Perihelion of Mercury's Orbit

A third prediction from Einstein's theory of general relativity is the excess precession of the perihelion of the orbit of Mercury of about 0.01° per century. This effect had been known and unexplained for some time, so in some sense its correct explanation represented an immediate success of the theory. It is a remarkable feature of Newton's law of gravity that it accounts for the orbits of the planets in detail, predicting that they should each be *closed* ellipses with the Sun located at one focus and the major axes always pointing in the same direction in space. In fact, however, the mutual gravitational interactions of the planets add a small time-varying force to the primary force of the Sun. The result is that the orbital motions are not quite closed ellipses but rotate slowly in the plane of the orbit; that is, the major axis of the ellipse slowly rotates about the Sun, as shown in Figure 2-21. In the absence of the mutual interactions of the planets, Newton's gravitational theory predicts the orbit to be a perfect ellipse, the distance r of the planet from the Sun being given by

$$r = r_{\min}\frac{1 + \varepsilon}{1 + \varepsilon \cos \phi} \qquad \textbf{2-55}$$

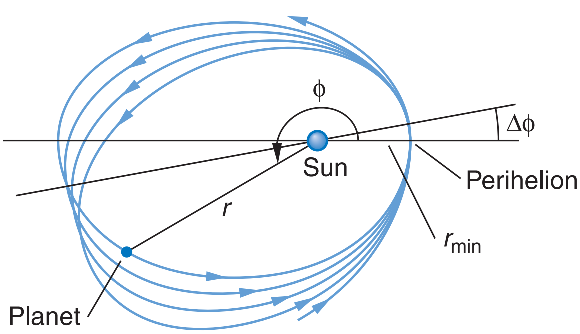

FIGURE 2-24 The elliptical planetary orbits have the Sun at one focus. The smaller gravitational forces due to the other planets cause the major axis to slowly rotate about the Sun, shifting the line from the Sun to the perihelion through an angle $\Delta\varphi$ each orbit. This shift is referred to as the precession of the perihelion. For Mercury, $\Delta\varphi = 9.55$ arc minutes per century.

where $r_{\min}$ = distance to the point of closest approach of the planet to the Sun, called the *perihelion*, and ε = eccentricity of the orbit, meaning that the maximum and minimum distances from the Sun are ε more or less, respectively, than the mean distance. ($\varepsilon = 0$ for a circle.) The eccentricities of the orbits of several planets are given in Table 2-2.

The rotation is described in terms of the progressive change in the direction of $r_{\min}$ in Figure 2-24 and is referred to as the *precession of the perihelion*. Newton's law of gravity permits the calculation of the interaction between every pair of planets, allowing these small effects on each planet's motion to be subtracted away, a process that should leave each planet with a closed, nonrotating elliptical orbit. However, the remarkable precision of astronomical measurements showed that this did not happen. In the case of Mercury, for example, the observed precession of the perihelion is 9.55 arc minutes/century, but, the calculated precision due to mutual interaction

Table 2-2 Precession of Planetary Orbits

Planet	n (orbits per century)	ε	r_{min} (AU)*	$n\,\Delta\varphi$ (arc seconds/century) General relativity	Observed
Mercury	415.2	0.206	0.307	43.0	43.1 ± 0.5
Venus	162.5	0.0068	0.717	8.6	8.4 ± 4.8
Earth	100.0	0.017	0.981	3.8	5.0 ± 1.2
Icarus**	89.3	0.827	0.186	10.0	9.8 ± 0.8

*Astronomical unit (AU) = mean Earth-Sun distance = 1.50×10^{11} m.
**Icarus is one of several thousand minor planets, or asteroids. It is included here because the perihelion of its orbit lies inside Mercury's orbit, closer to the Sun than any other asteroid.

with the other planets amounts to only 8.85 arc minutes/century.[21] This leaves a discrepancy of about 0.7 arc minute or 42 arc seconds/century. (The observed discrepancy is 43.1 arc seconds/century. See Table 2-2.)

The existence of the discrepancy was known before Einstein developed general relativity, and he ended his first paper on the general theory by explaining the origin of and correctly calculating the discrepancy.[22] General relativity modified Equation 2-49 to include $\Delta\varphi$:

$$r = r_{min}\frac{1 + \varepsilon}{1 + \varepsilon\cos(\varphi - \Delta\varphi)} \tag{2-56}$$

and predicted that $\Delta\varphi$, the precession per revolution, would be given by

$$\Delta\varphi = \frac{6\pi GM}{c^2(1 - \varepsilon^2)R} \tag{2-57}$$

where R equals the semi-major axis of the orbit and M is the mass of the central object, the Sun in this case.

As you might expect, Mercury, with the smallest orbit among the planets (i.e., the smallest R) and largest eccentricity, would show the largest relativistic $\Delta\varphi$. Substituting the appropriate values into Equation 2-51 yields $\Delta\varphi = 43.0$ arc seconds/century, in nearly perfect agreement with observation. (Table 2-2 includes the results for a few other planets too.) Einstein was so elated at the result that he wrote to Arnold Sommerfeld,

> The wonderful thing that happened was that not only did Newton's theory result from it [general relativity] *as a first approximation*, but also the perihelion motion of Mercury, *as a second approximation*.