Resonances and Excited States

Particles that are unstable against decay by the strong interaction have mean lives of the order of 10^{-23} s and therefore cannot be detected by ordinary means. For example, if such a particle moves with nearly the speed of light, it can travel a distance of only about $r = c\tau = (3 \times 10^8 \text{ m/s})(10^{-23} \text{ s}) = 3 \times 10^{-15} \text{ m} = 3 \text{ fm}$ (about the diameter of a nucleus), far too short to leave a track in a spark or time projection chamber. The existence of such particles is inferred from *resonances* in the scattering cross sections of one hadron on another or from the energy distribution of nuclear reaction products. The first process is the nuclear analog of the atomic Franck-Hertz effect described in Section 4-5.

Figure 12-14 shows the cross section versus energy for the scattering of π^+ and π^- mesons by protons. There is a strong resonance in the cross section at a π^+ kinetic energy of 195 MeV (in the laboratory frame). This corresponds to a total energy in the center-of-mass frame (including the rest energies of the π^+ and p) of 1232 MeV. The width of this resonance in the CM frame is about 100 MeV, which corresponds to a lifetime of the state of the order of $\tau = \hbar/\Delta E \approx 10^{-23}$ s. Despite the brief lifetime of this resonance state, such a state is now considered to be a particle that is in many ways as fundamental as those in Table 12-3, which are stable against hadronic decay. The particle is designated as $\Delta^{++}(1232)$. It has zero strangeness, since both p and π have zero strangeness. Furthermore, the isospin is $3/2$ since $I = 1/2$ and $I_3 = +1/2$ for the proton and $I = 1$ and $I_3 = +1$ for the π^+. The spin and parity can be inferred from angular distribution measurements. The $\Delta(1232)$ is an excited state of the nucleon, one of nine Δ resonances, all of which have in common $S = 0$ and $I = 3/2$.

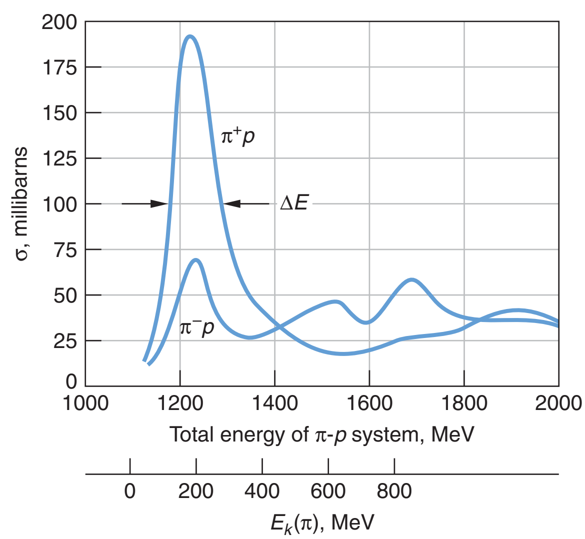

FIGURE 12-14 Cross section for scattering of π^+ and π^- mesons by protons. The resonance at a pion energy of 195 MeV, corresponding to a total center-of-mass energy (including rest energy) of 1232 MeV, indicates the existence of a new particle called the Δ particle. Other resonances in the $\pi^- + p$ scattering indicate other particles of greater rest energy. ΔE is the width of the curve at half the peak height.

Figure 12-14 also shows the cross section for the scattering of π^- on protons. The resonance at total CM energy of 1232 MeV also shows in this experiment, but its cross section is not nearly as great as that of (π^+, p) scattering. There are also additional resonances in the (π^-, p) cross section not seen in the (π^+, p) scattering. This is because the (π^-, p) system is a mixture of isospin states. Since $I = 1$ for the pion and $1/2$ for the nucleon, a system of pion plus nucleon can have either $I = 3/2$ or $I = 1/2$. Since $I_3 = +1$ for the π^+ and $I_3 = +1/2$ for the p, the (π^+, p) resonance can only have $I = 3/2$. However, $I_3 = -1$ for the π^-, so that (π^-, p) is a mixture of both $I = 3/2$ and $I = 1/2$. The resonance with $I = 3/2$ is the $\Delta(1232)$. The resonances at total CM energy of 1520 MeV and 1675 MeV are designated N because they have the same isospin and strangeness as the nucleon. The 17 N resonances that have been found to date all have $S = 0$ and $I = 1/2$ and, like the Δ resonances, are considered to be excited states of the nucleon. The $\Delta(1232)$ resonance was the first such particle found. It was discovered by Fermi and his coworkers in 1951 and is referred to as the *Fermi resonance.*

