}$ $\quad \frac{9}{10} \div \frac{3}{5} = \frac{9}{10} \times \frac{5}{3} = \frac{45}{30} = 1\tfrac{1}{2}$

$$4\tfrac{1}{4} \div 2\tfrac{1}{2} = \frac{17}{4} \div \frac{5}{2} = \frac{17}{4} \times \frac{2}{5} = \frac{34}{20} = 1\tfrac{7}{10}$$

PRACTICE:

1.9. Find the quotients (divide).

a. $\frac{3}{4} \div \frac{1}{2}$ **b.** $\frac{1}{2} \div \frac{3}{4}$ **c.** $\frac{21}{32} \div \frac{1}{8}$ **d.** $21\tfrac{1}{4} \div 3$ **e.** $11 \div \frac{2}{3}$ **f.** $\frac{49}{1000} \div 3\tfrac{1}{2}$

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F.2 Decimals

Equal Decimals

RULE: Annexing zeros to the last digit to the right of the decimal point gives an equal decimal. Removing zeros on the right end of a decimal gives an equal decimal.

EXAMPLES: $0.4 = 0.40 = 0.400000$ $\quad 2.073 = 2.07300$ $\quad 4 = 4.0$ $\quad 8.750 = 8.75$

$0.08 = 0.080$ $\quad 73.200 = 73.2$ $\quad 19.00 = 19$

PRACTICE:

2.1. Write two decimals that are equal to each given number.

a. 435.06 **b.** 1.4 **c.** 927.0400 **d.** 17 **e.** 0.680

Adding and Subtracting Decimals

RULE: Write in vertical form, with the decimal points aligned. Then add or subtract as though they were whole numbers, aligning the decimal point in the answer with the other decimal points.

EXAMPLES: $34.2 - 7.6 \rightarrow \begin{array}{r} 34.2 \\ \underline{-7.6} \\ 26.6 \end{array}$ $\qquad 200 - 63.08 \rightarrow \begin{array}{r} 200.00 \\ \underline{-63.08} \\ 136.92 \end{array}$

PRACTICE: (Notice that you can make up exercises, do them, and then check with a calculator.)

2.2. Calculate by hand.

a. $0.05 + 1.9$ **b.** $175.3 - 11.94$ **c.** $68.3 + 4 + 19.84 - 72.756$

Multiplying Decimals

RULE: To multiply two decimals, multiply as though they were whole numbers, count the total number of decimal places in the numbers being multiplied (the factors), and then place the decimal point that many places from the right end of the answer.

EXAMPLES: $0.2 \times 0.49 \rightarrow 2 \times 49 = 98$, 3 total decimal places in 0.2 and 0.49 $\rightarrow 0.098$

$1.52 \times 0.075 \rightarrow 152 \times 75 = 11{,}400$, 5 total decimal places $\rightarrow 0.11400 = 0.114$

PRACTICE: (Notice that you can make up exercises, do them, and then check with a calculator; keep in mind that calculators usually do not show unnecessary zeros.)

2.3. Find the products.

a. 0.2×0.3 **b.** 4.8×75 **c.** 12.39×0.14 **d.** 19.88×4.23

Dividing Decimals

Terms and notation: Dividend ÷ divisor = quotient; in working form,

$$\text{divisor}\overline{)\text{dividend}}^{\;\text{quotient}}$$

RULE: To divide two decimals, move the decimal point in the divisor to make the divisor a whole number, and move the decimal point the same number of places in the dividend (you may have to annex 0's). Do the division as though the numbers were whole numbers, keeping digits carefully aligned, and put the decimal point in the answer (the quotient) right above its new location in the dividend.

EXAMPLES: $11.2 \div 0.28 \rightarrow 1120 \div 28 = 40$ $\quad$ $11.256 \div 0.28 \rightarrow 1125.6 \div 28 = 40.2$
$336.4 \div 2.32 \rightarrow 33640 \div 232 = 145$
$0.69336 \div 9.63 \rightarrow 69.336 \div 963 = 0.072$
$100 \div 6.3 \rightarrow 1000 \div 63 =$ approximately 15.873016

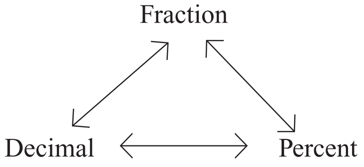
PRACTICE: (Again notice that you can make up exercises, do them, and then check with a calculator.)

2.4. Find the quotients. If there does not seem to be an exact answer, give the quotient to six decimal places.

a. $120 \div 2.5$ **b.** $36.344 \div 15.4$ **c.** $3.6344 \div 15.4$ **d.** $3.782 \div 0.0775$
e. $14 \div 200$ **f.** $27 \div 0.04$ **g.** $0.008 \div 0.2$

F.3 Fraction, Decimal, and Percent Conversions

You likely know that $\frac{1}{4} = 0.25 = 25\%$. These three forms—fraction, decimal, and percent—are "dialects" for the same number. Viewed as dialects that require "translation," there are six translations, as indicated in the drawing below. You should be able to start with any form and translate into the other two forms.



Fraction to Decimal

RULE: Divide the numerator by the denominator. Annex a decimal point and zeros as needed. Many times the quotient (the answer) is not exact, but the last decimal place given is possibly rounded, depending on the value in the next place.

EXAMPLES: $\frac{7}{16} = 7 \div 16 = \ldots = 0.4375$

$\frac{9}{11} = 9 \div 11 = \ldots = 0.8181818181\ldots$ (*forever*)

PRACTICE:

3.1. Write each fraction as a decimal. If the decimal appears to go on forever, give eight decimal places.

a. $\frac{3}{8}$ **b.** $\frac{1}{16}$ **c.** $\frac{3}{16}$ **d.** $\frac{1}{7}$ **e.** $\frac{2}{7}$ **f.** $\frac{1}{9}$ **g.** $\frac{7}{9}$ **h.** $\frac{1}{12}$
i. $\frac{1}{6}$ **j.** $\frac{12}{7}$ **k.** $\frac{1}{25}$ **l.** $\frac{27}{25}$ **m.** $\frac{22}{7}$ **n.** $\frac{1}{13}$

Decimal to Fraction

RULE: Write a fraction that has as its numerator the same digits as the decimal but no decimal point, and a denominator that suggests the smallest place value in the decimal (e.g., if the smallest place value is hundred*th*s, write 100 in the denominator). The place values, going to the right from the decimal point, are tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, and so on. Many times the resulting fraction can be simplified.

EXAMPLES: $1.35 = \frac{135}{100}$ (The smallest place value is hundredths, so use denominator 100.)
And, if desired, $\frac{135}{100} = \frac{27}{20} = 1\frac{7}{20}$.

$0.7421 = \frac{7421}{10{,}000}$ $\quad$ $2.1674932 = \frac{21{,}674{,}932}{10{,}000{,}000}$ (Can be simplified.)

PRACTICE:

3.2. Write a fraction and then simplify it as much as possible.

a. 3.75 **b.** 0.092 **c.** 0.0004 **d.** 4.68

Decimal to Percent

RULE: Move the decimal point two places to the right and put the % sign on the end. You may have to annex a zero or two to the original number. You do not usually write the decimal point in the percent expression unless there are more digits to the right.

EXAMPLES: $1.07 = 107\%$ $\quad$ $0.2 = 0.20 = 20\%$ $\quad$ $3 = 3.00 = 300\%$ $\quad$ $1.5 = 150\%$
$0.0067 = 0.67\%$

PRACTICE:

3.3. Write each as a percent.

a. 4.25 **b.** 3.146 **c.** 1 **d.** 0.62 **e.** 0.045 **f.** 0.00003

Percent to Decimal

RULE: Move the decimal point two places to the left and remove the % sign. You may have to insert one or more zeros.

EXAMPLES: $34\% = 34.\% = 0.34$ $\quad$ $110\% = 110.\% = 1.1$ $\quad$ $7.75\% = 0.0775$
$0.12\% = 0.0012$

PRACTICE:

3.4. Write each percent as a decimal.

a. 88% **b.** 33.3% **c.** 105% **d.** 1.5% **e.** 500% **f.** 5.5%

Fraction to Percent

RULE: Change the fraction to a decimal, as earlier, and then change the decimal to a percent, as above.

EXAMPLES: $\frac{5}{6} = 0.8333\ldots = 83.333\ldots\%$ $\quad$ $\frac{33}{40} = 0.825 = 82.5\%$
$\frac{15}{8} = 1.875 = 187.5\%$

PRACTICE:

3.5. Write each fraction as a percent.

a. $\frac{8}{5}$ **b.** $\frac{5}{8}$ **c.** $\frac{17}{20}$ **d.** $\frac{952}{1140}$ **e.** $\frac{113}{125}$ **f.** $\frac{140}{32}$

Percent to Fraction

RULE: Write the percent as a decimal, as earlier, and then write the decimal as a fraction, as above. The fraction often can be simplified.

EXAMPLES: $32.5\% = 0.325 = \frac{325}{1000}\left(= \frac{13}{40}\right)$ $\quad$ $0.72\% = 0.0072 = \frac{72}{10{,}000}\left(= \frac{9}{1250}\right)$

PRACTICE:

3.6. Write each percent as a fraction. Simplify the fraction if you need practice.

a. 40% **b.** $66\frac{2}{3}\%$ **c.** 165% **d.** 11.5% **e.** 0.25%

MIXED PRACTICE:

3.7. Write each of the given fractions, decimals, or percents in the other two forms.

a. $\frac{9}{10}$ **b.** 2.3 **c.** 56% **d.** $\frac{18}{15}$ **e.** 0.8 **f.** 1.58%

F.4 Solving a proportion

A proportion is an equation like $\frac{24}{36} = \frac{6}{9}$ (occasionally written as 24:36 = 6:9). Often there is an unknown value to find in a proportion, as in $\frac{x}{36} = \frac{6}{9}$. We will review two ways to find the missing value. The first approach involves noticing multiplication or division (but not addition or subtraction) relationships among the values. Look at the following examples. Notice that one arrow ends at the unknown and suggests the relationship to look for between the other two numbers.

