

## Classical Theory of Electrical Conduction

In the Drude model referred to in Section 10-2 of the textbook, a metal is pictured as a regular three-dimensional array of atoms or ions with a large number of electrons free to move throughout the whole metal. In copper, for example, there is approximately one free electron per copper atom. The density of free electrons can be measured using the Hall effect (see Section 10-6). In the absence of an electric field, the free electrons move about the metal much like gas molecules in a container. Unlike an ordinary gas, however, thermal equilibrium is maintained by collisions of electrons with the lattice ions rather than by collisions of electrons with each other. Since the velocity vectors of the electrons are randomly oriented, the average velocity due to this thermal energy is zero. On the other hand, the average speed of electrons is quite high and can be calculated from the equipartition theorem. The result is the same as that for ideal gas molecules, with the electron mass  $m_e$  substituted for the molecular mass in Equation 8-10 in the textbook. For example, at temperature  $T = 300$  K, the average speed is

$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m_e}} = \sqrt{\frac{8(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{\pi(9.11 \times 10^{-31} \text{ kg})}} = 1.08 \times 10^5 \text{ m/s} \quad \text{EC-1}$$

### Derivation of Ohm's Law

The conduction electrons in a metal are in random thermal motion with average velocity  $\langle v \rangle$ . Although there is a distribution of velocities about this average just as with the molecules in a gas, for simplicity in this discussion we will consider only the single velocity  $\langle v \rangle$ . If an electric field  $\epsilon$  is applied, all of the electrons acquire an added drift velocity  $v_d$  in the same direction, that is, opposite to that of the electric field. This newly acquired kinetic energy is soon lost by each electron via inelastic collisions with lattice ions, following which the electron is re-accelerated by the electric field. Let  $\tau$  be the average time before an electron, picked at random, makes its next collision. Because the collisions are random, this time does not depend on the time elapsed since the electron's last collision. If we look at an electron immediately after it makes a collision, the average time before its next collision will be  $\tau$ . Thus,  $\tau$ , called the *collision time* or the *relaxation time*, is the average time between collisions. It is also the average time since the *last* collision of an electron picked at random. As long as  $\tau$  (see Example 10-4 in the textbook) is small compared with  $\langle v \rangle$  as calculated in Equation EC-1, as it certainly is, then

$$\tau = \frac{\lambda}{\langle v \rangle} \quad \text{EC-2}$$

where  $\lambda$  is the mean free path (see the CCR unit *Mean Free Path*). From Newton's second law the velocity resulting from acceleration in the electric field after a time  $\tau$ , called the *drift velocity*, is

$$\mathbf{v}_d = -e \frac{\mathbf{E}}{m_e} \tau \quad \text{EC-2}$$

Thus, the average drift velocity between two collisions is

$$\overline{\mathbf{v}_d} = -\frac{e\mathbf{E}\tau}{2m_e} = -\frac{e\mathbf{E}\lambda}{2m_e\langle v \rangle} \quad \text{EC-3}$$

Since the random thermal motion of the electrons results in no net transport of charge, the *current density*  $j$  results only from the average drift velocity of the electrons. If there are  $N$  free electrons per unit area, the magnitude of the current density is

$$j = |-Ne\overline{v}_d| = \frac{e^2 N E \lambda}{2m_e \langle v \rangle} = \left( \frac{e^2 N \lambda}{2m_e \langle v \rangle} \right) E = \sigma E \quad \text{EC-4}$$

Noting that none of the factors that make up  $\sigma$  is dependent on  $\varepsilon$ , Equation EC-4 says that the current density is proportional to the strength of the electric field; it is one version of Ohm's law. We see that this is true as follows:

$$j = \frac{I}{A} = \left( \frac{e^2 N \lambda}{2m_e \langle v \rangle} \right) \frac{V}{\ell}$$

where  $I$  is the current,  $A$  is the cross-sectional area of the conducting metal,  $V$  is the electric potential across the conductor, and  $\ell$  is the length of the conductor. Rearranging yields

$$I = \left( \frac{e^2 N \lambda}{2m_e \langle v \rangle} \right) \frac{VA}{\ell} = \left( \frac{e^2 N \lambda}{2m_e \langle v \rangle} \right) \frac{A}{\ell} V$$

