Interference Fringes

Interference is a characteristic of waves. It is the result of the superposition of two or more waves at a particular point in space. If the waves have the same frequency and wavelength but differ only in their relative phases, the resultant wave's amplitude at that point will depend on the differences of the phases.

For simplicity, let's consider two superimposing waves as an example. The peaks and valleys of the two waves can match exactly (constructive interference) or exactly mismatch (destructive interference) or be anywhere between these two situations (see Figure IF-1*a*). Referring now to Figure IF-1*b*, at the air-soap film interface *A* the ray r_i with wavelength λ incident nearly perpendicular to the surface is split into two coherent rays, one, r_2, transmitted into the film and one, r_1 reflected from the top surface of the film. The latter ray experiences a 180-degree phase change because the index of refraction of the second medium $n_{\text{film}} > n_{\text{air}}$, the index of refraction of the first medium. So *at point A* the transmitted and reflected rays exactly mismatch or, as we say, are exactly out of phase. The ray r_2 transmitted into the soap film reflects from the film-air interface at *B* without a change of phase because air is now the second medium and $n_{\text{air}} < n_{\text{film}}$. The ray within the film, whose wavelength is now $\lambda' = \lambda/n_{\text{film}}$, travels an additional distance $2t$ before returning to point *A*. This distance corresponds to an additional phase change Δ given by

$$\delta = \frac{2t}{\lambda'}2\pi = \frac{2t}{\lambda'}360° \qquad \textbf{IF-1}$$

On returning to *A*, the total phase difference between r_1 and r_2 will be $180° + \delta$.

For the example illustrated in Figure IF-1*b*, if $t = 1000$ nm and $\lambda = 700$ nm (red light), then

$$\delta = \frac{2(1000 \text{ nm})(360°)}{(700 \text{ nm}/1.30)} = 1337°$$

The total phase difference between r_1 and r_2 at point *A* will be

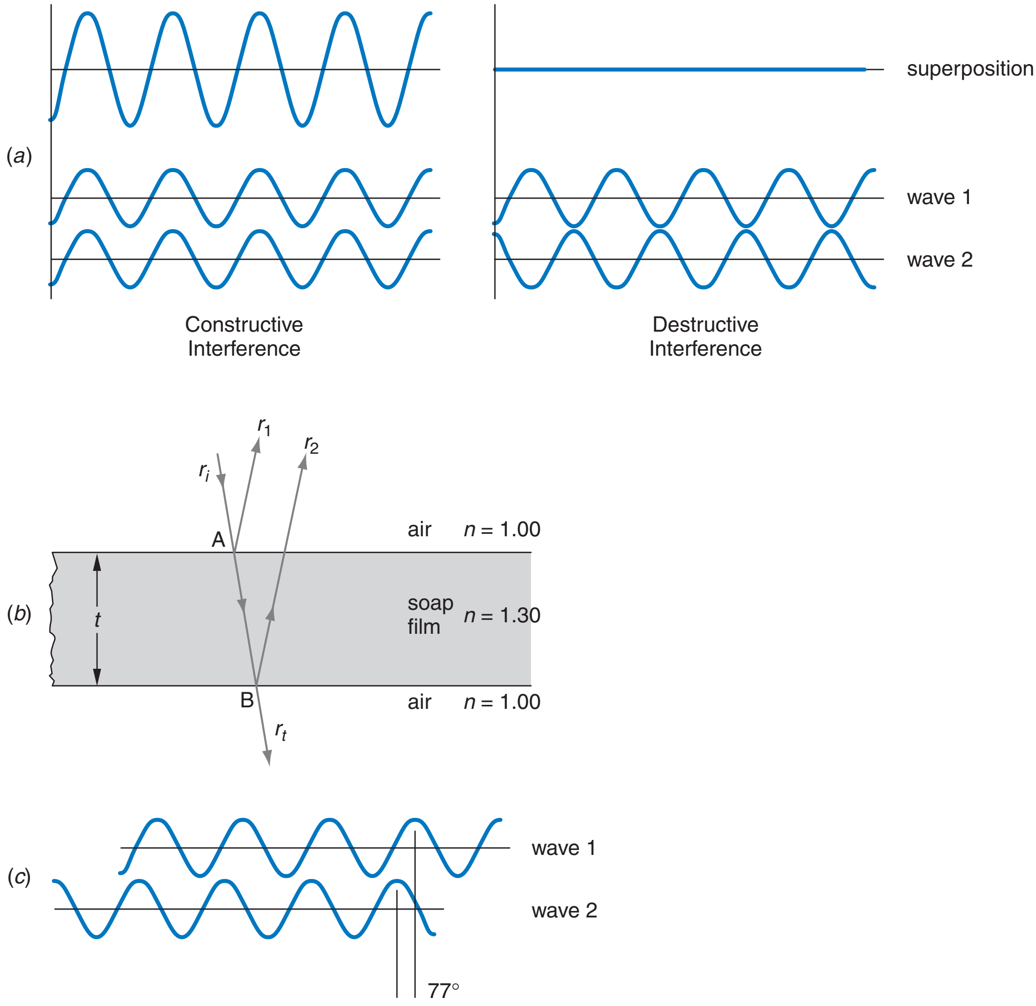
$$180° + \delta = 180° + 1337° = 1517° = 4(360°) + 77°$$