EXAMPLES:

$\times\frac{2}{3}\ \left(\frac{6}{9} = \frac{x}{36}\right)$ so $\times\frac{2}{3}$

$x = \frac{2}{3} \times 36 = 24$

so $\times 4$

$\frac{x}{36} = \frac{6}{9}$

$\times 4$

$x = 6 \times 4 = 24$

$\times 5$

$\frac{8}{47} = \frac{40}{y}$

so $\times 5$

$y = 47 \times 5 = 235$

$\div 2$

$\frac{50}{88} = \frac{25}{n}$

so $\div 2$

$n = 44$

Often there is no obvious relationship among the numbers. But we have a second approach. A rule follows from this observation: In $\frac{24}{36} = \frac{6}{9}$, notice that $24 \times 9 = 216$ and $36 \times 6 = 216$, so $24 \times 9 = 36 \times 6$. In $\frac{24}{36} = \frac{6}{9}$, if you draw a line from the 24 to the 9 and a line from the 36 to the 6, you make an X-shaped "cross." This "cross-multiplying" gives $24 \times 9 = 36 \times 6$, and leads to this rule for solving a proportion.

RULE: To solve a proportion, cross-multiply and solve the resulting equation. The unknown value can be in any position in the proportion.

EXAMPLE: Solve $\frac{18}{45} = \frac{28}{x}$. Cross-multiply to get $18x = 45 \times 28$, or $18x = 1260$. Then divide both sides of the equation by 18 to get $x = 70$.

PRACTICE:

4.1. Solve each proportion. Use your knowledge of multiplication or division relationships when you can [as in parts (a–c)]. Cross-multiply to solve when you do not see any relationships.

a. $\frac{48}{100} = \frac{x}{25}$ **b.** $\frac{x}{120} = \frac{27}{40}$ **c.** $\frac{x}{150} = \frac{16}{75}$ **d.** $\frac{15}{38} = \frac{9}{x}$ **e.** $\frac{25.2}{x} = \frac{18}{35}$

F.5 Whole-Number and Negative Exponents

Exponents are used a lot in mathematics because they provide an excellent shorthand for a repeated multiplication. For example, expressions like x^3 for $x \cdot x \cdot x$ or 2^5 for $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ obviously save time. When there is no exponent visible, it is understood to be 1, if needed. There are several rules for working with exponents, and the rules lead to definitions for exponents that are 0 or negative and do not fit the repeated multiplication idea.

Further definitions: $x^0 = 1$, for any value of x different from 0.

$x^{-n} = \frac{1}{x^n}$, for n any whole number and any x not 0.

Think of the exponents in the following rules as whole numbers.

RULES: **a.** $x^m x^n = x^{m+n}$ **EXAMPLES:** $x^3 x^4 = x^{3+4} = x^7$; $10^3 \cdot 10^2 = 10^5$

b. $(x^m)^n = x^{mn}$ **EXAMPLES:** $(x^4)^5 = x^{4 \cdot 5} = x^{20}$; $(10^2)^3 = 10^6$

c. $(xy)^n = x^n y^n$ **EXAMPLE:** $(3y)^2 = 3^2 y^2 = 9y^2$

d. $\dfrac{x^m}{x^n} = x^{m-n}$ **EXAMPLES:** $\dfrac{y^8}{y^2} = y^{8-2} = y^6$; $\dfrac{10^5}{10^3} = 10^{5-3} = 10^2$

PRACTICE:

5.1. Use the rules to simplify the following. The 10 could be any number, so you can make up variations easily.

a. $10^6 \cdot 10^6$ **b.** $(10^6)^6$ **c.** $10^2 \cdot 10^3$ **d.** $(10^2)^3$ **e.** $\dfrac{10^{12}}{10^3}$

f. $b^4 \cdot b^3$ **g.** $(b^4)^3$ **h.** $b^{-2}b^3$ **i.** $(b^{-2})^3$ **j.** $\dfrac{b^6}{b^3}$

k. $\dfrac{(3x)^5}{3x^2}$ **l.** $\dfrac{(2xy)^3}{(3y)^2}$

m. $(1.4 \times 10^4) \cdot (3 \times 10^{-3})$ **n.** $(2 \times 10^5) \cdot (3.2 \times 10^3)$

F.6 Properties of Operations

Mathematicians have identified several patterns of the operations (addition and multiplication, in particular) as being fundamental. These patterns are usually called properties (rather than rules) in mathematics. As an example, the fact that 568×437 and 437×568 give the same answer illustrates what is called the *commutative property of multiplication,* or sometimes just *commutativity of multiplication.* So we would know that $568 \times 437 = 437 \times 568$, even without doing the multiplication and finding that each equals 248,216. The properties are useful in many calculations. Here is a list of most of the properties that appear in the elementary school curriculum. These properties apply to any kind of numbers.

- ***Properties of addition***

  Commutativity of addition: $a + b = b + a$ for any choice of numbers a and b.
  Associativity of addition: $a + (b + c) = (a + b) + c$ for any numbers a, b, and c.
  Zero is the identity for addition: $0 + a = a$ and $a + 0 = a$ for any number a.
  Existence of additive inverses: For each number a, there is another number, denoted ${}^-a$, such that $a + {}^-a = 0$ and ${}^-a + a = 0$. Each of a and ${}^-a$ is the additive inverse of the other. For example, 7 and ${}^-7$ are additive inverses of each other, because $7 + {}^-7 = 0$.

- ***Properties of multiplication***

  Commutativity of multiplication: $a \times b = b \times a$ for any choice of numbers a and b.
  Associativity of multiplication: $a \times (b \times c) = (a \times b) \times c$ for any numbers a, b, and c.
  One (1) is the identity for multiplication: $1 \times a = a$ and $a \times 1 = a$ for any number a.
  Existence of multiplicative inverses: For each nonzero number a, there is another number b such that $a \times b = 1$ and $b \times a = 1$. For example, 6 and $\frac{1}{6}$ are multiplicative inverses of each other, because $6 \times \dfrac{1}{6} = 1$. Similarly, $\dfrac{5}{6}$ and $\dfrac{6}{5}$ are multiplicative inverses of each other, because $\dfrac{5}{6} \times \dfrac{6}{5} = 1$; $\dfrac{5}{6}$ and $\dfrac{6}{5}$ are often called reciprocals of each other.

- ***A property involving both addition and multiplication***

  Distributivity of multiplication over addition: $a \times (b + c) = (a \times b) + (a \times c)$ for any numbers a, b, and c. The form $(b + c) \times a = (b \times a) + (c \times a)$ is also useful. (This property is sometimes abbreviated as "distributivity," even though there are other distributive properties.)

EXAMPLES:

$8 + (92 + 49)$ can be calculated by $(8 + 92) + 49$ (associativity of addition).

$6\frac{4}{5} \times \frac{1}{2}$ and $\frac{1}{2} \times 6\frac{4}{5}$ have the same answer (commutativity of multiplication).

$25 \times (4 \times 72.7)$ can be calculated by $(25 \times 4) \times 72.7$ (associativity of multiplication).

$158 + {}^{-}158 = 0$ (additive inverses)

$\left(29 \times \frac{7}{8}\right) \times \frac{8}{7} = 29 \times \left(\frac{7}{8} \times \frac{8}{7}\right) = 29 \times 1 = 29$ (first, associativity of multiplication, then multiplicative inverses, and finally, 1 is the identity for multiplication)

$\frac{1}{2} \times 6\frac{4}{5}$ can be calculated by $\left(\frac{1}{2} \times 6\right) + \left(\frac{1}{2} \times \frac{4}{5}\right)$ (distributivity of multiplication over addition, because $6\frac{4}{5} = 6 + \frac{4}{5}$). $(7 + 40) + 3 = (40 + 7) + 3 = 40 + (7 + 3)$ (first commutativity, and then associativity, of addition)

$59.132 + 0 = 59.132$ (zero is the identity for addition)

PRACTICE:

6.1. In each part, apply the given property to the given expression and then compute.

a. associativity of addition, $97 + (3 + 228)$

b. associativity of multiplication, $7 \times (3.1 \times 10^5)$

c. commutativity of addition, $97 + (3 + 228)$

d. commutativity of multiplication, 9.2×10^3

e. distributivity of multiplication over addition, $6 \times (10 + 4)$

f. distributivity of multiplication over addition, $(75 \times 13) + (25 \times 13)$

g. commutativity of multiplication, $9\frac{6}{7} \times \frac{1}{3}$

h. distributivity of multiplication over addition, $(8 \times 991) + (8 \times 9)$

i. associativity of addition, $\left(\frac{27}{28} + \frac{3}{4}\right) + \frac{1}{4}$

j. identity for *multiplication,* $\left(1 \times \frac{8}{9}\right) + 0$

6.2. In each part, tell which property has been applied.

a. $(2 \times 87) \times 25 = (87 \times 2) \times 25$ (Caution.)

b. $(87 \times 2) \times 25 = 87 \times (2 \times 25)$

c. $40 + (30 + 6) = (40 + 30) + 6$

d. $100\% \times 39.98 = 39.98$

e. $(10 + 3) \times 21 = (10 \times 21) + (3 \times 21)$

f. $(10 + 3) \times (20 + 1) = (10 \times (20 + 1)) + (3 \times (20 + 1))$

g. $10 \times (20 + 1) = (10 \times 20) + (10 \times 1)$

h. $\frac{1}{2} \times (200 \times 68.374) = \left(\frac{1}{2} \times 200\right) \times 68.374$

i. $\frac{3}{5} \times 1\frac{2}{3} = \frac{3}{5} \times \frac{5}{3} = 1$

j. ${}^{-}\left(\frac{11}{16}\right) + 0 = {}^{-}\left(\frac{11}{16}\right)$

F.7 Order of Operations

To avoid lots of parentheses, there is a commonly accepted convention on how to calculate something like $3 + 4 \times 5$. (Parentheses are allowed, of course, for complete clarity.) $3 + 4 \times 5 = 23$, not 35, by the convention.

