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CHAPTER

31

# Statistical Process Control

## In this chapter, we cover...



Organizations are (or ought to be) concerned about the quality of the products and services they offer. A key to maintaining and improving quality is systematic use of *data* in place of intuition or anecdotes. In the words of Stan Sigman, former CEO of Cingular Wireless, "What gets measured gets managed."[1]

Because using data is a key to improving quality, statistical methods have much to contribute. Simple tools are often the most effective. A scatterplot and perhaps a regression line can show how the time to answer telephone calls to a corporate call center influences the percent of callers who hang up before their calls are answered. The design of a new product as simple as a multivitamin tablet may involve interviewing samples of consumers to learn what vitamins and minerals they want included and using randomized comparative experiments in designing the manufacturing process. An experiment might discover, for example, what combination of moisture level in the raw vitamin powder and pressure in the tablet-forming press produces the right tablet hardness.

Quality is a vague idea. You may feel that a restaurant serving filet mignon is a higher-quality establishment than a fast-food outlet that serves hamburgers. For statistical purposes, we need a narrower concept: *consistently meeting standards appropriate for a specific product or service.* By this definition of quality, the expensive restaurant may serve low-quality filet mignon, whereas the fast-food outlet serves high-quality hamburgers. The hamburgers are freshly grilled, are served at the right temperature, and are the same every time you visit. Statistically minded



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management can assess quality by sampling hamburgers and measuring the time from order to being served, the temperature of the burgers, and their tenderness.

This chapter focuses on just one aspect of statistics for improving quality: *statistical process control.* The techniques are simple and are based on sampling distributions (Chapters 15 and 20), but the underlying ideas are important and a bit subtle.

## 31.1 Processes

In thinking about statistical inference, we distinguish between the *sample* data we have in hand and the wider *population* that the data represent. We hope to use the sample to draw conclusions about the population. In thinking about quality improvement, it is often more natural to speak of *processes* rather than populations. This is because work is organized in processes. Some examples are

- Processing an application for admission to a university and deciding whether or not to admit the student.
- Reviewing an employee's expense report for a business trip and issuing a reimbursement check.
- Hot forging to shape a billet of titanium into a blank that, after machining, will become part of a medical implant for hip, knee, or shoulder replacement.

Each of these processes is made up of several successive operations that eventually produce the output—an admission decision, reimbursement check, or metal component.

### Process

A **process** is a chain of activities that turns inputs into outputs.

We can accommodate processes in our sample-versus-population framework: think of the population as containing all the outputs that would be produced by the process if it ran forever in its present state. The outputs produced today or this week are a sample from this population. Because the population doesn't actually exist now, it is simpler to speak of a process and of recent output as a sample from the process in its present state.

## 31.2 Describing Processes

*flowchart*

*cause-and-effect diagram*

The first step in improving a process is to understand it. Process understanding is often presented graphically using two simple tools: flowcharts and cause-and-effect diagrams. A **flowchart** is a picture of the stages of a process where the steps in the process are represented by various types of boxes that are connected by arrows to show the sequence of the process. A **cause-and-effect diagram** organizes the logical relationships between the inputs and stages of a process and an output. They are often used during brainstorm sessions when trying to determine the cause of a quality problem or issue. Sometimes the output is successful completion of the process task; sometimes it is a quality problem that we hope to solve. A good starting outline for a cause-and-effect diagram appears in Figure 31.1. The main branches organize the causes and serve as a skeleton for detailed entries. You can see why these are sometimes called "fishbone diagrams." An example will illustrate the use of these graphs.[2]

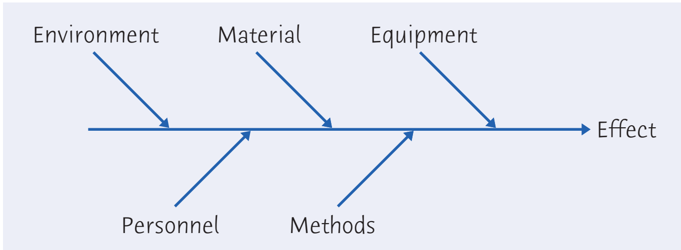



**FIGURE 31.1**
An outline for a cause-and-effect diagram. To complete the diagram, group causes under these main headings in the form of branches.