Thus, on returning to *A*, r_2 will be 77 degrees out of phase with (behind) r_1 (see Figure IF-1*c*).

We can also use Equation IF-1 to determine the minimum thickness that the soap film (or a thin sample of any transparent material) can have in order to exhibit constructive interference. Writing Equation IF-1 as

$$t = \frac{\delta\lambda'}{2(360°)}$$

For constructive interference δ must at minimum be 360°, so $t_{\text{min}} = \lambda'/2$. For the situation illustrated in Figure IF-1*b* and described above, $t_{\text{min}} = (700 \text{ nm}/1.30)/2 = 269$ nm.



IF-1 (*a*) In a particular region the peaks and valleys of waves 1 and 2 at exactly the same places result in constructive interference. The amplitude of the resultant wave is the sum of the amplitudes of the component waves 1 and 2. Here waves 1 and 2 are exactly in phase. (*b*) When the peaks and valleys of wave 1 occur at the same places as the valleys and peaks of wave 2, respectively, the amplitude of the superposition is zero. In this event waves 1 and 2 are exactly out of phase. (*c*) On recombining at point B, r_2 is four complete waves ($4 \times 360°$) plus 77° behind wave r_1. The amplitude of the resultant wave (not shown) will be the point-by-point sum of those of waves 1 and 2.

Continuing on, if we now form a wedge-shaped thin film of air (or water or any transparent fluid) between two glass plates and illuminate the upper surface with parallel light of wavelength $\lambda = 700$ nm normal to the surface as shown in Figure IF-2*a*, alternating bright (constructive interference) and dark (destructive interference) bands or *fringes* appear as sketched in Figure IF-2*b*. In this case the fringes are parallel and uniformly spaced. By measuring distances and counting fringes, we can determine several kinds of information. For example, the number of fringes per centimeter can be computed. Notice that the edge C where the two glass plates are in contact is a dark fringe since the ray reflected from the bottom plate at that edge experiences a

180-degree phase shift. The mth dark fringe will occur where the air wedge thickness is t' (see Figure IF-2a) and the path difference between r_1 and r_2 is $2t'$:

$$2t' = m\lambda \quad \Rightarrow \quad m = 2t'/\lambda \qquad \textbf{IF-2}$$

The angle θ between the plates is given by

$$\theta = t_0/L = 10^{-5}\,\text{m}/5.0 \times 10^{-2}\,\text{m} = 2.0 \times 10^{-4}\,\text{rad}$$

If the distance from C to the mth fringe is x, then

$$\theta = t'/x \quad \Rightarrow \quad t' = x\theta$$

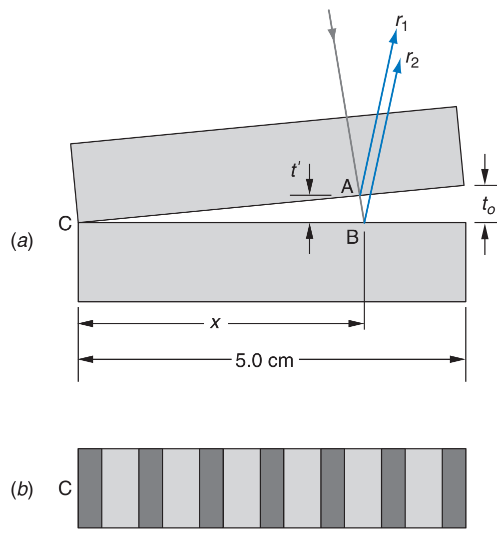
Substituting t' into Equation IF-2 and dividing by x yields

$$m = \frac{2x\theta}{\lambda} \quad \Rightarrow \quad \frac{m}{x} = \frac{2\theta}{\lambda} = \frac{2(2.0 \times 10^{-4}\,\text{rad})}{(700\,\text{nm} \times 10^{-9}\,\text{m/nm})}$$