RULE: First do the work inside any grouping symbols, like parentheses or the terms in a fraction, then attend to any exponents. Next do the multiplications and divisions as you encounter them, from left to right. Finally, do the additions and subtractions as you encounter them, from left to right. (Mnemonic: Please Excuse My Dear Aunt Sally)

EXAMPLE: Evaluate $7 + 4 \times 600 \div 5 - 6 \times (2 + 3)^2$. Following the rule, and doing just one step at a time in this example, we get

$$\begin{aligned} &7 + 4 \times 600 \div 5 - 6 \times 5^2 \\ &= 7 + 4 \times 600 \div 5 - 6 \times 25 \\ &= 7 + 2400 \div 5 - 6 \times 25 \\ &= 7 + 480 - 150 = 487 - 150 = 337 \end{aligned}$$

EXAMPLE: Evaluate $5x^2 - 3(x - 4)^2$ when $x = 6$. If we substitute and follow the rule, we get

$$\begin{aligned} &5 \times 6^2 - 3 \times (6 - 4)^2 \\ &= 5 \times 6^2 - 3 \times 2^2 \\ &= 5 \times 36 - 3 \times 4 \\ &= 180 - 12 = 168 \end{aligned}$$

PRACTICE:

7.1. Evaluate each of the following:

a. $3 \times 10^2 + 5 \times 10 + 2$ **b.** $417 - 2(10 - 3)^2$ **c.** $\frac{5}{9} \times (212 - 32)$

d. $\frac{3 + 15}{6}$ **e.** $\frac{1}{2} + \frac{1}{2}(5 - 2)^2$

7.2. Evaluate the expressions.

a. $2x^3 - 7x - (10 - 2)$, when $x = 3$ **b.** $25x - (x + 2)(x - 3)^2$, when $x = 5$

c. $\frac{16 + 3x}{2}$, when $x = 3$ **d.** $\frac{2x^2 + 18}{9}$, when $x = 6$

F.8 Signed Number Arithmetic

Unless you have recently reviewed the rules for adding, subtracting, multiplying, and dividing signed numbers, your memory of these rules may have faded (especially if you have not done any recent or technical work involving negative numbers). Recall that positive numbers are often written without any sign: $^+8.3$ can be interpreted as 8.3.

RULES for ***addition*** (two cases):

a. Both numbers have the *same sign*—add the numbers without regard to their sign, and give the sum the same sign as the original numbers: $^+10 + {}^+5 = {}^+15$, $^-6 + {}^-2 = {}^-8$.

b. The two numbers have *different signs*—ignore the signs and find the difference in the numbers; give the answer the sign of the larger number without regard to sign.

EXAMPLES: **a.** $^+10 + {}^+5 = {}^+15$ $^-6 + {}^-2 = {}^-8$, $5 + {}^+3 = 8$

b. $^+8 + {}^-3 = {}^+(8 - 3) = {}^+5$ $^-7 + {}^+4 = {}^-(7 - 4) = {}^-3$

RULE for ***subtraction***: Change the sign of the number being subtracted, and add.

EXAMPLES: $6 - {}^-2 = 6 + 2 = 8$ $^-7 - 4 = {}^-7 + {}^-4 = {}^-11$

$^-6 - {}^-4 = {}^-6 + 4 = {}^-2$ $^-4 - {}^-6 = {}^-4 + 6 = 2$

The subtraction rule also works when the two numbers are positive, but it is sometimes more complicated than necessary: $12 - {}^+9 = 12 + {}^-9 = {}^+(12 - 9) = {}^+3$, but you know already that $12 - 9 = 3$.

RULES for ***multiplication*** and ***division*** (two cases):

a. Both numbers have the *same sign*—multiply or divide as usual; the answer will be positive.

b. The two numbers have *different signs*—multiply or divide as usual; the answer will be negative.

EXAMPLES: **a.** $^-8 \times {}^-2 = 16$ $\quad ^-8 \div {}^-2 = 4$ $\quad (^-3)^2 = {}^-3 \times {}^-3 = {}^+9$

b. $5 \times {}^-3 = {}^-15$ $\quad ^-4 \times 3 = {}^-12$ $\quad 16 \div {}^-2 = {}^-8$ $\quad ^-12 \div 2 = {}^-6$

PRACTICE:

8.1. Find the sums (add).

a. $4 + {}^-2$ **b.** $2 + {}^-4$ **c.** $517 + {}^-517$ **d.** $^-3 + {}^-9$

e. $^-50 + 10 + {}^-20$ **f.** $3\frac{1}{2} + {}^-1\frac{1}{2}$ **g.** $^-6.8 + {}^-3.2$ **h.** $^-5 + 1\frac{1}{4}$

8.2. Find the differences (subtract).

a. $4 - {}^-2$ **b.** $2 - {}^-4$ **c.** $^-3 - {}^-9$ **d.** $^-50 - 10 - {}^-20$

e. $^-6.8 - 3.2$ **f.** $^-5 - 1\frac{1}{3}$ **g.** $^-9.6 - {}^-2.4$ **h.** $15 - {}^-6\frac{1}{2}$

8.3. Find the products (multiply).

a. $6 \times {}^-3$ **b.** $^-14 \times 2$ **c.** $^-3 \times {}^-10$ **d.** $^-2 \times {}^-2 \times {}^-3$

e. $(^-5)^2$ **f.** $^-3(^-1)^2$ **g.** $(^-2)^4$ **h.** $^-2^4$ (Caution.)

8.4. Find the quotients (divide).

a. $24 \div {}^-3$ **b.** $^-18 \div 6$ **c.** $^-12 \div 4$ **d.** $^-12 \div {}^-4$

e. $10 \div {}^-20$ **f.** $\frac{^-16}{8}$ **g.** $\frac{20}{^-2}$ **h.** $\frac{^-15}{^-5}$

Answers for Appendix F

F.1 Fractions (Including Mixed Numbers)

1.1 a. $\frac{18}{5}$ **b.** $\frac{24}{25}$ **c.** $\frac{3}{4}$ **d.** $\frac{49}{60}$ **e.** $\frac{21}{8}$ **f.** $\frac{1}{3}$ **g.** $\frac{11}{15}$ **h.** $\frac{31}{15}$ **i.** $\frac{7}{9}$

j. $\frac{13}{25}$ **k.** $\frac{147}{250}$ **l.** $\frac{3}{36} = \frac{1}{12}$ **m.** $\frac{3}{40}$ **n.** $\frac{3}{4}$

1.2 a. $\frac{63}{70}$ **b.** $\frac{24}{36}$ **c.** $\frac{96}{180}$ **d.** $\frac{99}{121}$ **e.** $\frac{27}{72}$ **f.** $\frac{49}{56}$

1.3. a. $\frac{3}{12} = \frac{3 \times 2}{12 \times 2} = \frac{6}{24}$; *or* $\frac{3}{12} = \frac{3 \times 3}{12 \times 3} = \frac{9}{36}$; *similarly* $= \frac{12}{48} = \frac{15}{60} = \ldots$;
also $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$. There are many other possibilities.

b. *Similarly,* $\frac{8}{7} = \frac{16}{14} = \frac{24}{21} = \frac{32}{28} = \ldots. = \frac{800}{700}$

1.4. None should be greater than, or less than, the given fraction. This question is to make certain that you realize that your answers are indeed *equal* to the given fraction (and to each other).

1.5. a. $6\frac{61}{72}$ **b.** $1\frac{7}{8}$ **c.** $3\frac{1}{4}$ (It is customary to simplify the fraction.)

d. $66\frac{2}{3}$ **e.** $18\frac{3}{4}$ **f.** $10\frac{2}{3}$ **g.** $12\frac{6}{7}$ **h.** $84\frac{1}{6}$

1.6. a. $\frac{15}{2}$ **b.** $\frac{58}{3}$ **c.** $\frac{423}{8}$ **d.** $\frac{17}{1}$ **e.** $\frac{119}{10}$

1.7. a. $\frac{41}{48}$ **b.** $8\frac{19}{24}$ **c.** $1\frac{1}{4}$ **d.** $118\frac{3}{5}$ **e.** $539\frac{5}{7}$ **f.** $4\frac{5}{12}$

1.8. a. $\frac{21}{32}$ **b.** 75 **c.** $\frac{45}{24} = 1\frac{7}{8}$ **d.** $306\frac{2}{3}$ **e.** $36\frac{3}{5}$ **f.** $\frac{4}{9}$

1.9. a. $1\frac{1}{2}$ **b.** $\frac{2}{3}$ **c.** $5\frac{1}{4}$ **d.** $7\frac{1}{12}$ **e.** $16\frac{1}{2}$ **f.** $\frac{7}{500}$

F.2 Decimals

2.1. a. For example, 435.060 and 435.0600 **b.** For example, 1.40 and 1.4000
c. For example, 927.04 and 927.040 and 927.04000 **d.** For example, 17.00 and 17.000
e. For example, 0.68 and 0.6800

2.2. a. 1.95 **b.** 163.36 **c.** 19.384

2.3. a. 0.06 **b.** 360 **c.** 1.7346 **d.** 84.0924

2.4. a. 48 **b.** 2.36 **c.** 0.236 **d.** 48.8 **e.** 0.07 **f.** 675 **g.** 0.04

F.3 Fraction, Decimal, and Percent Conversions

3.1. Some of the answers here are given to more than eight decimal places.

a. 0.375 (The initial zero is often written to alert one so that the decimal point is not overlooked.)

b. 0.0625 **c.** 0.1875 **d.** 0.142857142857142857. . . (forever)

e. 0.285714285714285714. . . (forever) **f.** 0.111111. . . (forever) **g.** 0.777. . . (forever)

h. 0.0833333. . . (forever) **i.** 0.166666. . . (forever) **j.** 1.714285714285. . . **k.** 0.04

l. 1.08 **m.** 3.142857142857. . . (forever) **n.** 0.076923076923. . .