## EXAMPLE 31.1 Hot Forging

Hot forging involves heating metal to a plastic state and then shaping it by applying thousands of pounds of pressure to force the metal into a die (a kind of mold). Figure 31.2 is a flowchart of a typical hot-forging process.[3]

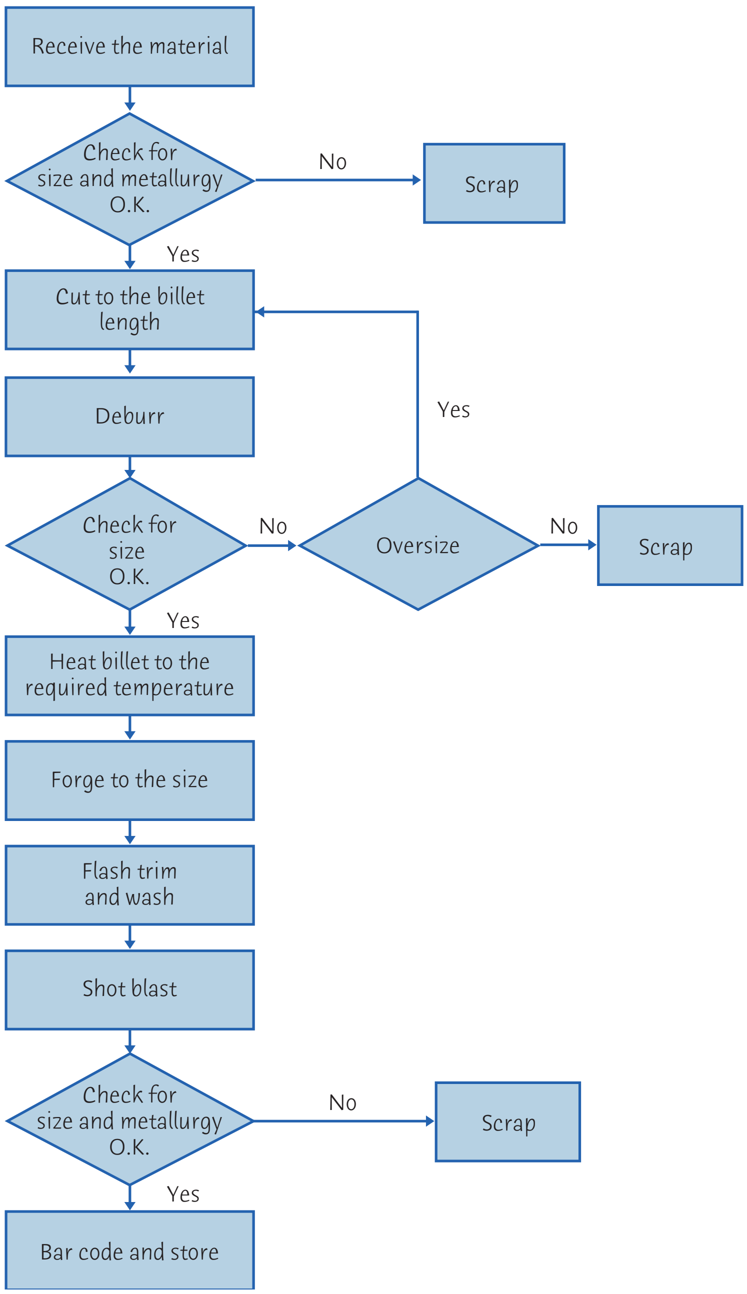



**FIGURE 31.2**
Flowchart of the hot-forging process, for Example 31.1. Use this as a model for flowcharts: decision points appear as diamonds and other steps in the process appear as rectangles. Arrows represent flow from step to step.

