Energetics of Alpha Decay

The energy released in α decay, Q, is determined by the difference in mass of the parent nucleus and the decay products, which include the daughter nucleus and the α particle. Consider the decay of ^{232}Th ($Z = 90$) into ^{228}Ra ($Z = 88$) plus an α particle. This is written as

$$^{232}\text{Th} \rightarrow {}^{228}\text{Ra} + \alpha \qquad (= {}^{228}\text{Ra} + {}^{4}\text{He}) \tag{11-33}$$

The energy Q is usually expressed in terms of atomic masses (which include the masses of the electrons) because, as explained earlier, these are the masses measured in mass spectroscopy. If M_P is the mass of the parent atom, M_D that of the daughter atom, and M_{He} that of the helium atom, the decay energy Q is given by conservation of mass-energy as

$$\frac{Q}{c^2} = M_P - (M_D + M_{\text{He}}) \tag{11-34}$$

Note that the mass of the two electrons in the He atom compensates for the fact that the daughter atom has two fewer electrons than the parent atom. Applying this to the example given in Equation 11-33, we see that the mass of the ^{232}Th atom is 232.038124 u. The mass of the daughter atom ^{228}Ra is 228.031139 u, and adding it to the 4.002603 u mass of ^{4}He, we get 232.033742 u for the total mass of the decay products. Equation 11-34 then yields $Q/c^2 = 0.004382$ u, which, when multiplied by the conversion factor 931.5 MeV$/c^2$, gives $Q = +4.08$ MeV. Thus, the rest energy of ^{232}Th is greater than that of ^{228}Ra + ^{4}He; therefore, ^{232}Th is unstable toward spontaneous α decay.

The kinetic energy of the α particle (for decays to the ground state of the daughter nucleus) is slightly less than the decay energy Q because of the small recoil energy of the daughter nucleus. If the parent nucleus is at rest when it decays, the daughter nucleus and the α particle must have equal and opposite momenta. If p is the magnitude of the momentum of either particle, the decay energy is

$$Q = \frac{p^2}{2M_D} + \frac{p^2}{2M_{\text{He}}} = \frac{p^2}{2M_{\text{He}}}\left(1 + \frac{M_{\text{He}}}{M_D}\right) \tag{11-35}$$

(Since we are not calculating mass *differences*, it doesn't matter whether we use nuclear masses or atomic masses.) Then, writing E_α for $p^2/2M_{\text{He}}$ and $M_{\text{He}}/M_D = 4/(A - 4)$, where A is the mass number of the parent nucleus, we have

$$E_\alpha = \frac{A - 4}{A} Q \tag{11-36}$$

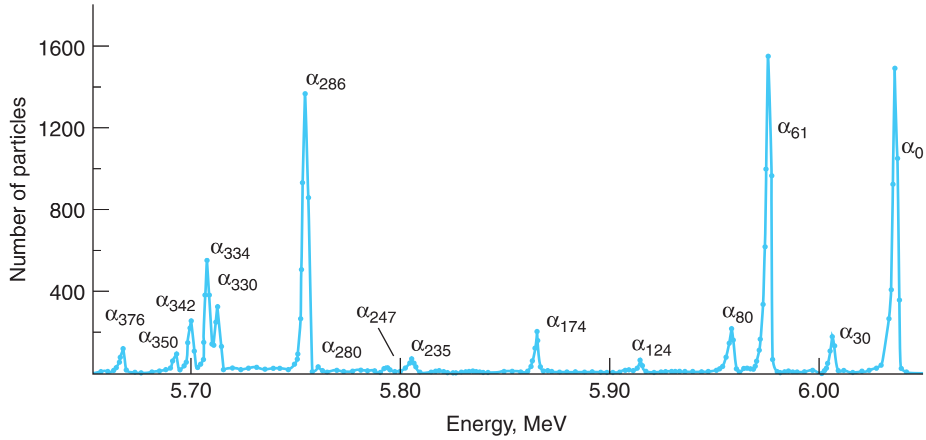

FIGURE 11-19 Alpha-particle spectrum from ^{227}Th. The highest-energy α particles correspond to decay to the ground state of ^{223}Ra with a transition energy of $Q = 6.04$ MeV. The next highest energy particles, α_{30}, result from transitions to the first excited state of ^{223}Ra, 30 keV above the ground state. The energy levels of the daughter nucleus, ^{223}Ra, can be determined by measurement of the α-particle energies.