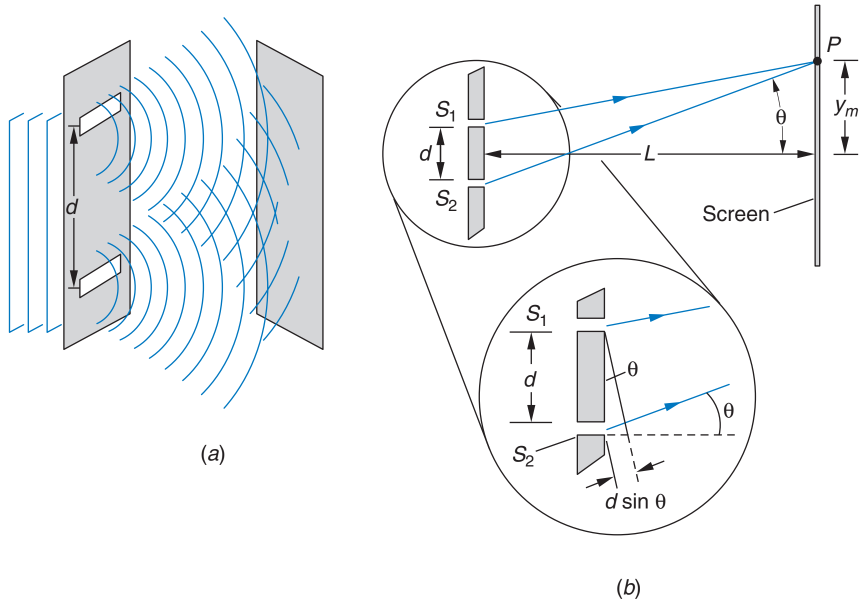
$$\frac{m}{x} = 570\,\text{fringes/m} = 5.7\,\text{fringes/cm}$$

By calibrating the air wedge with light of known wavelength, which determines θ, the wavelength λ emitted by an unknown source can be found simply by counting the number of fringes from edge C to some point x measured along the sample. (Of course, there are much better ways of measuring λ.) In addition, replacing the air in the wedge with another transparent fluid (gas or liquid) makes possible, within limits, the determination of the index of refraction of the fluid since λ in Equation IF-2 is now replaced by $\lambda' = \lambda/n_{\text{fluid}}$. (There are also much better ways of measuring n.)

The observation of interference fringes is possible only if the superimposing waves are from two or more coherent sources. Another simple example is that of light diffracted by two closely spaced, parallel slits as illustrated in Figure IF-3a, known as Young's double slit experiment. When parallel light of wavelength λ illuminates the slits, they act as a pair of coherent sources. From Figure IF-3b, on reaching a distant screen at point P, the path difference between the two rays r_1 through slit S_1 and r_2 through S_2 is $d \sin\theta$. An interference maximum (bright fringe) will appear at P when



IF-2 (a) An air wedge formed between two glass flats. The thickness of the wedge varies linearly along the x axis, producing a linear increase in the optical path difference from left to right along the wedge. (b) Michelson used such an arrangement in the interferometer to produce parallel, straight-line fringes.



IF-3 (*a*) Illuminated by parallel light, two (or more) closely spaced, parallel slits act as coherent sources. (*b*) An arrangement for Young's double slit experiment.

$$d \sin \theta = m\lambda \qquad \text{for } m = 0, 1, 2, \ldots \qquad \textbf{IF-3}\boldsymbol{a}$$

and an interference minimum (dark fringe or intensity minimum) when

$$d \sin \theta = \left(m - \frac{1}{2}\right)\lambda \qquad \text{for } m = 1, 2, 3, \ldots \qquad \textbf{IF-3}\boldsymbol{b}$$

The maxima and minima occur as a result of a phase difference δ between the two rays given by

$$\delta = \frac{d \sin \theta}{\lambda} 2\pi = \frac{d \sin \theta}{\lambda} 360° \qquad \textbf{IF-4}$$

Note the similarity to Equation IF-1. In both cases $\delta/2\pi$ is the fraction of a wavelength represented by the path difference of the superimposing waves. In Figure IF-3*b* the distance y_m to the *m*th bright fringe measured from the central point of the screen is related to the angle θ by

$$\tan \theta = \frac{y_m}{L}$$

If the distance to the screen L is large compared to the spacing between the slits d, the small angle approximation $\tan \theta \approx \sin \theta \approx y_m/L$ can be used in Equation IF-3*a*, yielding

$$y_m = m\frac{\lambda L}{d} \qquad m = 0, 1, 2, \ldots \qquad \textbf{IF-5}$$

The interference of coherent waves, both electromagnetic and particle, is of central importance in performing many experiments and in understanding many physical phenomena. Refer to *Physics for Scientists and Engineers*, 6th edition, by Tipler and Mosca for a more complete discussion of interference.