

Mean Free Path

The conduction electrons in a metal are in random thermal motion with average velocity $\langle v \rangle$. Although there is a distribution of velocities about this average just as with the molecules in a gas, for simplicity in this discussion, we will consider only the single velocity $\langle v \rangle$. If an electric field ε is applied, all of the electrons acquire an added drift velocity \mathbf{v}_d in the same direction, that is, opposite to that of the electric field. This newly acquired kinetic energy is soon lost by each electron via inelastic collisions with lattice ions. Let τ be the average time before an electron, picked at random, makes its next collision. Because the collisions are random, this time does not depend on the time elapsed since the electron's last collision. If we look at an electron immediately after it makes a collision, the average time before its next collision will be τ . Thus, τ , called the *collision time* or the *relaxation time*, is the average time between collisions. It is also the average time since the *last* collision of an electron picked at random. The drift velocity \mathbf{v}_d is the average velocity of an electron picked at random since the average thermal velocity is zero. Since the acceleration is $e\varepsilon/m_e$, the drift velocity is

$$\mathbf{v}_d = -e \frac{E}{m_e} \tau$$

The average distance the electron travels between collisions is called the *mean free path* λ . It is the product of the average speed $\langle v \rangle$ and the average time between collisions τ (see Figure FP-1):

$$\lambda = \langle v \rangle \tau$$

In terms of the mean free path, the drift velocity is

$$\mathbf{v}_d = -\frac{eE\lambda}{m_e \langle v \rangle}$$

Using this result with Equation EC-4 (see the CCR unit *Classical Theory of Electrical Conduction*), we can then write for the electric current I

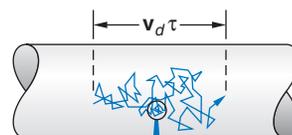
$$I = NeAv_d = \frac{Ne^2\lambda EA}{m_e \langle v \rangle}$$

and for the current density j

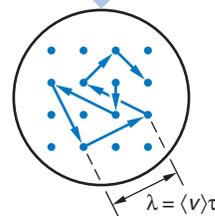
$$j = \frac{I}{A} = \frac{nNe^2\lambda E}{m_e \langle v \rangle}$$

Comparing Equations FP-4 and FP-5 with Ohm's law (see the CCR unit *Classical Theory of Electrical Conduction*), we can write for the resistivity ρ and the conductivity σ

FP-1



FP-2

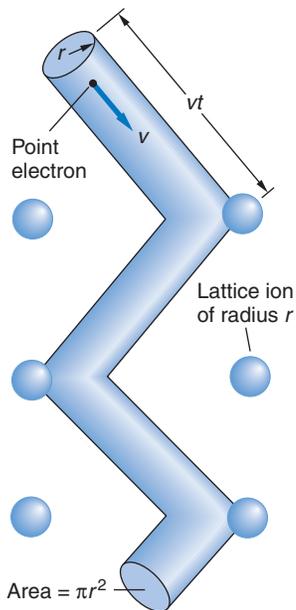


FP-3

FP-4

FP-1 Path of electron in a wire. Superimposed on the random thermal motion is a slow drift at speed v_d in the direction of the electric force $e\varepsilon$. The mean free path λ , the average time between collisions τ , and the average speed $\langle v \rangle$ are related by $\lambda = \langle v \rangle \tau$.

FP-5



FP-2 Model of an electron moving through the lattice ions in copper. The electron, which is considered to be a point, collides with a lattice ion if it comes within a distance r of the center of the ion, where r is the radius of the ion. If the electron has speed v , it collides with all the ions in the cylindrical volume $\pi r^2 vt$ in time t .

$$\rho = \frac{m_e \langle v \rangle}{Ne^2 \lambda} \quad \text{and} \quad \sigma = \frac{1}{\rho} = \frac{Ne^2 \lambda}{m_e \langle v \rangle} \quad \text{FP-6}$$

We can relate the mean free path to the size of the ions in the metal lattice. Consider one electron moving with speed v through a region of stationary ions (see Figure FP-2). Since this speed is related to its thermal energy, it is essentially unaffected by collisions. Assuming that the size of the electron is negligible, we see that it will collide with an ion if it comes within a distance r from the center of the ion, where r is the radius of the ion. In some time t , the electron moves a distance vt and collides with every ion in the cylindrical tube of volume $\pi r^2 vt$ surrounding the path of the electron. (After each collision, the direction of the electron changes, so the path is really a zigzag one, like that shown in Figure FP-1.) The number of ions in this volume and hence the number of collisions in the time t is $n_a \pi r^2 vt$ where n_a is the number of ions per unit volume. The total path length divided by the number of collisions is the mean free path λ .

$$\lambda = \frac{vt}{n_a \pi r^2 vt} = \frac{1}{n_a \pi r^2} \quad \text{FP-7}$$

EXAMPLE FP-1 Mean Free Path and Relaxation Time of Electrons

Estimate the mean free path and the relaxation time of electrons in copper. The number of copper ions per unit volume is 8.47×10^{22} ions/cm³.

SOLUTION

1. The mean free path λ is related to the number of copper ions per unit volume n_a and the radius r of a copper ion by Equation FP-7:

$$\lambda = \frac{1}{n_a \pi r^2}$$

2. Using $r \approx 10^{-10} \text{ m} = 10^{-8} \text{ cm}$ as the radius of a copper ion, substituting these into Equation FP-7 gives

$$\begin{aligned} \lambda &= \frac{1}{\pi (8.47 \times 10^{22} \text{ ions/cm}^3) (10^{-8} \text{ cm})^2} \\ &= 3.8 \times 10^{-8} \text{ cm} = 3.8 \times 10^{-10} \text{ m} = 0.38 \text{ nm} \end{aligned}$$

3. The relaxation time τ is given by Equation FP-1. In terms of the mean free path λ and the average speed $\langle v \rangle$ of the ions, this is

$$\tau = \frac{\lambda}{\langle v \rangle}$$

4. Substituting λ from above and the value of $\langle v \rangle$ for copper ions computed in Equation EC-1 (see the CCR unit *Classical Theory of Electrical Conduction*), we have that

$$\tau = \frac{3.8 \times 10^{-9} \text{ m}}{1.1 \times 10^5 \text{ m/s}} = 3.5 \times 10^{-15} \text{ s}$$