or

$$V = I \left( \frac{2m_e \langle v \rangle}{e^2 N \lambda} \right) \frac{\ell}{A} = IR \quad \text{EC-5}$$

where  $R$  is the resistance given by

$$R = \left( \frac{2m_e \langle v \rangle}{e^2 N \lambda} \right) \frac{\ell}{A} = \rho \frac{\ell}{A} = \frac{1}{\sigma} \frac{\ell}{A} \quad \text{EC-6}$$

where  $\rho$  is the *resistivity* and  $\sigma$  is the *conductivity*. Equation EC-5 is the familiar form of Ohm's law.

The current in a wire is the amount of charge passing through a cross-sectional area  $A$  per unit time. The current  $I$  is related to the number of electrons per unit volume  $N$ , the drift velocity  $\mathbf{v}_d$ , the charge on the electrons  $e$ , and the cross-sectional area  $A$  by Equation EC-7 below. We will assume that each electron moves with a drift velocity  $\mathbf{v}_d$ . In a time  $\Delta t$  all the particles in the volume  $A\mathbf{v}_d\Delta t$ , shaded in Figure EC-1, pass through the area element. If the number of electrons per unit volume is  $N$  and the number in this volume is  $NA\mathbf{v}_d\Delta t$ , then the total charge  $\Delta Q$  is

$$\Delta Q = eNAv_d\Delta t$$

And the current is thus also given by

$$I = \frac{\Delta Q}{\Delta t} = eNAv_d \quad \text{EC-7}$$

The drift velocity is found as illustrated in Example 10-4 in the textbook.

According to Ohm's law (Equation EC-5), the current in a conducting wire segment is proportional to the voltage drop across the segment:

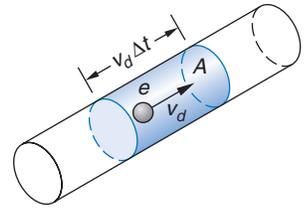
$$I = \frac{V}{R}$$

where the resistance  $R$  is independent of both  $I$  and  $V$ . It is proportional to the length  $\ell$  of the wire segment and inversely proportional to the cross-sectional area  $A$  of the segment, as indicated in Equation EC-6. From the definition of  $j$  and Equation EC-7 we can then express  $v_d$  as

$$v_d = \frac{j}{Ne} \quad \text{EC-8}$$

For materials that obey Ohm's law, the resistivity  $\rho$  and the conductivity  $\sigma$  must be independent of  $\varepsilon$ .

All homogeneous materials, both conductors (e.g., metals) and semiconductors (e.g., germanium and silicon), obey Ohm's law over some range of applied electric field; however, at high electric fields the behavior of all materials, including metals, departs from Ohm's law. At first glance it is surprising that any material obeys Ohm's law since, in the presence of an electric field a free electron experiences a force of magnitude  $e\varepsilon$ . If this were the only force acting on the electron, it would have acceleration  $e\varepsilon/m$  and its velocity would steadily increase. However, Ohm's law implies that there is a steady-state situation in which the drift velocity of the electron is proportional to the field  $\varepsilon$  because the current  $I$  is proportional to  $\varepsilon$  (Equation EC-4) and also to  $v_d$  (Equation EC-7); that is,  $v_d = j/ne = \sigma E/ne$ . In the classical model, it is assumed that a free electron is accelerated for a short time and then collides with a lattice ion. After the collision, the velocity of the electron is assumed to be completely unrelated to that before the collision. The justification for this assumption is that, as we have seen (Example 10-4), the drift velocity is very small compared to the average thermal velocity. The objective of the classical theory of conduction is to find an expression for  $\rho$  in terms of the properties of metals. Doing so is aided by consideration of the mean free path (see the CCR unit *Mean Free Path*).



**EC-1** In time  $\Delta t$ , all the charges in the shaded volume pass through  $A$ . If there are  $n$  charge carriers per unit volume, each with charge  $e$ , the total charge in this volume is  $\Delta Q = nev_d A \Delta t$ , where  $v_d$  is the drift velocity of the charge carriers. The total current is then  $I = \Delta Q / \Delta t = nev_d A$ .