3.2 a. $\frac{15}{4} = 3\frac{3}{4}$ **b.** $\frac{92}{1000} = \frac{23}{250}$ **c.** $\frac{4}{10{,}000} = \frac{1}{2500}$ **d.** $\frac{468}{100} = \frac{117}{25} = 4\frac{17}{25}$

3.3. a. 425% **b.** 314.6% **c.** 100% **d.** 62% **e.** 4.5% **f.** 0.003%

3.4. a. 0.88 **b.** 0.333 **c.** 1.05 **d.** 0.015 **e.** 5 **f.** 0.055

3.5. a. $1.6 = 160\%$ **b.** $0.625 = 62.5\%$ **c.** $0.85 = 85\%$

d. $\frac{952}{1140} \approx 0.8350877 = 83.50877\%$ **e.** $0.904 = 90.4\%$ **f.** $4.375 = 437.5\%$

3.6. a. $0.40 = \cdots = \frac{2}{5}$ **b.** $0.66\frac{2}{3} = \frac{66\frac{2}{3}}{100} = \cdots = \frac{2}{3}$ **c.** $1.65 = \frac{165}{100} = \frac{33}{20} = 1\frac{13}{20}$

d. $0.115 = \frac{115}{1000} = \frac{23}{200}$ **e.** $0.0025 = \frac{25}{10{,}000} = \frac{1}{400}$

3.7. a. 0.9, 90% **b.** $\frac{23}{10} = 2\frac{3}{10}$, 230% **c.** 0.56, $\frac{56}{100} = \frac{14}{25}$

d. 1.2, 120% **e.** 80%, $\frac{8}{10} = \frac{4}{5}$ **f.** 0.0158, $\frac{158}{10{,}000} = \frac{79}{5000}$

F.4 Solving a Proportion

4.1. a. $25 = 100 \div 4$, so $x = 48 \div 4$. $x = 12$. The equation from cross-multiplying is $48 \times 25 = 100x$, or $1200 = 100x$, which gives $x = 12$ by dividing both sides of the equation by 100.

b. $x = 81$ **c.** $x = 32$ **d.** $x = 22\frac{4}{5}$ **e.** $x = 49$

F.5 Whole-Number and Negative Exponents

5.1. a. 10^{12} (not 100^{12}) **b.** 10^{36} **c.** 10^5 **d.** 10^6 **e.** 10^9

f. b^7 **g.** b^{12} **h.** b^1 or just b **i.** b^{-6} **j.** b^3

k. 3^4x^3 or $81x^3$ **l.** $\frac{8x^3y}{9}$ **m.** 4.2×10^1 (scientific notation), or 42

n. 6.5×10^8 (scientific notation), or 640000000

F.6 Properties of Operations

6.1. a. $(97 + 3) + 228 = 100 + 228 = 328$

b. $(7 \times 3.1) \times 10^5 = 21.7 \times 10^5 = 21.7 \times 100{,}000 = 2{,}170{,}000$

c. $97 + (228 + 3)$ or $(3 + 228) + 97$ or even $(228 + 3) + 97$. Applying commutativity here does not help in computing, in contrast to using associativity as in part (a).

d. $10^3 \times 9.2 = 9200$

e. $(6 \times 10) + (6 \times 4) = 60 + 24 = 84$

f. $(75 + 25) \times 13 = 100 \times 13 = 1300$

g. $\frac{1}{3} \times 9\frac{6}{7} = \left(\frac{1}{3} \times 9\right) + \left(\frac{1}{3} \times \frac{6}{7}\right) = 3 + \frac{2}{7} = 3\frac{2}{7}$

h. $8 \times (991 + 9) = 8 \times 1000 = 8000$

i. $\frac{27}{28} + \left(\frac{3}{4} + \frac{1}{4}\right) = \frac{27}{28} + \frac{4}{4} = \frac{27}{28} + 1 = 1\frac{27}{28}$

j. $\frac{8}{9} + 0$, which then, with the additive identity, gives $\frac{8}{9}$

6.2 **a.** Commutativity of multiplication **b.** Associativity of multiplication

c. Associativity of addition **d.** 1 is the identity for multiplication

e. Distributivity of multiplication over addition

f. Distributivity of multiplication over addition (the 20 + 1 is like a single number)

g. Distributivity of multiplication over addition

h. Associativity of multiplication **i.** Multiplicative inverses

j. 0 is the additive identity

F.7 Order of Operations

7.1. **a.** 352 **b.** 319 **c.** 100 **d.** 3 **e.** 5

7.2. **a.** 25 **b.** 97 **c.** $12\frac{1}{2}$ **d.** 10

F.8 Signed Number Arithmetic

8.1. **a.** 2 **b.** $^{-}2$ **c.** 0 **d.** $^{-}12$ **e.** $^{-}60$ **f.** 2 **g.** $^{-}10$ **h.** $^{-}3\frac{3}{4}$

8.2. **a.** 6 **b.** 6 **c.** 6 **d.** $^{-}40$ **e.** $^{-}10$ **f.** $^{-}6\frac{1}{3}$ **g.** $^{-}7.2$ **h.** $21\frac{1}{2}$

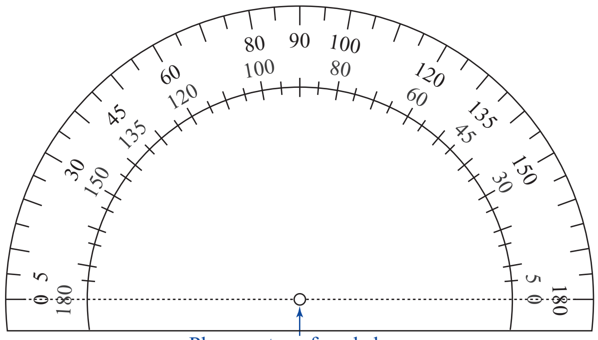
8.3. **a.** $^{-}18$ **b.** $^{-}28$ **c.** 30 **d.** $^{-}12$ **e.** 25 **f.** $^{-}3$ **g.** 16 **h.** $^{-}16$

8.4. **a.** $^{-}8$ **b.** $^{-}3$ **c.** $^{-}3$ **d.** 3 **e.** $^{-}\frac{1}{2}$ **f.** $^{-}2$ **g.** $^{-}10$ **h.** 3

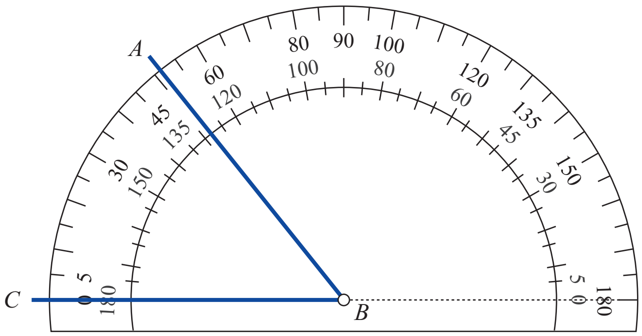
Appendix G

Using a Protractor to Measure Angle Size

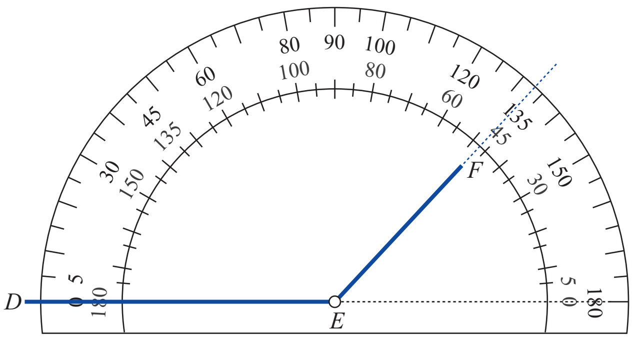
One tool for measuring angle size is called the **protractor.** Different kinds of protractors are available, but a common type of protractor is partially illustrated below. One complication is that this protractor has two scales going from 0 to 180, so you must be attentive to which scale you are using. In this drawing, the scale marks are 5° apart and only a few are labeled; on an actual protractor, the marks may be 1° or even $\frac{1}{2}°$ apart. Notice also that an important location, marked here by a point (but sometimes by an arrowhead), is halfway between the two 0 marks. The vertex of the angle to be measured *must* be at this point.



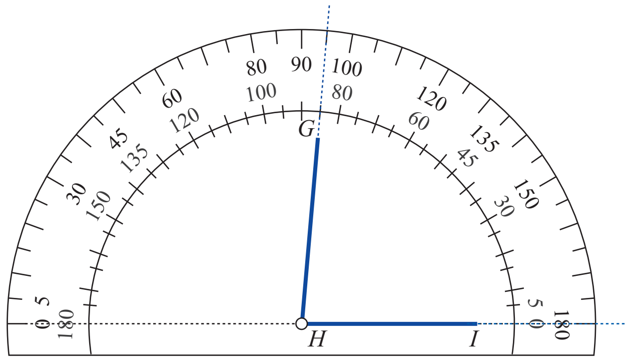
To measure an angle, say, angle ABC as shown below, place the protractor so that the vertex of the angle, B for this angle, is at the special point. Align the protractor so that one side (side CB here) of the angle goes through a 0 point on one of the scales. The other side of the angle gives the size of the angle, which is about 52° here. Be sure to read the measure on the same scale containing your 0 point.



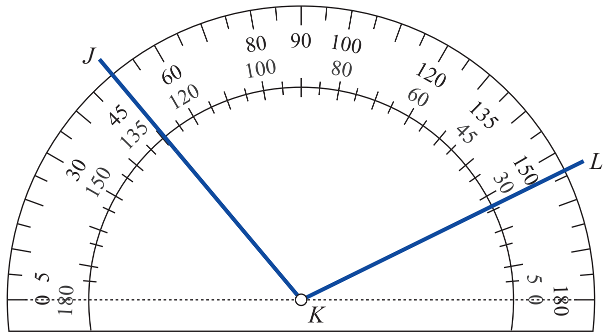
Angle DEF has a size of about 133°. Notice that side EF had to be extended so that the scale marking could be read. Such extensions are common, especially when using a large protractor.



The inner scale is also useful, as in finding that angle *GHI* has a size of about 85°. Notice that both sides of the angle had to be extended to align one side with 0 and then to read the angle size.



If you are careful to use the same scale for both readings, you can subtract scale readings to find an angle size, as in finding the size of angle *JKL* shown below. (Notice that the vertex of the angle still has to be at the special point.) Using the outer scale, the angle size is about $154 - 50 = 104°$; using the inner scale instead, the angle size is about $130 - 26 = 104°$.



Appendix H

Using the TI-73

Whether or not you use the TI-73 calculator will depend on availability and what type of technology your instructor decides to use in your class. This book includes several appendices to help you learn different forms of technology, and you most often need to learn to use only one.

The lessons here are highly procedural. They simply tell you how to use the TI-73 calculator to obtain statistical information on data sets you have. To better understand these procedures would take a great deal of time. The lessons given should be sufficient to generalize to different data sets, and this is all you need to be able to do for now. *You must use them exactly as given.* The nine lessons need not be done all at once, or in order. The following lessons are included here:

1. **Getting started**
2. **Generating five random three-digit numbers**
3. **Other simple simulations can be run with the TI-73: Try these**
4. **Making lists (such as those used in statistics to generate different types of displays)**
5. **Making a histogram**
6. **Making a box plot using the same data on GPAs and gender data**
7. **Finding the mean and standard deviation for GPA data**
8. **Making a scatter plot**
9. **Finding the regression line and the correlation coefficient**

Usually, **bold** type refers to buttons (keys) and type like Helvetica refers to material on the calculator screen.

