# Classical Relativity

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Galileo was the first to recognize the concept of acceleration when, in his studies of falling objects, he showed that the rate at which the velocity changed was always constant, indicating that the motion of the falling body was intimately related to its *changing* velocity. It was this observation, among others, that Newton generalized into his second law of motion:

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} \qquad \textbf{CR-1}$$

where $d\mathbf{v}/dt = \mathbf{a}$ is the acceleration of the mass $m$ and $\mathbf{F}$ is the net force acting on it. (Recall that letters and symbols printed in boldface type are vectors.) Newton's first law of motion, the law of inertia, is also implied in Equation 1-1: the velocity of an object acted on by no net force does not change; that is, its acceleration is zero.

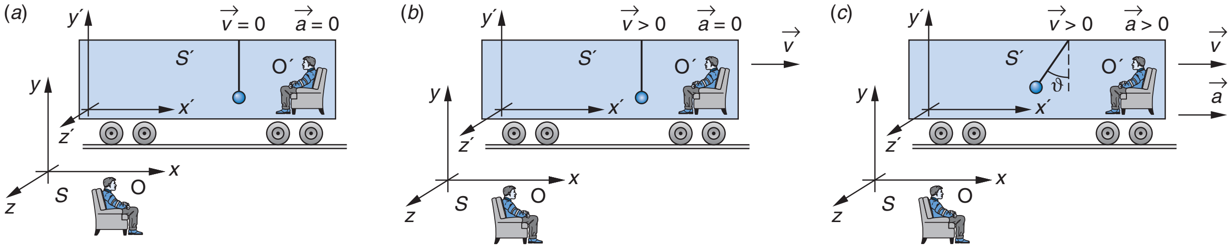
## Frames of Reference

An important question regarding the laws of motion, one that concerned Newton himself and one that you likely studied in first-year physics, is that of the reference frame in which they are valid. It turns out that they work correctly only in what is called an *inertial reference frame*, a reference frame in which the law of inertia holds. Newton's laws of motion for mechanical systems are *not* valid in systems that accelerate relative to an inertial reference frame; that is, an accelerated reference frame is not an inertial reference frame. Figures CR-1 and CR-2 illustrate inertial and non-inertial reference frames.

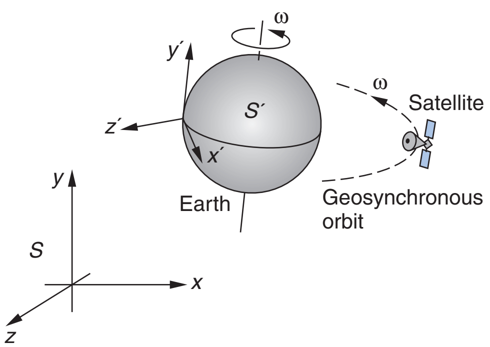
## Galilean Transformation

Newton's laws brought with them an enormous advance in the relativity of the laws of physics. The laws are *invariant*, or unchanged, in reference systems that move at constant velocity with respect to an inertial frame. Thus, not only is there no special or favored position for measuring space and time, *there is no special or favored velocity for inertial frames of reference*. All such frames are equivalent. If an observer in an inertial frame $S$ measures the velocity of an object to be $\mathbf{u}$ and an observer in a reference frame $S'$ moving at constant velocity $\mathbf{v}$ in the $+x$ direction with respect to $S$ measures the velocity of the object to be $\mathbf{u}'$, then $\mathbf{u}' = \mathbf{u} - \mathbf{v}$, or in terms of the coordinate systems in Figure CR-3,

$$u'_x = u_x - v \qquad u'_y = u_y \qquad u'_z = u_z \qquad \textbf{CR-2}$$




**CR-1** A mass suspended by a cord from the roof of a railroad boxcar illustrates the relativity of Newton's second law $\mathbf{F} = m\mathbf{a}$. The only forces acting on the mass are its weight $m\mathbf{g}$ and the tension $\mathbf{T}$ in the cord. (*a*) The boxcar sits at rest in $S$. Since the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the boxcar (i.e., the system $S'$) are both zero, both observers see the mass hanging vertically at rest with $\mathbf{F} = \mathbf{F}' = 0$. (*b*) As $S'$ moves in the $+x$ direction with $\mathbf{v}$ constant, both observers see the mass hanging vertically but moving at $\mathbf{v}$ with respect to $O$ in $S$ and at rest with respect to the $S'$ observer. Thus, $\mathbf{F} = \mathbf{F}' = 0$. (*c*) As $S'$ moves in the $+x$ direction with $\mathbf{a} > 0$ with respect to $S$, the mass hangs at an angle $\theta > 0$ with respect to the vertical. However, it is still at rest (i.e., in equilibrium) with respect to the observer in $S'$, who now "explains" the angle $\theta$ by adding a pseudoforce $\mathbf{F_p}$ in the $-x'$ direction to Newton's second law.




**CR-2** A geosynchronous satellite has an orbital angular velocity equal to that of Earth and, therefore, is always located above a particular point on Earth; that is, it is at rest with respect to the surface of Earth. An observer in $S$ accounts for the radial, or centripetal, acceleration $\mathbf{a}$ of the satellite as the result of the net force $\mathbf{F_G}$. For an observer O′ at rest on Earth (in $S'$), however, $\mathbf{a}' = 0$ and $\mathbf{F_G}' \neq m\mathbf{a}'$. To explain the acceleration being zero, observer O′ must add a pseudoforce $\mathbf{F_p} = -\mathbf{F_G}$.