Another method of detection is based on the dependence of the energy distribution of decay particles on the number of particles. Consider the decay $A \longrightarrow B + C$. Suppose that we detect only the particle C. As we have previously discussed, conservation of energy and momentum imply a unique energy for particle C. On the other hand, if this decay involves three particles in the final state such as $A \longrightarrow B + C + D$, we expect a distribution of energies for particle C as shown in Figure 12-15*a,* which can be calculated from statistics. If the decays proceed via a two-step process such as

$$A \longrightarrow B + E$$

$$\text{followed by} \quad E \longrightarrow C + D \qquad \textbf{12-28}\boldsymbol{a}$$

then the energy of particle C will be unique, as shown in Figure 12-15*b,* since E and then C are both products of two-particle decays. If, however, some of the decays are three-particle events while others proceed in two steps like Equation 12-28*a*, then the energy distribution of particle C will look like Figure 12-15*c*. Figure 12-16 shows the energy distribution of the π^+ observed in the reaction

$$K^- + p \longrightarrow \pi^+ + \pi^- + \Lambda^0 \qquad \textbf{12-28}\boldsymbol{b}$$

(This is just part of a very complicated analysis of this reaction.) The solid curve is that expected if there are three particles in the final state. The large peak at kinetic energy of about 300 MeV implies the existence of an unseen particle with a very short

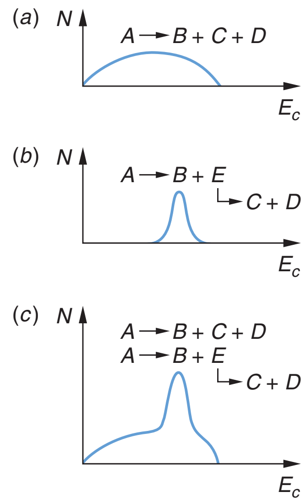

FIGURE 12-15 Energy distribution of particle C if (a) the reaction yields three particles $A \longrightarrow B + C + D$, ($b$) the reaction yields two particles, one of which then decays, as in $A \longrightarrow B + E$, $E \longrightarrow C + D$, or (c) the reaction sometimes yields three particles, as in (a), and sometimes two particles, as in (b). The existence of an unseen particle E can be inferred from a peak in the energy distribution as in (b) or (c).

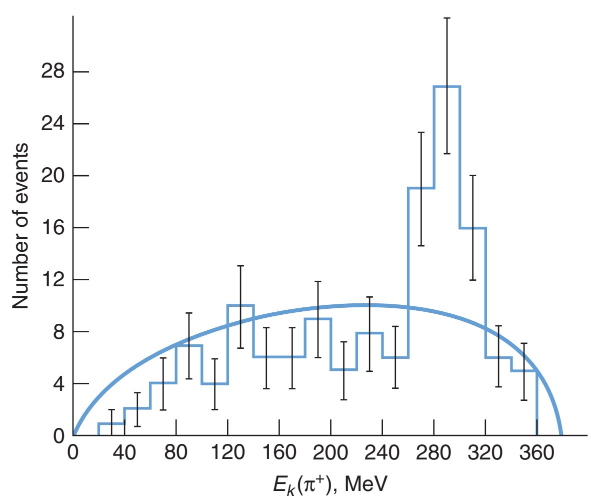

FIGURE 12-16 Kinetic energy distribution of the π^+ mesons in the reaction $\kappa^- + p \longrightarrow \pi^- + \pi^+ + \Lambda^0$. The solid curve is that expected from statistics if there are three particles in the final state. The peak indicates that the reaction proceeds in two steps, either $\kappa^- + p \longrightarrow \pi^+ + \Sigma^-$ followed by $\Sigma^- \longrightarrow \pi^- + \Lambda^0$ or $\kappa^- + p \longrightarrow \pi^- + \Sigma^+$; $\Sigma^+ \longrightarrow \pi^+ + \Lambda^0$. The rest energy of the $\Sigma^\pm$ resonance particles is 1385 MeV.

lifetime. The reaction observed is actually a mixture of two reactions, each of which proceeds in two steps. These reactions are

$$K^- + p \rightarrow \pi^+ + \Sigma^-$$

$$\text{followed by} \quad \Sigma^- \rightarrow \pi^- + \Lambda^0 \qquad \textbf{12-28}\boldsymbol{c}$$

and

$$K^- + p \rightarrow \pi^- + \Sigma^+$$

$$\text{followed by} \quad \Sigma^+ \rightarrow \pi^+ + \Lambda^0 \qquad \textbf{12-28}\boldsymbol{c}$$

In Equations 12-28*a* and *b* the second-step pions correspond to the particle *C* in the above discussion. The $\Sigma\pm$ are the unseen particle *E*. The energy balance yields its mass. Table 12-8 lists some of the mesons and baryons that are unstable to decay via the strong interaction. Many more have been discovered, some of whose properties are not yet completely established.