Getting started

1. Study the keyboard to see what is available. Notice that several buttons have something printed in yellow just above them. These buttons have dual functions. To use the buttons to access the yellow print above the button, press **2nd** (the only yellow button) and then the button.
2. The arithmetic operation buttons are red, as is the **ENTER** button (*very* frequently used) at the bottom right. There are also red arrow keys to enable moving around on the screen. Just below the screen are five red buttons; four have **2nd** functions.
3. To turn on the TI-73, use the **ON** button. To turn off, press the yellow **2nd** button, then the **ON** button.
4. To adjust the display, press and release the **2nd** button, and then use the **red up and down arrows.**
5. To clear out anything reserved in memory, go through this reset process: Press **ON,** press **2nd** (which gives you access to the functions described in yellow over the regular buttons), then **MEM** (the 0 button), then **7** (or scroll down to 7), and then **1** (for ALL RAM). You will see a RESET window. Press **2.** You should receive a message that RAM is cleared. THIS CLEARS OUT ANY DATA SETS IN THE CALCULATOR. DO NOT PERFORM THIS STEP IF YOU HAVE DATA YOU WANT TO SAVE.

Generating five random three-digit numbers

(Clear memory if necessary. See item 5 in Lesson 1.)

1. Click on **MATH** (just below **2nd**). Across the top you will see MATH NUM PRB LOG. Use the red arrows to go to PRB.
2. Use a red cursor to go down to **2:** randInt(.
3. Press **ENTER.**
4. Using the comma button and the) button, complete the line to read randInt(100,999). The 100,999 gives random numbers between 100 and 999—that is, any three-digit number. If you want numbers of a different sort, say, just one-digit numbers, then fill in with (0,9). Notice that the comma and open and close parentheses have their own special buttons.
5. Press **ENTER,** and a random 3-digit number will appear. Continue pressing **ENTER** until you have 5 numbers.

This process can easily be adapted to obtain random numbers of any size you wish.

Other simple simulations can be run with the TI-73: Try these

(Clear memory if necessary. See item 5 in Lesson 1.)

1. Simulate the tossing of 4 coins, where 0 is Heads and 1 is Tails,

    Follow the steps above but this time instead of stopping at randInt(go to coin(. If you want to throw 4 coins, complete the line to say coin(4) and hit **ENTER.**

   Assume that 0 is Heads and 1 is Tails. Keep a tally on paper: Number of Heads.

   Thus, if the calculator has (0 1 0 0), put 3 for Number of Heads.
2. Continue hitting **ENTER** until you have the number of trials you want. Count those with 3 heads, 2 heads, 1 head, 0 heads, and predict how many times, after 100 trials, you will have 3 heads (or whatever outcome you are interested in).
3. Simulate the tossing of 2 dice 4 times. This time scroll to dice(and press **ENTER** (or press **7**). Finish the line as dice(4,2), then press **ENTER.** You will see 4 numbers, each representing the sum of the two dice on a roll. You can once again keep a tally of the results and use this tally to make predictions.

Making lists (such as those used in statistics to generate different types of displays)

In this and the next exercise, you will make and use lists from the *60 Students* data set that can be found in your data sets (Appendix D). Specifically, we will be using the study time and GPA lists in this example. Below are the GPAs and frequencies for students who studied fewer than 25 hours per week (F) and students who studied 25 hours or more (M) per week. (You could also have found these numbers from the *60 Students* file.)

GPA	F	M	GPA	F	M	GPA	F	M	GPA	F	M
0.8	1	0	1.7	0	0	2.6	3	2	3.5	0	4
0.9	0	0	1.8	1	0	2.7	1	2	3.6	1	2
1.0	0	0	1.9	1	2	2.8	1	1	3.7	0	0
1.1	0	0	2.0	1	1	2.9	2	4	3.8	0	1
1.2	0	0	2.1	1	1	3.0	1	0	3.9	3	0
1.3	0	0	2.2	2	0	3.1	4	2	4.0	1	1
1.4	0	0	2.3	1	0	3.2	3	2			
1.5	0	0	2.4	2	1	3.3	0	0			
1.6	0	0	2.5	1	2	3.4	1	0			

1. Go through the Getting Started routine in Lesson 1.
2. Click on the **LIST** button. You will see several columns. The first 6 columns are named L1 − L6. Suppose we want to name our columns so that we know which column is which. Use the right arrow to reach . . . .7. Across the bottom of the screen, you will see NAME=.

   Naming a list: To name this column, we will use text, which can be done by clicking on **2nd,** then on **TEXT** (which is the **MATH** button just below, but now is text because of clicking on **2nd**). Use the red arrows to spell GPA by finding G, clicking **ENTER,** finding P, clicking **ENTER,** finding A, and clicking **ENTER.** Then use the down arrow to DONE and click **ENTER.** You will again see the lists. Click on **ENTER** again to place the name at the top of the column.
3. ***Entering data:*** Use the down arrow to the first entry place. Click on **LIST** again. Type in 0.8 (because this is the lowest GPA we have). This number will appear at the bottom. Click **ENTER** and the number will show in the list. Type in **0.9, ENTER, 1.0, ENTER,** and so on through all 33 GPAs.

   If you make an error: Continue to the next entry, then use the arrow to get back to the error, type in the replacement number, and **ENTER.** Use **CLEAR** or **2nd** and then **QUIT** to begin over. Or, if you get a message ERR: DATA TYPE), use the **down arrow** to 2:GoTo and click on **ENTER** to get you back to where you were.
4. Make two more columns this way. You might call them FFR for fewer than 25 hours per week frequencies and MFR for 25 hours or more per week frequencies. Be sure to place 0 beside any GPAs that are 0 in the table above. You should have the same number of entries in each column; if you do not have the same number, you've made an entry error that needs to be corrected.

Making a histogram

1. Turn off any stat plots that are on the calculator this way: Click on **LIST,** then **2nd,** then **PLOT** (above the red **Y=** button just below the screen on the left), then **4,** and you will see a message PlotsOff. Click on **ENTER** and you will see DONE.
2. To view the stats plot menu, again click on **2nd,** then **PLOT,** then **1,** and then **ENTER.**
3. To choose a histogram, use the down arrow to Type, then the right arrow until you come to the histogram symbol (right after the circle graph symbol), and press **ENTER.** Next we need to name the Xlist, which in this case will be GPA. Use the down arrow, then click again on **2nd** and on **STAT** (the **LIST** button), and scroll down to GPA and click **ENTER.** Do similarly for the "Freq" and in this case use just the FFR for fewer than 25 hours per week frequencies.
4. Display a histogram by clicking **ZOOM,** then **7,** and then **ENTER.** You will see a histogram. By clicking on **TRACE** (red button on top) and then the red arrows, you can find the min and max of the value on the horizontal axis (the GPA) of each bar.
5. But these may not be the min and max you want. So click on **WINDOW** (next to **ZOOM**). Suppose we want to group the GPAs as 0.5–0.9, 1–1.4, 1.5–2, and so on. For Xmin, type in **0.5.** Click the arrow down, and type in **4** for the Xmax. Go down two more lines to Xsc1, and type in **0.5** for the size of the interval. The Ymin and Ymax are the minimum and maximum frequencies once they are placed in the intervals. Obviously, the Ymin is **0.** Type this in. For the max of the frequencies, (Ymax) 12 seems like a safe bet for the max of the grouped data. Type in **12.**
6. We are ready to try again. Click on **GRAPH,** and you have your histogram. Click on **TRACE** to learn about each bar. Write down this information so that you can compare it to the next set of GPAs, for students who studied 25 or more hours per week.

REPEAT THIS PROCESS TO MAKE A HISTOGRAM FOR GPAs of students who studied 25 or more hours per week. Compare the information to that for GPAs of students who studied fewer than 25 hours per week.

Making a box plot using the same data on GPAs and study data

Repeat the same process as in Lesson 5 through Step 4, *but* this time, choose box plot rather than histogram. After pressing **ZOOM,** the **TRACE** will again provide relevant information.

You should be able to find all five relevant points.

The five-number summary is 0.8, 2.35, 2.85, 3.2, 4 for FFRs' (Fewer Frequencies') GPAs.

Finding the mean and standard deviation for GPA data

1. Clear your screen: **2nd,** then **QUIT** (above **MODE),** and then **CLEAR.**
2. Press **2nd** and then **STAT**; use the red arrow to move right to **MATH,** then press **3** for **MEAN.** A screen will appear that says only mean(.
3. We want the screen to look like this: mean(GPA). Click on **2nd,** then **STAT,** and then scroll down the name of lists until you come to GPA. Press **ENTER** and then press) (which is the close parentheses symbol).
4. Click on **ENTER** and you will see the mean. It is 2.4.
5. Use this procedure to find the standard deviation. It is 0.97. Notice the other numbers that can also be found from the **MATH** menu in step 2.

Making a scatter plot

We will once again use the three lists we have entered to make two scatter plots.

1. Clear the screen (**2nd, QUIT, CLEAR**). Turn off all stat plots that are on the calculator: Click on **2nd,** then **PLOT,** then **4,** and then **ENTER.**
2. Click on **2nd** and then **PLOT** to open the stats plots. Click on **1,** then **ENTER,** and then move down to Type; the first icon there is a scatter plot, so click **ENTER.** Move down to the Xlist, click on **2nd,** then **STAT,** and scroll down the list to GPA. **ENTER.** Move to Ylist, click on **2nd,** then **STAT,** scroll down the list to FFR, and press **ENTER.** Scroll to the last line for a mark to represent FFR, press **ENTER.** (The scatter plot for these data alone can be seen: **ZOOM, 7.**)
3. We now need a second set of data to plot. Click on **2nd,** then **PLOT** to open the stats plots. Click this time on **2,** then **ENTER,** and then move down to Type; the first icon there is a scatter plot, so click **ENTER.** Move down to the Xlist, click on **2nd,** then **STAT,** scroll down the list to GPA. **ENTER.** Move to Ylist, click on **2nd,** then **STAT,** scroll down the list to MFR, and press **ENTER.** Scroll to the last line for a mark to represent MFR, choose a new mark, and press **ENTER.**
4. Our scatter plot can next be seen. Press **ZOOM,** then **7.** You will see two different types of marks: one for the GPAs of students who study fewer than 25 hours per week and one for GPAs of students who study 25 or more hours per week.

Finding the regression line and the correlation coefficient

1. Clear out the memory, as in Lesson 1, step 5 (**2nd, MEM,** 7, 1, 2).
2. Enter two lists for L1 and L2 (see Lesson 4). Use the following lists unless you have your own data; remember that each list must have the same number of cases.