Since A is much greater than 4 for most nuclides that decay by α decay, E_α is nearly equal to Q. For the ^{232}Th decay discussed above, the α particle carries away about 98 percent of the decay energy Q, or about 4.01 MeV (see Problem 11-25). Note that α decay as illustrated by Equation 11-35 also conserves electric charge and the number of nucleons.

If all the α decays proceeded from the ground state of the parent nucleus to the ground state of the daughter nucleus, the emitted α particles would all have the same energy, related to total energy available Q by Equation 11-36. When the energies of the emitted α particles are measured with high resolution, a spectrum of energies is observed, as shown for the decay of ^{227}Th to ^{223}Ra in Figure 11-19. The peak in the spectrum labeled α_0 corresponds to α decays to the ground state of the daughter nucleus, with total energy of $Q = 6.04$ MeV, as calculated from Equation 11-34. The peak labeled α_{30} indicates α particles with energy 30 keV less than those of maximum energy, indicating that the decay is to an excited state of the daughter nucleus at 30 keV above the ground state. (Unless the parent nucleus was recently produced in a reaction or previous decay, it will be in the ground state. The energy spectrum of α particles then indicates the energy levels in the daughter nucleus as in the case shown in Figure 11-19.) This interpretation of the α-particle energy spectrum is confirmed by the observation of a 30 keV γ ray emitted as the daughter nucleus decays to its ground state. Because the half-life for γ decay is very short, this decay follows essentially immediately after the α decay. Figure 11-20 shows an energy-level diagram for ^{223}Ra obtained from the measurement of the α-particle energies in the decay of ^{227}Th and from the observed γ-decay energies. Only the lowest-lying levels and some of the γ-ray transitions are indicated.

EXAMPLE 11-11 **^{232}Th Decay by Proton Emission?** Show that ^{232}Th, which we have seen to be unstable to α decay, cannot reduce its energy by spontaneously emitting a proton, that is, it is stable for proton emission.

SOLUTION

The emission of a proton would be represented by the following equation:

$$^{232}\text{Th} \rightarrow {}^{231}\text{Ac} + p \qquad (= {}^{231}\text{Ac} + {}^{1}\text{H})$$

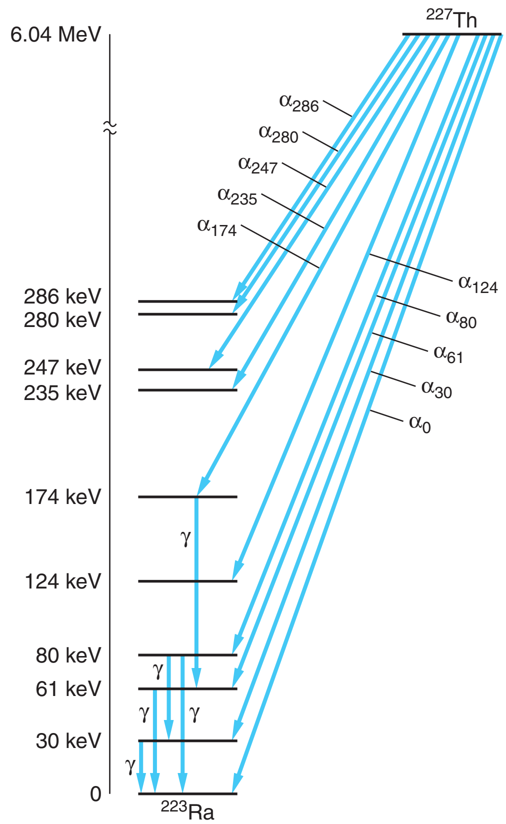

FIGURE 11-20 Energy levels of ^{223}Ra determined by measurement of α-particle energies from ^{227}Th, as shown in Figure 11-19. Only the lowest-lying levels and some of the γ-ray transitions are shown.

This process would be similar to decay, with M_H replacing M_{He} in Equation 11-34. Thus,

$$
\begin{aligned}
Q/c^2 &= M_{Th} - (M_{Ac} + M_H) \\
&= 232.038051 \text{ u} - (231.038551 + 1.007825) \text{ u} \\
&= -0.007825 \text{ u} \\
&= -7.29 \text{ MeV}/c^2
\end{aligned}
$$

and we then have $Q = -7.29$ MeV. Thus, for proton decay of ^{232}Th the mass of the decay products is larger than that of the parent atom, so spontaneous decay via that mode is forbidden by energy conservation.