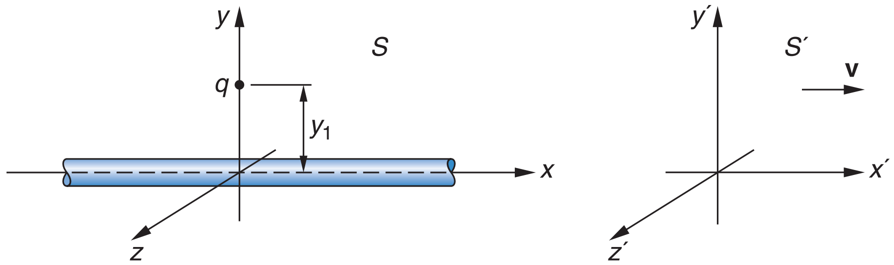
Speed of Light

In about 1860 James Clerk Maxwell discovered that the experimental laws of electricity and magnetism could be summarized in a consistent set of four concise mathematical statements, the Maxwell equations, one consequence of which was the prediction of the possibility of electromagnetic waves. It was recognized almost immediately, indeed by Maxwell himself, that the Maxwell equations did not obey the principle of Newtonian relativity; that is, the equations were not invariant when transformed between inertial reference frames using the Galilean transformations. That this is the case can be seen by considering Figure SL-1, which shows an infinitely long wire with a uniform negative charge density λ per unit length and a point charge q located a distance y_1 above the wire. The wire and charge are at rest in the S frame. A second reference frame S' moves at constant speed v in the $+x$ direction with respect to S. An observer at rest in S' sees the wire and charge q moving in the $-x'$ direction at speed v. The observers in S and S' thus have *different forms* for the electromagnetic force acting on the point charge q near the wire, implying that Maxwell's equations are not invariant under a Galilean transformation.

A fair question at this point would be, Why does anyone care that Maxwell's electromagnetic laws are not invariant between inertial systems the way Newton's laws of mechanics are? Scientists of the time probably *wouldn't* have cared a great deal except that Maxwell's equations predict the existence of electromagnetic waves whose speed would be a particular value $c = 1/(\mu_0\epsilon_0)^{\frac{1}{2}} = 3.00 \times 10^8$ m/s. The excellent agreement between this number and the measured value of the speed of light and between the predicted polarization properties of electromagnetic waves and those observed for light provided strong confirmation of the assumption that light was an electromagnetic wave and, therefore, traveled at speed c.

That being the case, it was postulated in the nineteenth century that electromagnetic waves, like all other waves, propagated in a suitable material medium. Called the *ether*, this medium filled the entire universe, including the interior of matter. (The Greek philosopher Aristotle had first suggested that the universe was permeated with "ether" 2000 years earlier.) It had the inconsistent properties, among others, of being extremely rigid (in order to support the stress of the high electromagnetic wave speed) while offering no observable resistance to motion of the planets, which was fully accounted for by Newton's law of gravitation. The implication of this postulate is that a light wave, moving with velocity $\mathbf{c}$ with respect to the ether, would, according to classical relativity, the Galilean transformation, travel at velocity $\mathbf{c}' = \mathbf{c} + \mathbf{v}$ with respect to a frame of reference moving through the ether at $\mathbf{v}$. This would require that Maxwell's equations have a different form in the moving frame so as to predict the speed of light to be c' instead of $c = 1/(\mu_0\epsilon_0)^{\frac{1}{2}}$. That would in turn reserve for the ether the status of a favored or special frame for the laws of electromagnetic theory. It should then be possible to design an experiment that would detect the existence of the favored frame.



SL-1 The observers in S and S' see identical electric fields $2k\lambda/y_1$ at a distance $y_1 = y'_1$ from an infinitely long wire carrying uniform charge λ per unit length. Observers in both S and S' measure a force $2kq\lambda/y_1$ on q due to the line of charge; however, the S' observer measures an additional force $-\mu_0\lambda v^2 q/(2\pi y_1)$ due to the magnetic field at y'_1 arising from the motion of the wire in the $-x'$ direction. Thus, the electromagnetic force does not have the same form in different inertial systems, implying that Maxwell's equations are *not* invariant under a Galilean transformation.

The problem with the ether postulate at the time it was made was not that it became a favored frame of reference for Maxwell's equations (Newton had postulated a similar status for the "fixed stars" for the laws of mechanics), but that, unlike the media through which other kinds of waves moved (e.g., water, air, solids), it offered no other evidence of its existence. Many experiments were performed to establish the existence of the ether, but nearly all of them suffered from the same serious limitation.