   L1: 1, 2, 3, 4, 4, 5, 6 and L2: 2, 1, 3, 3, 4, 6, 8

3. Set the decimal notation to two places to avoid long decimals. Press **MODE,** the down arrow, and the right arrow three times, so that it ends on 2. Press **ENTER.**

4. Arrange for a scatter plot, as in Lesson 8: Click on **2nd** and then **PLOT** to open the stats plots. Click on **1,** then **ENTER,** and then move down to Type; the first icon there is a scatter plot, so click **ENTER.** Move down to the Xlist, click on **2nd,** then **STAT,** scroll to L1. **ENTER.** Move to Ylist, click on **2nd,** then **STAT,** scroll down the list to L2, and press **ENTER.** Scroll to the last line for a mark to represent the points, and press **ENTER.**
5. Press **2nd** and then **STAT.** Right arrow to **CALC,** then **5,** then **2nd,** then **VARS,** then **2,** then **2,** and then **ENTER.** Your screen should say LinReg y = ax + b a = 1.27 b = $^{-}$.69. Thus, the (linear) regression equation (the line of best fit) is $y = 1.27x + {}^{-}0.69$. To find the correlation coefficient, press **2nd,** then **VARS,** and then **3.** Move with the right arrow across the screen to EQ. Press **5** to obtain *r;* the correlation coefficient. Press **ENTER.** You will see that *r* is 0.91. Pressing **ZOOM 7** gives the scatter plot and the graph of the regression line.

Appendix I

Using *Excel*

Lessons dealing with the following topics are included here:

1. **Getting started**
2. **Generating random numbers for a simulation or sampling**
3. **Making a circle graph (or pie chart in *Excel*'s terminology)**
4. **Making a bar graph**
5. **Finding quartiles, means, and standard deviations**
6. **Making a scatter plot**
7. **Adding a line of best fit, or regression line (called a trendline in *Excel*)**

Excel was created for business users rather than for instructional purposes, so we are introducing only its very basic techniques. If you are new to *Excel,* you might first skim through all of the following to get an overview of some of the features of *Excel.*

These instructions assume that you are familiar with using a mouse and with copying-and-pasting. The instructions are for use with Microsoft *Excel X* for Macintosh computers, but they should suffice for use with other versions.

Getting started

Open *Excel* by double-clicking on the *Excel* icon. If the data are already in a file, double-click on the file icon.

Entering your own data: (You may have to go to **File** and **New Workbook.**) In the menu bar at the top, click on **File** and drag down to **New Workbook.** You get a large array of boxes, called cells. The cells are identified by the column letter and row number, as in A1 for the cell that is highlighted when the new workbook is created. You can type directly into the highlighted cell, moving to another cell either with the mouse or with the Tab key.

For an example, here is how to enter the transportation data for the whole school from Section 30.1: 182 walk, 166 rode a bus, and 72 came by car. Type the word "walk" in the first cell (A1), press Tab (or use the mouse) to get to B1, and type 182 into B1. Continue with the other data on the next two lines, again using the A and B columns. Notice that the category is given first and then the number of children in that category.

Generating random numbers for a simulation or sampling

Open *Excel.* Click on an empty cell at the top of an empty column. Go to **Insert,** and down to **Function.** In the new screen, called **Paste Function,** go to the **Function** category and click on **All.** Then in **Function Name,** scroll to **RAND.** Click OK (and again, on another OK), and you will have a random number between 0 and 1 (including 0 but not 1) in the cell you selected.

Select your random number, and drag down the column to select other empty cells until you have as many cells as you want random numbers. (You will be filling the cells with other random numbers.) Go to **Edit,** then **Fill . . . Down.** The cells will fill with new random numbers. Ignore the 0 and decimal point. If you need four-digit numbers, consider just the first four digits after the decimal point for each of the numbers in your list.

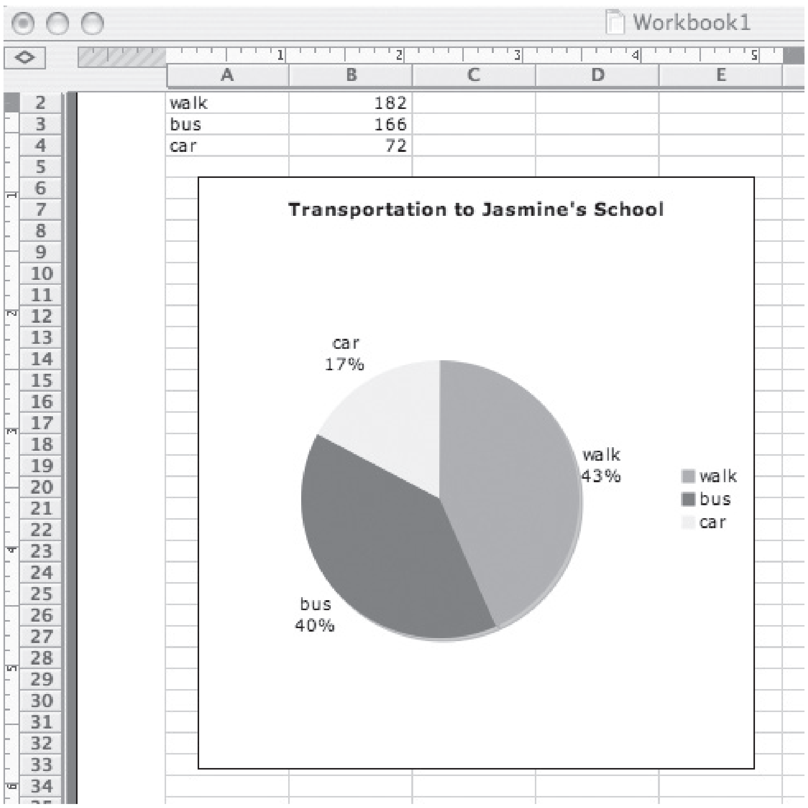
Alternatively, other approaches will generate a first random *whole* number in a selected cell. Type the following in the selected cell and press Return (notice the =):

=TRUNC(10*RAND(),0)	(Multiplying RAND by 10 gives a number from 0 up to 10. TRUNC means truncate, or cut off. The 0 says to have 0 decimal places. So this instruction gives a random number from 0 through 9.)
=TRUNC(100*RAND(),0)	(This gives a two-digit random number from 0 through 99. A one-digit answer like 8 should be interpreted as 08.)
=TRUNC(6*RAND(),)+1	(This gives a random number from 1 to 6, inclusive, as though you were simulating the toss of a die. Do you see how it works?)

In each case, you can select your random number and drag down that column to select other cells to put random numbers in. Once they are selected, use **Edit,** then **Fill** . . . **Down** to get a list of other random numbers. The list can be as long as you like.

Making a circle graph (or pie chart in *Excel*'s terminology)

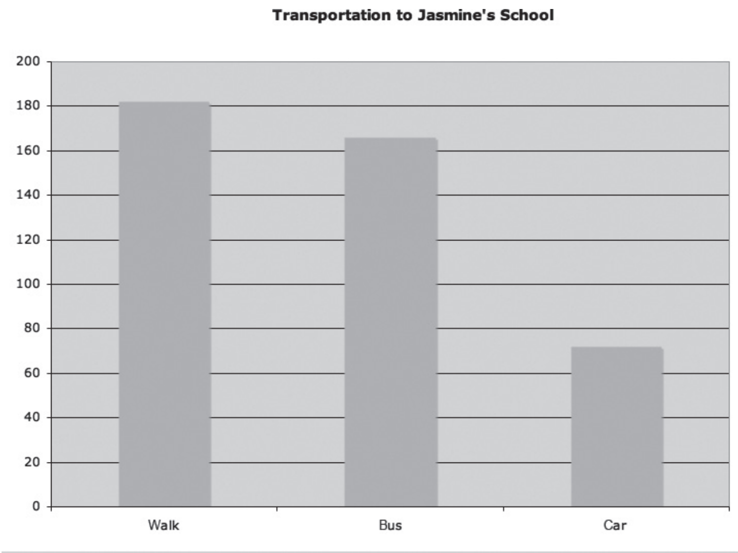
Continuing from Lesson 1 with the transportation data, click and drag to select all the data in the six cells. In the menu bar, click on **Insert** and drag down to **Chart.** Click on **Pie,** and then **Next.** (If you wish, you can experiment with the different versions of pie charts visible by clicking on one, and then clicking on the "Press and Hold . . ." button.) Click on **Next** in the next insert (our data set has the categories in one column and the counts in the next). Notice the tab labels that invite you to type in a title for the chart (Titles), where to place the (optional) legend (Legend), and how to label the regions (Data Labels). Make your choices. Any text with the graph at this point is probably too tiny to read, but it gives you



an idea of where things will appear in the final circle graph. Press **Next.** For most of our purposes, having the final graph appear on the sheet with the data is all right. (We can copy and paste the graph elsewhere, move it around on the sheet, or print the sheet.) Press **Finish.** After moving the graph to the left, the work to this point might look like that on the previous page. When the graph is selected (dark squares in the corners and midpoints), **Chart** appears in the menu bar and allows you to make changes.

Making a bar graph

We will use the same transportation data from Lesson 1 (or Lesson 3) to illustrate how to create a bar graph. Again, click and drag to select all the data. Click on **Insert** and then **Chart.** Bars in columns are the default choice, so click on **Next.** Again, the data typed in are arranged in columns, so just click on **Next.** As with the circle graph, the screen now shows several tabs with different items that can be changed. Try them. Once you are finished, double-click on **Next** and then **Finish.** By double-clicking on the background area in the graph, you can change the background, for example. With our choices, the final bar graph looks like the one below, copied, pasted, and reduced in size (in *Excel* the graph is selected when you click on it so that dark squares appear at the corners and midpoints).



With supplementary software (Analysis ToolPak), *Excel* can also give histograms. The steps involved, however, are somewhat involved and not likely to be used by children.

Finding quartiles, means, and standard deviations

Excel generates the quartiles, the mean, and the standard deviation by using what *Excel* calls "functions." You can use the Insert-Function procedure, as in generating random numbers, but choosing different functions. Or you can just type the following in an empty cell (and hit return or enter). For example, suppose the file of 30 exam scores from Section 30.3 has been opened or the scores have been typed into a worksheet, with the scores in the cells from A1 to A30. Then, for each of the following, select a blank cell and type what is below. Press the return or enter key, and *Excel* will calculate the result and put it in the cell. Notice the "=" sign and the colon in A1:A30.

