q/m from the Zeeman Effect

In 1896 Pieter Zeeman discovered that spectral lines emitted by atoms were split into three closely spaced lines when the atoms were placed in an external (to the atoms) magnetic field (see Figure ZE-1 and the More section *The Zeeman Effect*). Of the three lines, one had the frequency (and wavelength) of the original, no-field spectral line, one was at a slightly lower frequency (longer wavelength), and the third was at a slightly higher frequency (shorter wavelength). The frequency differences $\pm\Delta f$ of the two new lines from the frequency of the original line were equal. This observation was explained by H. A. Lorentz using classical mechanics and classical electromagnetic theory.[1] He treated the additional motion of the electron in the atom due to the external magnetic field as a simple harmonic vibration resulting from an elastic restoring force acting to return the electron to some equilibrium position. The vibration frequency of electron was, by electromagnetic theory, that given to the emitted electromagnetic wave resulting from the harmonic acceleration of the charged electron and was equal to

$$f = \sqrt{\frac{a}{m}} \qquad \textbf{ZE-1}$$

where m is the electron mass and a is a positive constant dependent on the properties of the particular atom.

If the external magnetic field $\mathbf{H}$ ($\mathbf{H} = \mathbf{B}/\mu$, where μ = permeability) is applied in the $+z$ direction, then a force is introduced given by

$$\mathbf{F} = \frac{q}{c}\mathbf{v} \times \mathbf{H} \qquad \textbf{ZE-2}$$

The components of $\mathbf{F}$ are

$$F_x = \frac{qH}{c}\frac{dx}{dt} \qquad F_y = \frac{qH}{c}\frac{dy}{dt} \qquad F_z = 0 \qquad \textbf{ZE-3}$$

and the equations of motion of the charge become

$$\text{for } x: \quad m\frac{d^2x}{dt^2} = -ax + \frac{qH}{c}\frac{dx}{dt}$$

[1]H. A. Lorentz, *The Theory of Electrons* (London: Macmillan & Co., 1909). This book records the lectures delivered by Lorentz at Columbia University during the spring 1906 term.

$$\text{for } y: \quad m\frac{d^2y}{dt^2} = -ay - \frac{qH}{c}\frac{dy}{dt}$$

$$\text{for } z: \quad m\frac{d^2z}{dt^2} = -az \qquad \textbf{ZE-4}$$

As Lorentz described, solving the z equation yields the original no-field frequency f_0. Solving the x and y equations and noting that $qH/mc \ll f_0$ leads to the two approximate solutions

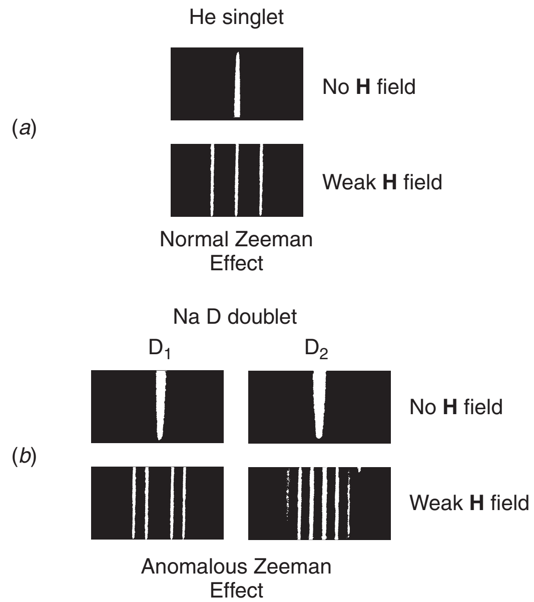
$$f_+ = f_0 + \frac{qH}{mc} \quad \text{and} \quad f_- = f_0 - \frac{qH}{mc} \qquad \textbf{ZE-5}$$

Thinking about this situation a bit more, you may recall that the magnetic force given by Equation ZE-2 is always perpendicular to both **v** and **H** and, therefore, does no work on the charge q. Hence, it does not change the energy (i.e., frequency of rotation) of the charge. However, as the **H** field is applied, there is a $d\mathbf{H}/dt$, which, via Maxwell's second law, *does* result is work being done on the charge as **H** increases from zero to its final value. That work is the energy acquired by the magnetic dipole moment associated with the charge's orbital motion in the magnetic field **H**. A complete classical solution yields, on substituting the electron's charge e for the general charge q,

$$\Delta f = \pm\frac{eH}{4\pi mc} \qquad \textbf{ZE-6}$$

The classical solution "explains" the normal Zeeman effect, which is exhibited by relatively few atoms, but gives no suggestion for an explanation of the anomalous Zeeman effect, which requires electron spin, unknown to Lorentz at the time, for its explanation (see the More section *The Zeeman Effect*).

Measurements of the Zeeman effect have provided a wealth of information on such topics ranging from atomic structure to the magnetic fields of the Sun. However, Zeeman's original application of his discovery, using Lorentz's theoretical explanation and the known values of the speed of light c and his external magnetic field **H**, was the determination of the charge-to-mass ratio e/m for the electron. This was a spectroscopic measurement of the wavelength (i.e., frequency) differences $\pm\Delta\lambda$ between the new spectral lines that appeared with the field "on" and the original line λ_0 with the field "off." His was the first such measurement, preceding Thomson's by about a year. Zeeman's measured value of e/m, about 1.6×10^{11} C/kg, compares well with the currently accepted value of 1.759×10^{11} C/kg.



ZE-1 (*a*) The red line in the He singlet spectrum at 667.8 nm exhibits the normal Zeeman effect when the excited He atoms are placed in a weak magnetic field. (*b*) The yellow Na D-lines at 589.0 nm (D_1) and 589.6 nm (D_2) provide an example of the anomalous Zeeman effect when the excited Na atoms are in a weak magnetic field.