EXAMPLE 12-9 Lifetime of Λ(1520) From the width of the resonance of the Λ(1520) given in Table 12-8, estimate the lifetime of the particle.

SOLUTION

The width (ΔE) tabulated in the table is the full width at half of the maximum height of the Λ(1520) resonance curve similar to that of the Δ shown in Figure 12-14. From the table the value is 16 MeV. The lifetime of the Λ(1520) is then given by

$$\tau = \frac{\hbar}{\Delta E} = \frac{1.055 \times 10^{-34}\,\mathrm{J \cdot s}}{(16\,\mathrm{MeV})(1.60 \times 10^{-13}\,\mathrm{J/MeV})} = 4.1 \times 10^{-23}\mathrm{s}$$

EXAMPLE 12-10 Decay of the Σ^+ (2030) The positively charged Σ(2030) listed in Table 12-8 decays according to the reaction $\Sigma^+ \rightarrow N^+ + \overline{K}^0$. Which of the *N* resonances in the table can be reached by this decay? What will be the combined energies of the N^+ and $\overline{K}^0$ in each case?

SOLUTION

The mass of the Σ(2030) is 2030 MeV$/c^2$ and that of the $\overline{K}^0$ (from Table 12-3) is 498 MeV$/c^2$. Thus, any *N* resonance can be reached whose mass is given by

$$\text{Mass}(N) \leq \text{mass}(\Sigma) - \text{mass}(\overline{K}^0)$$

$$\text{Mass}(N) \leq 2030 - 498 = 1532\,\mathrm{MeV}/c^2$$

Only one *N* resonance in Table 12-8 has a mass less than this amount, the *N* (1470). The combined kinetic energy of the decay products is then

$$E_k = 2030\,\mathrm{MeV} - 1470\,\mathrm{MeV} = 560$$

Table 12-8 A partial list of resonance particles

Particle	Mass (MeV/c^2)	Width (MeV)	Primary decay mode	T	B	S	$J^{P\dagger}$
Meson resonances							
$\rho(770)$	770	153	$\pi\pi$	1	0	0	1^-
$\omega(783)$	783	10	$\pi^+\pi^-\pi^0$	1	0	0	1^-
$\omega(1670)$	1666	166	$\rho\pi$	0	0	0	3^-
$J/\psi(3100)$	3097	0.06	Hadrons	0	0	0	1^-
$K^*(890)$	892	51	$K\pi$	$\frac{1}{2}$	0	+1	1^-
$K^*(1420)$	1425	100	$K\pi$	$\frac{1}{2}$	0	+1	3^-
Baryon resonances							
$\Delta(1232)$	1232	120	$N\pi$	$\frac{3}{2}$	1	0	$\frac{3}{2}^+$
$\Delta(1620)$	1620	140	$N\pi\pi$	$\frac{3}{2}$	1	0	$\frac{1}{2}^-$
$\Delta(1700)$	1685	250	$N\pi\pi$	$\frac{3}{2}$	1	0	$\frac{3}{2}^-$
$N(1470)$	1470	300	$N\pi$	$\frac{1}{2}$	1	0	$\frac{1}{2}^+$
$N(1670)$	1670	160	$N\pi\pi$	$\frac{1}{2}$	1	0	$\frac{5}{2}^-$
$N(1688)$	1688	145	$N\pi$	$\frac{1}{2}$	1	0	$\frac{5}{2}^+$
$\Lambda(1405)$	1405	40	$\Sigma\pi$	0	1	−1	$\frac{1}{2}^-$
$\Lambda(1520)$	1520	16	$N\overline{K}$	0	1	−1	$\frac{3}{2}^-$
$\Lambda(1670)$	1670	30	$N\overline{K}$	0	1	−1	$\frac{1}{2}^-$
$\Sigma(1385)$	1382	35	$\Lambda\pi$	1	1	−1	$\frac{3}{2}^-$
$\Sigma(1670)$	1670	50	$\Sigma\pi$	1	1	−1	$\frac{3}{2}^-$
$\Sigma(2030)$	2030	175	$N\overline{K}$	1	1	−1	$\frac{7}{2}^+$
$\Xi(1530)$	1532	9	$\Xi\pi$	$\frac{1}{2}$	1	−2	$\frac{3}{2}^+$
$\Xi(1820)$	1823	30	$\Lambda\overline{K}$	$\frac{1}{2}$	1	−2	$\frac{3}{2}^?$
$\Xi(2030)$	2030	20	$\Sigma\overline{K}$	$\frac{1}{2}$	1	−2	?