=QUARTILE(A1:A30,1)	(gives the first quartile score, 49.25)
=QUARTILE(A1:A30,2)	(gives the median. 59.5)
=QUARTILE(A1:A30,3)	(gives the third quartile score, 66.75)
=AVERAGE(A1:A30)	(gives the mean, 57.8)
=STDEVP(A1:A30)	(gives the standard deviation, 13.4)

Excel sometimes calculates more precisely or in a slightly different way from our way. For example, *Excel* gives 49.25 and 66.75 for the first and third quartile scores, whereas we would use 49 and 67. STDEVP, rather than a more natural-looking STDEV, gives our version. (STDEV is calculated by dividing by $n - 1$ rather than n, but STDEVP uses n.) *Excel* also has =MAX and =MIN, but these are usually easily determined without computer help (unless there is a very large number of values). *Excel* will also order data. Select by clicking and dragging through the values you wish to put in order. Go to **Data** in the menu, and choose **Sort. Sort** is risky if there are several columns of data in your file.

If too many decimal places, or too few, are showing, select the cell, then click on **Format** in the menu, and drag to **Cells.** Then using the Numbers tab and the Numbers choice, you can adjust the number of decimal places visible.

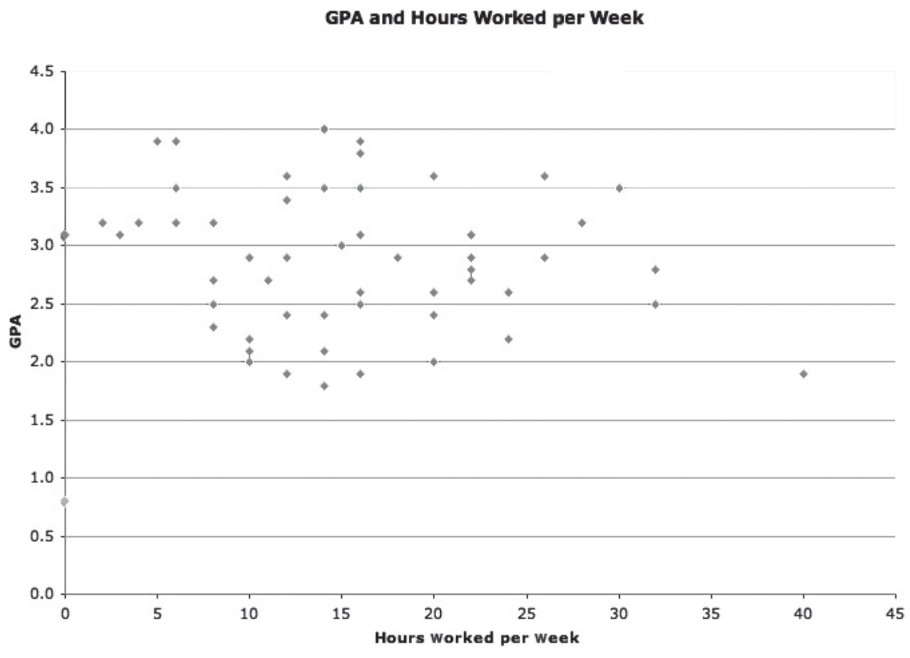
Making a scatter plot

For our example, we will assume that the *60 Students* file has been opened in *Excel,* and we want the scatter plot of hours worked per week on the x-axis and GPA on the y-axis.

The two columns for those data are not adjacent in the table. Perhaps the easiest way to deal with that is to select the x-value column (hours worked per week) by clicking and dragging through the data in the column, using **Edit** from the menu bar, and choosing **Copy.** Then go to the head of some blank column and use **Edit** again to **Paste.** Do the same for the y variable (GPA), pasting it in the column immediately to the *right* of the new x-values column.

Click and drag to select the values in both these new columns, starting at the upper left and dragging to the lower right of the values. *Excel* treats the first column as the x-values and the second as the y-values.

Now go to the menu bar for **Insert** and then **Chart.** Click on **XY (Scatter),** then **Next,** and then **Next** again. As with other charts, this screen offers tabs, for example, to insert a title and label the axes. When you have made your choices, press **Next** and then **Finish.** As before, the graph appears on the worksheet, but it can be moved or copied. If you would like more detailed tick marks on the axes, double-click on the axis, and use the Patterns tab to insert tick marks. Double-clicking on the background color allows you to change its color. A final version might look like the one below, which was copied-and-pasted from *Excel.*



Adding a line of best fit, or regression line (called a trendline in *Excel*)

You can add a line of best fit to an *Excel* scatter plot this way. While in *Excel,* select the chart (black squares at the corners and midpoints). Then **Chart** appears in the menu bar. Click on **Chart** and drag down to **Add Trendline.** You will be asked for the Type (Linear). Use the Options tab if you would like to display the equation and/or the R^2 value to the scatter plot. The equation and R^2 value are movable (frequently, they are obscured by the data or the line of best fit). A final version might look like the one below.

Appendix J

Using the *Illuminations* Website

Whether or not you use the *Illuminations* website will depend on your access to the Internet and on what type of technology your instructor decides to use with your class. *Illuminations* is one of the easiest types of technology to use because the website consists of a series of applets, written with precollege students in mind. An *applet* is a small computer program written to focus on one or a few functions.

The *Illuminations* website has numerous online activities; we will use only a few. You will want to look at several of the activities, particularly for grades K–2 and grades 3–5 (in addition to those we will use here), because this website will likely be useful when you are teaching. The National Council of Teachers of Mathematics, the sponsor of the *Illuminations* website, has other items of interest to teachers of mathematics at http://www.nctm.org.

Note: We expect the *Illuminations* website to be available for years to come. However, as computer operating systems and Web browsers change, the *Illuminations* applets may need updating and so may be unavailable for short periods of time.

The remainder of this Appendix has the following lessons:

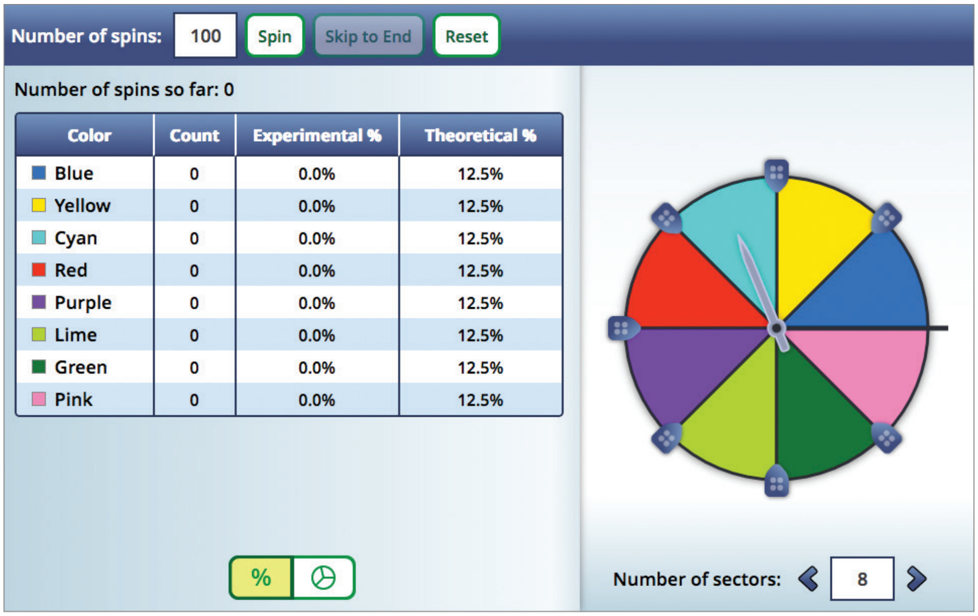
1. **Getting started**
2. **Running a simulation**
3. **Making a circle graph**
4. **Making a bar graph**
5. **Making a histogram and finding the mean and standard deviation**
6. **Making a box plot and finding the five-number summary**
7. **Making a scatter plot and finding the line of best fit and correlation coefficient**
8. **Running a second simulation (for use with Chapter 32)**

Getting started

1. The *Illuminations* website is found on the website for the National Council of Teachers of Mathematics (NCTM). You can access it at http://illuminations.nctm.org. You will see buttons for Lessons and Interactives.
2. We will be using the **Interactives,** so click on that.
3. Under the pull-down menu of **Interactives,** you will see **All Interactives.** Click here to see the range of activities and browse through the available ones, clicking on those of interest.
4. Now return to the upper right of the screen and click the box for **Interactives;** type in which of the activities you want to use, then click on **Search.**

Running a simulation

1. Go to **Adjustable Spinner.** Click on the box labeled **Instructions** and read the instructions. Suppose you want to simulate tossing three coins to find the probability that all will fall heads.
2. For this type of simulation, you will need to know ahead of time the possible ways three coins can fall. They are HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT. Because each of these possible outcomes is equally likely, you already know the theoretical probability of HHH is $\frac{1}{8}$, 0.125, or 12.5%.



