Multipole Radiation

Electromagnetic (EM) radiation is produced by oscillating electric charges for electric radiation and oscillating currents or magnetic moments for magnetic radiation (see Figure MR-1). In general, EM radiation is classified in *multipoles*, expressed in the shorthand notation EL or ML of order 2^L. For example, electric dipole radiation is labeled $E1$, meaning electric radiation of order $2^1 = 2$; electric quadrupole radiation is represented by $E2$, meaning electric radiation of order $2^2 = 4$; electric octupole radiation by $E3$, meaning of order $2^3 = 8$; and so on. By the same token, magnetic radiation of order $2^1 = 2$ is labeled $M1$ and so on. For the multipole of order L, $L\hbar$ units of orbital angular momentum are carried away by the emitted photon. Since the photon has intrinsic angular momentum (spin) of $1\hbar$, there is no monopole $E0$ or $M0$ radiation, the latter also prohibited by the absence of magnetic monopoles.

The concept of multipole radiation arises from the terms of the series (multipole) expansion for the electric and magnetic potentials. For example, for a distribution of electric charges with density $\rho(\mathbf{r})$, the electric potential at a point $\mathbf{R}$ far from the distribution is given by

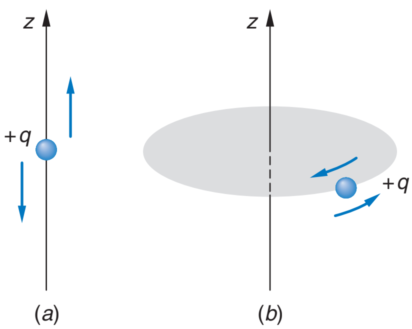
$$\varphi(R) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{R^{n+1}}\int r^n \mathrm{P}_n(\cos\vartheta)\rho(r)\,d\mathbf{r} \qquad \textbf{MR-1}$$

where $\mathrm{P}_n(\cos\vartheta)$ are the Legendre polynomials, $\cos\vartheta = \hat{\mathbf{r}}\cdot\hat{\mathbf{R}}$ (see Figure MR-2), and $n = 2L$. The first term in the expansion is the contribution of the monopole moment (zero), the second term is the contribution of the dipole moment, the third that of the quadrupole moment, and so on. Note that the moments all depend on the location of the origin. Thus, an electric dipole may have a quadrupole moment if the origin selected is not at the center between the charges. Figure MR-3 illustrates some simple electric multipole distributions.

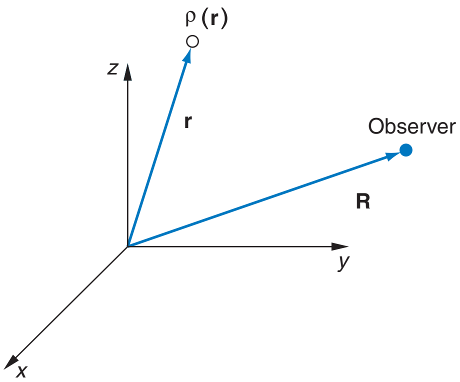
Oscillations of multipole charge distributions generate EM radiation. Generally, one or sometimes two multipoles dominate the emitted radiation. This is related to the relative values of the decay constants λ for successive multipole orders:

$$\frac{\lambda(EL+1)}{\lambda(EL)} \approx \frac{\lambda(ML+1)}{\lambda(ML)} \sim 10^{-3} \qquad \textbf{MR-2}$$

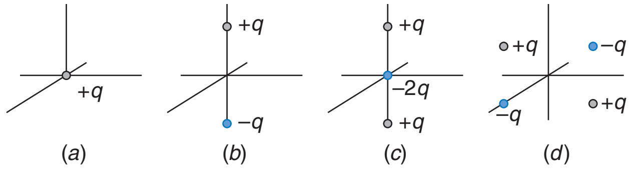
Thus, the decay constant for emission of $E1$ radiation is approximately 1000 times larger than that for $E2$ radiation, which is about 1000 times that for $E3$ radiation, and so on. In addition, the decay constant for emission of $E1$ radiation is about 100 times larger than that for the emission of $M1$ radiation. In both atomic and nuclear EM decays the emission of higher multipole orders becomes important only when $E1$ transitions are forbidden by selection rules.



MR-1 Oscillating (*a*) electric and (*b*) magnetic dipole moments.



MR-2 Relative orientation of the distribution $\rho(\mathbf{r})$ and the observer. $|\mathbf{R} - \mathbf{r}|$ is large.



MR-3 Electric (*a*) monopole, (*b*) dipole, (*c*) linear quadrupole, and (*d*) simple quadrupole.