A process improvement team, after making and discussing this flowchart, came to several conclusions:

- Inspecting the billets of metal received from the supplier adds no value. We should insist that the supplier be responsible for the quality of the material. The supplier should put in place good statistical process control. We can then eliminate the inspection step.
- Can we buy the metal billets already cut to rough length and ground smooth by the supplier, thus eliminating the cost of preparing the raw material ourselves?
- Heating the metal billet and forging (pressing the hot metal into the die) are the heart of the process. We should concentrate our attention here.

The team then prepared a cause-and-effect diagram (Figure 31.3) for the heating and forging part of the process. The team members shared their specialist knowledge of the causes in their areas, resulting in a more complete picture than any one person could produce. Figure 31.3 is a simplified version of the actual diagram. We have given some added detail for the "Hammer stroke" branch under "Equipment" to illustrate the next level of branches. Even this requires some knowledge of hot forging to understand. Based on detailed discussion of the diagram, the team decided what variables to measure and at what stages of the process to measure them. Producing well-chosen data is the key to improving the process.

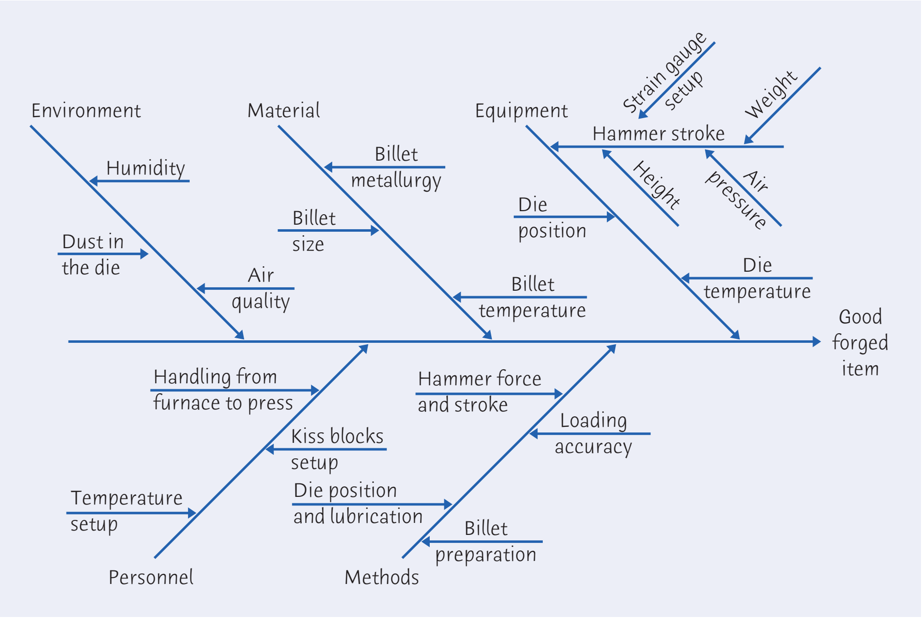



**FIGURE 31.3** Simplified cause-and-effect diagram of the hot-forging process, for Example 31.1. Good cause-and-effect diagrams require detailed knowledge of the specific process.

We will apply statistical methods to a series of measurements made on a process. Deciding what specific variables to measure is an important step in quality improvement. Often, we use a "performance measure" that describes an output of a process. A company's financial office might record the percent of errors that outside auditors find in expense account reports or the number of data entry errors per week. The personnel department may measure the time to process employee insurance claims or the percent of job offers that are accepted. In the case of complex processes, it is wise to measure key steps within the process rather than just final outputs. The process team in Example 31.1 might recommend that the temperature of the die and of the billet be measured just before forging.

## APPLY YOUR KNOWLEDGE

**31.1 Describe a Process.** Choose a process that you know well. If you lack experience with actual business or manufacturing processes, choose a personal process such as ordering something over the Internet, paying a bill online, or recording a TV show on a DVR. Make a flowchart of the process. Make a cause-and-effect diagram that presents the factors that lead to successful completion of the process.