3. You will need eight sectors of equal size, so go to the right arrow and click on it until the number of sectors is 8. If you move the buttons on the circle or the dots on the circle, you can change the size of the sectors, but for this experiment we want the sectors to be of equal size. You will see that each sector is 12.5% of the circle. Your screen will look something like what is shown above. Suppose we assign blue to represent HHH.
4. Go to **Number of Spins** and type in 100 (or any number you wish). You can skip actually seeing all 100 spins if you select **Skip to End.**
5. In the Probability box, you will see the theoretical probabilities (each 12.5%, which is equivalent to $\frac{1}{8}$) and the experimental probabilities, which should all be close to the theoretical probabilities. Do 100 spins, then clear (by clicking on **Reset**), and do 100 spins again 4 more times. Write down the probability of HHH (blue) each time. Then do 10,000 spins 5 times. Were the probabilities of blue for 10,000 spins closer to 12.5% than they were with 100 spins? Click on **Show results frame.**

   Notice here how close the experimental and theoretical probabilities are. Try again with 1000 spins and see whether or not the experimental probabilities better approximate the theoretical probabilities. (Recall that probabilities involve *the long run.*)

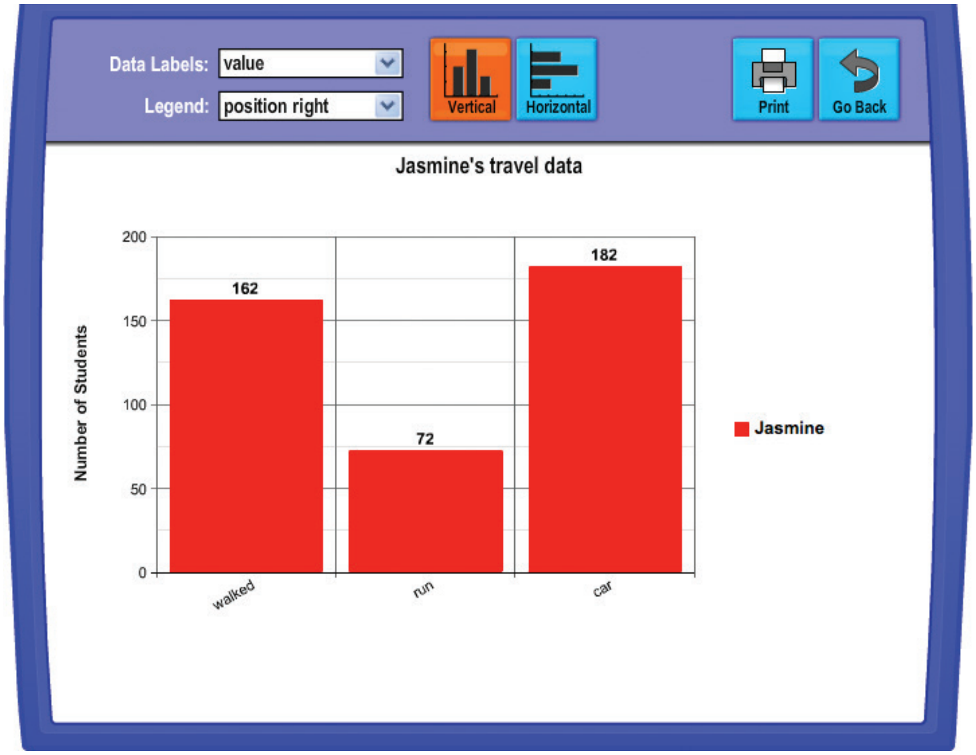
Making a circle graph

Suppose we want to make a circle graph of the data Jasmine collected at her school on the different ways that students came to school: Of the total 420, 182 walked, 166 road the bus, and 72 came by car.

In **Interactives,** go to **Data Grapher** and select **Pie.** Enter the title "Jasmine's travel data." Below that, enter Jasmine's data: Replace row 1 with "Walked" and enter 182. Click on the bottom with + and horizontal lines to add a row. Enter "Bus" and 166. Continue for the third row and enter "Car" and 72. Next click **Preview.** You will see the circle graph representing Jasmine's data. Select Data labels: % of total. Select Legend: position right.

Making a bar graph

We can use the same data to make a bar graph by going to **Data Grapher.** Fill in your data as for a circle graph. Then choose **Graph Data.** You can change the height of the bars by changing the value in the **Maximum Value on Bar Graph** and change bar width by moving the button on **Bar Width,** or by using the scroll bars by the two axes. Your graph should look similar to the following one:



Making a histogram and finding the mean and standard deviation

1. Go to the **Advanced Data Tool** activity. Read the **Instructions** and select **Histogram.** This time we will use one of the data sets provided. Go to the data files, and find the one for *60 Students.* We will use the GPA data. Type the data into the **Histogram,** adding columns as needed.
2. Rename the histogram in the space where you are asked to describe your data. Call it GPAs.
3. Click on **Preview.** A histogram of the data will appear. You may want to change the interval size. Try 0.2. (You may need to reset the interval and the frequency.)
4. Think about the information shown in the graph. Notice that the mean and the standard deviation are given.
5. Reset the interval size to see what happens. (Remember to click **OK.**)

   Note: The **Histogram Tool** does not always work well with Macs, depending on the browser you use. For example, the histogram may be cut off at the side edges. This does not affect how the histogram applet works, although you may not be able to see all of the data that you enter.

Making a box plot and finding the five-number summary

1. Go to the **Advanced Data Tool** activity. Read the **Instructions.** We will again use the data file on student GPAs in the file on *60 Students.*
2. Click on the **Box Plotter** tool, then enter the GPA data into the cleared space as in the preceding lesson. You may have to use the scroll bar at the bottom to see the data.
3. Rename the box plot GPAs.

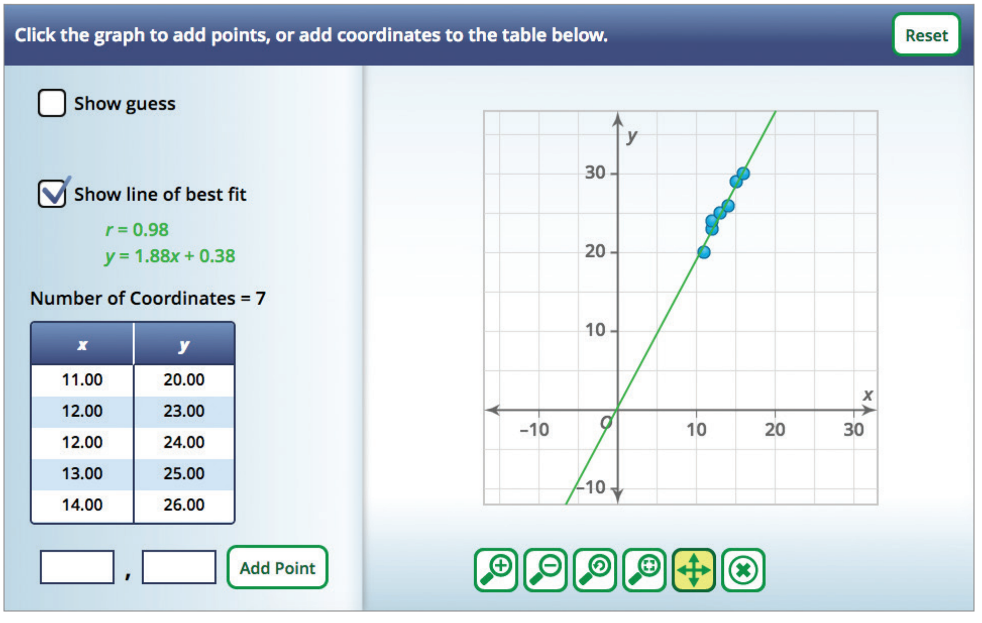
4. Click on **Preview.** A box plot of the data will appear. The median and quartiles (Q1 and Q3) are also provided. Find the minimum and maximum GPAs and you will have the five-number summary.

Making a scatter plot and finding the line of best fit and correlation coefficient

Here is a table of data relating packages of macaroni in stock on a given day and servings of macaroni served that day.

Packages	Servings
11	20
12	23
12	24
13	25
14	26
15	29
16	30

1. Go to the **Line of Best Fit** tool. Read the directions.
2. You will see an empty graph. You can place points on the graph either by clicking on the points or by filling in the pairs of numbers in the window at the bottom. (Points can be removed by first clicking on icon with an X and then clicking on the points.) At the bottom, fill in the pairs of points from the table, one pair per line.
3. The data points will appear on the graph. This graph is called a *scatter plot,* even though that name does not appear on this screen.
4. What line do you think would best fit these points? Click on **Show Guess** and a line will appear with large dots. You can move the line around by using those dots or entering an equation. Do it. Try to find the best fit you can.
5. How good is your guess? Click on **Show Line of Best Fit** and a green line will appear. The line is mathematically calculated to best fit the data points. Your screen should look like this:



6. The correlation coefficient r is 0.98. The closer this number is to 1, the more confident you can be that packages and number of servings are related. You can even estimate number of servings from number of packages.
7. Click off the lines or add a point (via the boxes) and see what happens if you add a new point that is substantially different (e.g., twice as many servings for the number of packages). Add this data point and once again predict the line of best fit and check your prediction. What is the correlation coefficient now?

Running a second simulation (for use with Chapter 32)

Once again go to the **Adjustable Spinner.**

1. Suppose you know that 63% of the students at a large high school are Hispanic. If 10 students enter the bookstore within a couple of minutes, would you be surprised if only 4 were Hispanic? What would be the probability of this happening? Of course, to find the probability of this happening, we should repeat this experiment many times. To do so, set the number of sectors to 2, and set the theoretical probabilities to 63% and 37%, where the first color represents Hispanics.
2. Click on **Spin** a few times. Compare the Experimental and Theoretical probabilities each time, to get a feel for what is happening. Notice that on any one spin, the experimental probabilities vary a great deal from the theoretical probability. After some testing, click on **Reset.**
3. Suppose we spin 10 times, over and over for 100 trials. First make a table as follows, up to 90% in the first column. (This should suffice; you may need to extend your table.) The second column should be filled with tally marks for the experimental probabilities for the percent of Hispanics in each spin. (The third column will be used later.)

	Number of 100 samples giving this percent	
Percent that are Hispanic (%)	**Sample size 10**	**Sample size 100**
30		
31		
32		
Etc.		

4. The second column represents the percentage that you *find.* For example, if you use a sample size 10 (by placing 10 in the **Number of spins** box), then the spinner will spin 10 times. If the first color shows up 40% of the time, make a tally mark in the *Sample size 10* column in the 40% row. Repeat this process 100 times so that you will have 100 tally marks in the *Sample size 10* column. *Be sure to click on* **New experiment** *each time so that you get a new percentage that does not include the last spin.*

   Count up the ticks on each line. One line in your table might look like this:

50%	### #### // 12

   Study the results. What was the range of percentages you found? Did they cluster around the entries closest to 63%, the theoretical probability in this case? What is the experimental probability of having 4 Hispanic students in a sample of 10 from a population that is 63% Hispanic?

5. Use the third column in your table. This time the heading you will have is *Sample size 100.* Set the **Number of spins** to 100. Click on **Reset** and repeat the process 100 times, again clicking on **Reset** each time and making tally marks each time in your table. Again make tick marks by the percentages that result each time.
6. Here is an extremely important question: *How does the spread of experimental findings differ in the two columns?* As the sample size gets larger, what happens to the spread of the results? Do you think you would be more or less likely to get 40% in a sample size of 10 or a sample size of 100? Make predictions about what you would expect for sample size 1000, with 100 spins. Would you feel more confident of the result when the sample size is 500 compared to a sample size of 2000? Explain why or why not.

The results of the preceding lesson can help you better understand Table 1 in Chapter 32.