If we recall that $u'_x = dx'/dt$, $u'_y = dy'/dt$, and so forth then, integrating each of the Equations CR-2, the *velocity FL_transformation* between $S$ and $S'$, yields Equations CR-3, the *Galilean transformation of coordinates*:

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad \textbf{CR-3}$$

assuming the origins of $S$ and $S'$ coincided at $t = 0$. Differentiating Equations CR-2 leads to

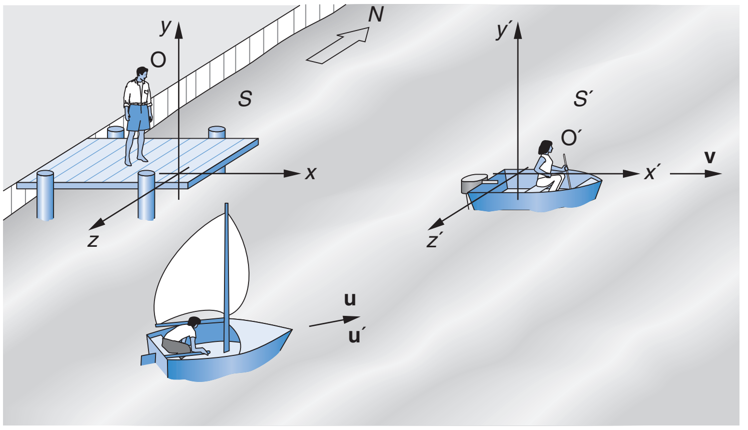
$$a'_x = \frac{du'_x}{dt} = \frac{du_x}{dt} = a_x \qquad a'_y = \frac{du'_y}{dt} = \frac{du_y}{dt} = a_y$$

$$a'_z = \frac{du'_z}{dt} = \frac{du_z}{dt} = a_z \qquad \textbf{CR-4}$$

and the conclusion that $\mathbf{a}' = \mathbf{a}$. Thus, we see that $\mathbf{F} = m\mathbf{a} = m\mathbf{a}' = \mathbf{F}'$ in Figure CR-3 and Figure CR-1*b* and, indeed, in every situation where the relative velocity $\mathbf{v}$ of the reference frames is constant. Constant relative velocity $\mathbf{v}$ of the frames means that $d\mathbf{v}/dt = 0$; hence, the observers measure identical accelerations for moving objects and agree on the results when applying $\mathbf{F} = m\mathbf{a}$. Note that $S'$ is thus also an inertial frame and neither frame is preferred or special in any way. This result can be generalized as follows:

*Any reference frame that moves at constant velocity with respect to an inertial frame is also an inertial frame. Newton's laws of mechanics are invariant in all reference systems connected by a Galilean transformation.*

The second of the preceding statements is the *Newtonian principle of relativity*. Note the tacit assumption in the foregoing that the clocks of both observers keep the same time, that is, $t' = t$.




**CR-3** The observer in $S$ on the dock measures **u** for the boat's velocity. The observer in $S'$ (in the motorboat) moving at constant velocity **v** with respect to $S$ measures $\mathbf{u}'$ for the velocity of the sailboat. The invariance of Newton's equations between these two systems means that $\mathbf{u}' = \mathbf{u} - \mathbf{v}$.

**EXAMPLE CR-1 Velocity of One Boat Relative to Another** What will a person in the motorboat in Figure CR-3 measure for the velocity of the sailboat? The motorboat is sailing due east at 3.0 m/s with respect to the dock. The person on the dock measures the velocity of the sailboat as 1.5 m/s toward the northeast. The coordinate system $S$ is attached to the dock and $S'$ is attached to the motorboat.

**SOLUTION**

1. The magnitude of the sailboat's velocity $\mathbf{u}'$ is given by

$$u' = \sqrt{u_x'^{\,2} + u_y'^{\,2} + u_z'^{\,2}}$$

2. The components of $\mathbf{u}'$ are given by Equation CR-2 with $v = 3.0$ m/s, $u_x = 1.5 \cos 45°$, $u_y = 0$, and $u_z = -1.5 \sin 45°$:

$$u_x' = 1.5 \cos 45 - 3.0$$
$$u_y' = 0$$
$$u_z' = -1.5 \sin 45$$

3. Substituting these into $u'$ above yields

$$u' = \sqrt{3.76 \text{ m}^2/\text{s}^2 + 1.13 \text{ m}^2/\text{s}^2}$$
$$= 2.2 \text{ m/s}$$

4. The direction of $\mathbf{u}'$ relative to north (the $-z$ axis) is given by

$$\theta' = \tan^{-1}(u_x'/u_z')$$

5. Substituting from above:

$$\theta' = \tan^{-1}(-1.94/-1.06)$$
$$= 61° \text{ west of north}$$

**Remarks:** *Note that the observers in S and S′ obtain different values for the speed and direction of the sailboat. It is the equations that are invariant between inertial systems, not necessarily the numbers calculated from them. Since neither reference frame is special or preferred, both results are correct!*