**31.2 Describe a Process.** Each weekday morning, you must get to work or to your first class on time. Make a flowchart of your daily process for doing this, starting when you wake. Be sure to include the time at which you plan to start each step.

**31.3 Process Measurement.** Based on your description of the process in Exercise 31.1, suggest specific variables that you might measure to

(a) assess the overall quality of the process.

(b) gather information on a key step within the process.

*Pareto charts*

**31.4 Pareto Charts.** **Pareto charts** are bar graphs with the bars ordered by height. They are often used to isolate the "vital few" categories on which we should focus our attention. Here is an example. A large medical center, financially pressed by restrictions on reimbursement by insurers and the government, looked at losses broken down by diagnosis. Government standards place cases into diagnostic related groups (DRGs). For example, major joint replacements (mostly hip and knee) are DRG 209.[4] Here is what the hospital found: DRG

| DRG | Percent of Losses |
|---|---|
| 104 | 5.2 |
| 107 | 10.1 |
| 109 | 7.7 |
| 116 | 13.7 |
| 148 | 6.8 |
| 209 | 15.2 |
| 403 | 5.6 |
| 430 | 6.8 |
| 462 | 9.4 |

What percent of total losses do these nine DRGs account for? Figure 31.4 is a Pareto chart of losses by DRG. Which DRGs should the hospital study first when attempting to reduce its losses?

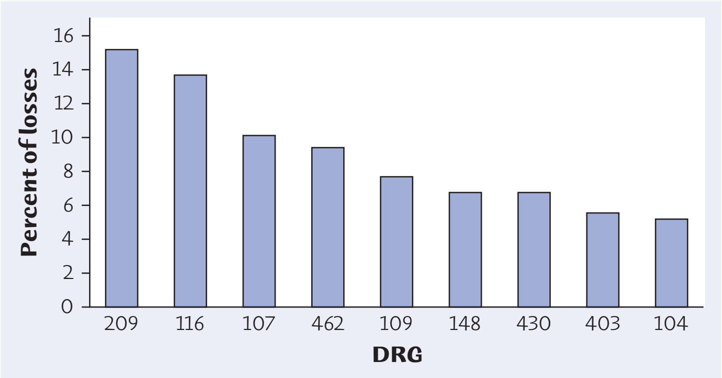



**FIGURE 31.4** Pareto chart of losses by DRG, for Exercise 31.4.

**31.5 Pareto Charts, continued.** Continue the study of the process of getting to work or class on time from Exercise 31.2. If you kept good records, you could make a Pareto chart of the reasons (special causes) for late arrivals at work or class. Make a Pareto chart that you think roughly describes your own reasons for lateness. That is, list the reasons from your experience and chart your estimates of the percent of late arrivals each reason explains.

**31.6 Pareto Charts.** In the 2014 midterm elections, voter turnout was exceptionally low. Only 41.9% of eligible voters (citizens who are at least 18 years old) cast ballots. The Census Bureau conducted a survey to determine the reasons people have for not voting.[5] Here is what the Census Bureau found: VOTE

| Reason for Not Voting | Percent of Reasons for Not Voting |
|---|---|
| Bad weather conditions | 0.4 |
| Did not like candidate or campaign issues | 7.6 |
| Don't know or refused to answer | 2.9 |
| Forgot to vote | 8.3 |
| Illness or disability | 10.8 |
| Inconvenient polling place | 2.3 |
| Not interested | 16.4 |
| Other reason | 9.1 |
| Out of town | 9.5 |
| Registration problems | 2.4 |
| Too busy, conflicting schedule | 28.2 |
| Transportation problems | 2.1 |

For what percent of the total do the top eight reasons account? Make a Pareto chart of the Percent of Reasons for Not Voting. To increase voter turnout, what are some things that might be done?