Let's use Fizeau's classic measurement of the speed of light to illustrate that limitation (see Figure SL-2). The time t for the light beam to make a round trip (wheel to mirror back to wheel) is $2L/c$; therefore, the speed of light would be

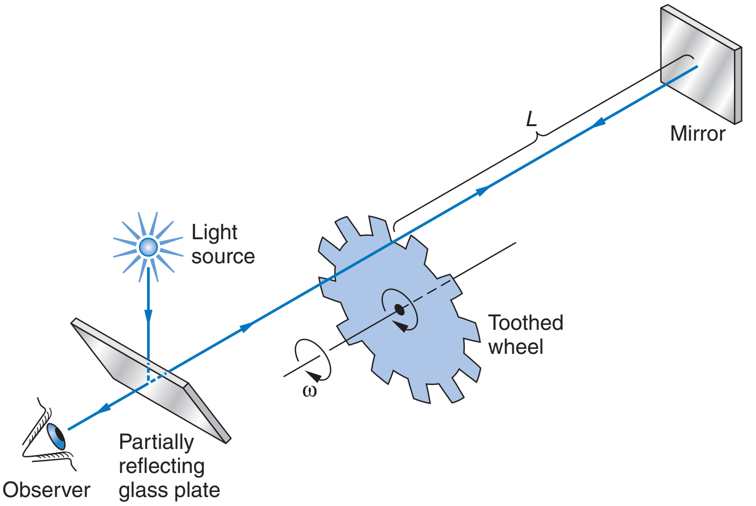
$$c = \frac{2L}{t}$$

However, the motion of Earth relative to the ether at some speed v (unknown) would affect the time measured in an "out and back" terrestrial measurement of the light's speed, such as Fizeau's. If the Earth moves toward the right in Figure SL-2 at speed v, then in the outbound leg the speed of light relative to the laboratory is $c' = c - v$ and in the return leg $c' = c + v$. The round-trip time t is then

$$t = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c}\frac{1}{1 - \dfrac{v^2}{c^2}}$$

$$= \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1} \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2} + \cdots\right) \qquad \textbf{SL-1}$$

where the term $(1 - v^2/c^2)^{-1}$ has been expanded using the binomial expansion in powers of the small quantity v^2/c^2 (see Appendix B2 in the textbook) and only the first two terms have been retained. Although the speed of Earth relative to the ether was unknown, one could reasonably expect that at some season of the year it should be at least equal to Earth's orbital speed around the Sun, about 30 km/s. Thus, the maximum observable effect would only be of the order of $v^2/c^2 = (3 \times 10^4/3 \times 10^8)^2 = 10^{-8}$, or about 1 part in 10^8. The experimental accuracy of Fizeau's measurement was too poor by a factor of about 10^4 to detect



SL-2 Fizeau measured the speed of light in 1849 by aiming a beam of light at a distant mirror through the gap between two teeth in a wheel, in effect changing the light beam into light pulses. A light pulse traveling at speed c would take $2L/c$ seconds to go from the wheel to the mirror and back to the wheel. If, during that time, rotation of the wheel moved a tooth into the light's path, the observer could not see the light. But if the angular velocity were such that the pulse arrived back at the wheel coincident with the arrival of the next gap, the observer saw the light.

this small an effect. A large number of experiments intended to detect the effect of Earth's motion on the propagation speed of light were proposed, but for all of them except one the accuracy possible with the apparatus available was, like Fizeau's, insufficient to detect the small effect. The one exception was the experiment of Michelson and Morley.

EXAMPLE 1-2 Earth's Orbital Speed Determine Earth's average orbital speed with respect to an inertial frame of reference attached to the center of the Sun. The mean value of Earth's orbit radius R is 1.496×10^8 km.

SOLUTION

1. The average orbital speed v is given in terms of the orbital circumference C and the time required to complete one orbit:

$$v = C/t$$

2. The circumference is given in terms of the orbit radius R. The mean value of R is a convenient unit of length used for distances within the solar system; it is called the *astronomical unit* (AU).

$$\begin{aligned} C &= 2\pi R \\ &= 2\pi(1.496 \times 10^8 \text{ km}) \\ &= 9.40 \times 10^8 \text{ km} \end{aligned}$$

3. Earth travels a distance equal to C in $t = 1 \text{ y} = 3.16 \times 10^7$ s. The average speed is then given by

$$\begin{aligned} v &= \frac{9.40 \times 10^8 \text{ km}}{3.16 \times 10^7 \text{ s}} \\ &= 29.8 \text{ km/s} \end{aligned}$$