## 31.3 The Idea of Statistical Process Control

The goal of statistical process control is to make a process stable over time and then keep it stable unless planned changes are made. You might want, for example, to keep your weight constant over time. A manufacturer of machine parts wants the critical dimensions to be the same for all parts. "Constant over time" and "the same for all" are not realistic requirements. They ignore the fact that *all processes have variation.* Your weight fluctuates from day to day; the critical dimension of a machined part varies a bit from item to item; the time to process a college admission application is not the same for all applications. Variation occurs in even the most precisely made product due to small changes in the raw material, the adjustment of the machine, the behavior of the operator, and even the temperature in the plant. Because variation is always present, we can't expect to hold a variable exactly constant over time. The statistical description of stability over time requires that the *pattern of variation* remain stable, not that there be no variation in the variable measured.

## Statistical Control

A variable that continues to be described by the same distribution when observed over time is said to be in statistical control, or simply **in control**.

**Control charts** are statistical tools that monitor a process and alert us when the process has been disturbed so that it is now **out of control**. This is a signal to find and correct the cause of the disturbance.

In the language of statistical quality control, a process that is in control has only **common cause** variation. Common cause variation is the inherent variability of the system, due to many small causes that are always present. When the normal functioning of the process is disturbed by some unpredictable event, **special cause** variation is added to the common cause variation. We hope to be able to discover what lies behind special cause variation and eliminate that cause to restore the stable functioning of the process.

***common cause***

***special cause***

## EXAMPLE 31.2 Common Cause, Special Cause

Imagine yourself doing the same task repeatedly—say, folding an advertising flyer, stuffing it into an envelope, and sealing the envelope. The time to complete the task will vary a bit, and it is hard to point to any one reason for the variation. Your completion time shows only common cause variation.

Now the telephone rings. You answer, and though you continue folding and stuffing while talking, your completion time rises beyond the level expected from common causes alone. Answering the telephone adds special cause variation to the common cause variation that is always present. The process has been disturbed and is no longer in its normal and stable state.

If you are paying temporary employees to fold and stuff advertising flyers, you avoid this special cause by not having telephones present and by asking the employees to turn off their cell phones while they are working.

Control charts work by distinguishing the always-present common cause variation in a process from the additional variation that suggests that the process has been disturbed by a special cause. A control chart sounds an alarm when it sees too much variation. The most common application of control charts is to monitor the performance of industrial and business processes. The same methods, however, can be used to check the stability of quantities as varied as the ratings of a television show, the level of ozone in the atmosphere, and the gas mileage of your car. Control charts combine graphical and numerical descriptions of data with use of sampling distributions.

## APPLY YOUR KNOWLEDGE

**31.7 Special Causes.** John recently lost weight, and during this time, he charted the number of calories consumed each day. His calorie consumption varied each day but was generally stable. There were some days when his calorie count was unusual. Sometimes his calorie intake was much higher and sometimes it was much lower than expected. Give several examples of special causes that might significantly increase or decrease John's calorie consumption on a given day.

**31.8 Common Causes, Special Causes, continued.** In Exercise 31.1, you described a process that you know well. What are some sources of common cause variation in this process? What are some special causes that might at times drive the process out of control?

**31.9 Common Causes, Special Causes, continued.** As described in Exercise 31.2, each weekday morning, you must get to work or to your first class on time. The time at which you reach work or class varies from day to day, and your planning must allow for this variation. List several common causes of variation in your arrival time. Then list several special causes that might result in unusual variation leading to either early or (more likely) late arrival.

## 31.4 $\bar{x}$ Charts for Process Monitoring

*chart setup*

*process monitoring*

When you first apply control charts to a process, the process may not be in control. Even if it is in control, you don't yet understand its behavior. You will have to collect data from the process, establish control by uncovering and removing special causes, and then set up control charts to maintain control. We call this the **chart setup** stage. Later, when the process has been operating in control for some time, you understand its usual behavior and have a long run of data from the process. You keep control charts to monitor the process because a special cause could erupt at any time. We will call this **process monitoring**.[6]

Although, in practice, chart setup precedes process monitoring, the big ideas of control charts are more easily understood in the process-monitoring setting. We will start there, then discuss the more complex chart setup setting.

Choose a quantitative variable $x$ that is an important measure of quality. The variable might be the diameter of a part, the number of envelopes stuffed in an hour, or the time to respond to a customer call. Here are the conditions for process monitoring.

### Process-Monitoring Conditions

Measure a quantitative variable $x$ that has a **Normal distribution**. The process has been operating in control for a long period, so that we know the **process mean** $\mu$ and the **process standard deviation** $\sigma$ that describe the distribution of $x$ as long as the process remains in control.

In practice, we must, of course, estimate the process mean and standard deviation from past data on the process. Under the process-monitoring conditions, we have very many observations, and the process has remained in control. The law of large numbers tells us that estimates from past data will be very close to the truth about the process. That is, at the process-monitoring stage, we can act as if we know the true values of $\mu$ and $\sigma$. *Note carefully that $\mu$ and $\sigma$ describe the center and variability of the variable $x$ only as long as the process remains in control.* A special cause may at any time disturb the process and change the mean, the standard deviation, or both.

To make control charts, begin by taking small samples from the process at regular intervals. For example, we might measure four or five consecutive parts or time the responses to four or five consecutive customer calls. There is an important idea here: *the observations in a sample are so close together that we can assume that the process is stable during this short period of time.* Variation within the same sample gives us a benchmark for the common cause variation in the process. *The process standard deviation $\sigma$ refers to the standard deviation within the time period spanned by one sample.* If the process remains in control, the same $\sigma$ describes the standard deviation of observations across any time period. Control charts help us decide whether this is the case.

We start with the **$\bar{x}$ chart** based on plotting the means of the successive samples. Here is the outline:

***$\bar{x}$ chart***

1. **Take samples** of size $n$ from the process at regular intervals. **Plot the means** $\bar{x}$ of these samples against the order in which the samples were taken.
2. We know that the sampling distribution of $\bar{x}$ under the process-monitoring conditions is Normal with mean $\mu$ and standard deviation $\sigma/\sqrt{n}$ (see page 359). Draw a solid **center line** on the chart at height $\mu$.
3. The 99.7 part of the 68–95–99.7 rule for Normal distributions (page 82) says that, as long as the process remains in control, 99.7% of the values of $\bar{x}$ will fall between $\mu - 3\sigma/\sqrt{n}$ and $\mu + 3\sigma/\sqrt{n}$. Draw dashed **control limits** on the chart at these heights. The control limits mark off the range of variation in sample means that we expect to see when the process remains in control.

***center line***

***control limits***

If the process remains in control and the process mean and standard deviation do not change, we will rarely observe an $\bar{x}$ outside the control limits. Such an $\bar{x}$ is therefore a signal that the process has been disturbed.

## EXAMPLE 31.3 Manufacturing Computer Monitors

DATA

**MONITRS**

A manufacturer of computer monitors must control the tension on the mesh of fine vertical wires that lies behind the surface of the viewing screen. Too much tension will tear the mesh and too little will allow wrinkles. Tension is measured by an electrical device with output readings in millivolts (mV). The manufacturing process has been stable with mean tension $\mu = 275$ mV and process standard deviation $\sigma = 43$ mV.

The mean 275 mV and the common cause variation measured by the standard deviation 43 mV describe the stable state of the process. If these values are not satisfactory—for example, if there is too much variation among the monitors—the manufacturer must make some fundamental change in the process. This might involve buying new equipment or changing the alloy used in the wires of the mesh. In fact, the common cause variation in mesh tension does not affect the performance of the monitors. We want to watch the process and maintain its current condition.

The operator measures the tension on a sample of four monitors each hour. Table 31.1 gives the last 20 samples. The table also gives the mean $\bar{x}$ and the standard deviation $s$ for each sample. The operator did not have to calculate these—modern measuring equipment often comes equipped with software that automatically records $\bar{x}$ and $s$ and even produces control charts. Figure 31.5(a) is a plot of the sample means versus sample number.

**TABLE 31.1 Twenty control chart samples of mesh tension (in millivolts)**

| Sample | Tension Measurements | | | | Sample Mean | Standard Deviation |
|---|---|---|---|---|---|---|
| 1 | 234.5 | 272.3 | 234.5 | 272.3 | 253.4 | 21.8 |
| 2 | 311.1 | 305.8 | 238.5 | 286.2 | 285.4 | 33.0 |
| 3 | 247.1 | 205.3 | 252.6 | 316.1 | 255.3 | 45.7 |
| 4 | 215.4 | 296.8 | 274.2 | 256.8 | 260.8 | 34.4 |
| 5 | 327.9 | 247.2 | 283.3 | 232.6 | 272.8 | 42.5 |
| 6 | 304.3 | 236.3 | 201.8 | 238.5 | 245.2 | 42.8 |
| 7 | 268.9 | 276.2 | 275.6 | 240.2 | 265.2 | 17.0 |
| 8 | 282.1 | 247.7 | 259.8 | 272.8 | 265.6 | 15.0 |
| 9 | 260.8 | 259.9 | 247.9 | 345.3 | 278.5 | 44.9 |
| 10 | 329.3 | 231.8 | 307.2 | 273.4 | 285.4 | 42.5 |
| 11 | 266.4 | 249.7 | 231.5 | 265.2 | 253.2 | 16.3 |
| 12 | 168.8 | 330.9 | 333.6 | 318.3 | 287.9 | 79.7 |
| 13 | 349.9 | 334.2 | 292.3 | 301.5 | 319.5 | 27.1 |
| 14 | 235.2 | 283.1 | 245.9 | 263.1 | 256.8 | 21.0 |
| 15 | 257.3 | 218.4 | 296.2 | 275.2 | 261.8 | 33.0 |
| 16 | 235.1 | 252.7 | 300.6 | 297.6 | 271.5 | 32.7 |
| 17 | 286.3 | 293.8 | 236.2 | 275.3 | 272.9 | 25.6 |
| 18 | 328.1 | 272.6 | 329.7 | 260.1 | 297.6 | 36.5 |
| 19 | 316.4 | 287.4 | 373.0 | 286.0 | 315.7 | 40.7 |
| 20 | 296.8 | 350.5 | 280.6 | 259.8 | 296.9 | 38.8 |

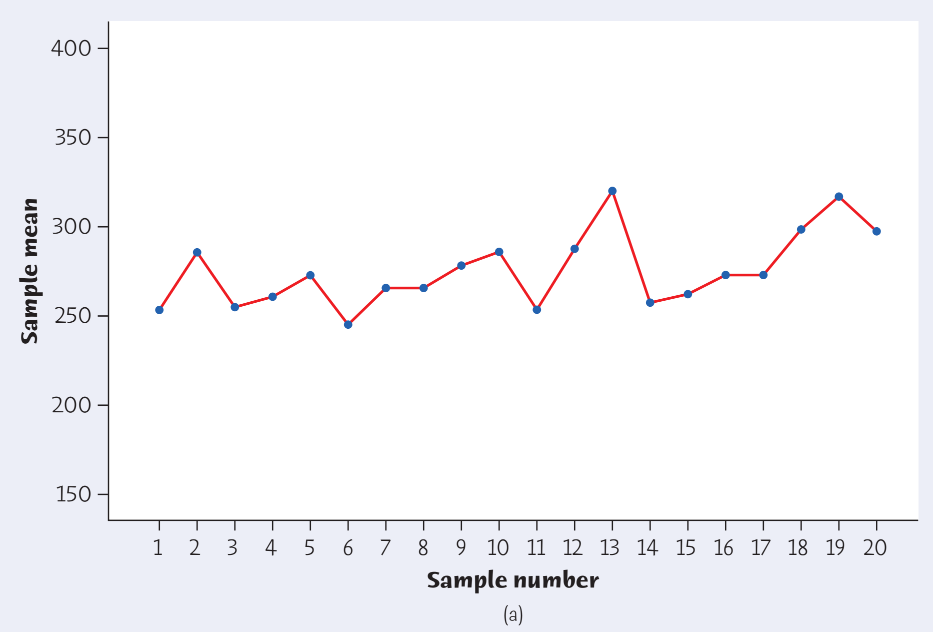



**FIGURE 31.5**
(a) Plot of the sample means versus sample number for the mesh tension data of Table 31.1. (b) $\bar{x}$ chart for the mesh tension data of Table 31.1. No points lie outside the control limits.

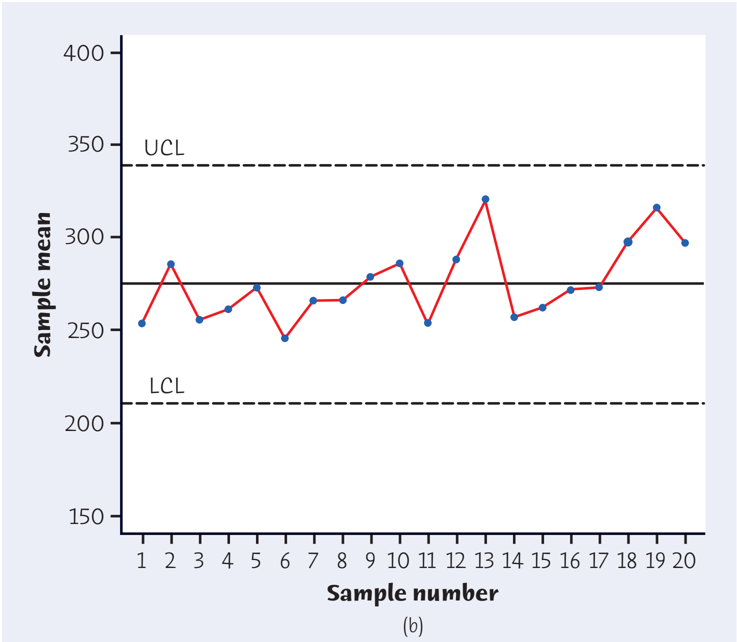



**FIGURE 31.5**
(*Continued*)

Figure 31.5(b) is an $\bar{x}$ control chart for the 20 mesh tension samples in Table 31.1. We have plotted each sample mean from the table against its sample number. For example, the mean of the first sample is 253.4 mV, and this is the value plotted for Sample 1. The center line is at $\mu = 275$ mV. The upper and lower control limits are

$$\mu + 3\frac{\sigma}{\sqrt{n}} = 275 + 3\frac{43}{\sqrt{4}} = 275 + 64.5 = 339.5 \text{ mV} \qquad \text{(UCL)}$$

$$\mu - 3\frac{\sigma}{\sqrt{n}} = 275 - 3\frac{43}{\sqrt{4}} = 275 - 64.5 = 210.5 \text{ mV} \qquad \text{(LCL)}$$

As is common, we have labeled the control limits UCL for upper control limit and LCL for lower control limit.

## EXAMPLE 31.4 Interpreting $\bar{x}$ Charts

Figure 31.5(b) is a typical $\bar{x}$ chart for a process in control. The means of the 20 samples do vary, but all lie within the range of variation marked out by the control limits. We are seeing the common cause variation of a stable process.

Figures 31.6 and 31.7 illustrate two ways in which the process can go out of control. In Figure 31.6, the process was disturbed by a special cause sometime between Sample 12 and Sample 13. As a result, the mean tension for Sample 13 falls above the upper control limit. It is common practice to mark all out-of-control points with an "x" to call attention to them. A search for the cause begins as soon as we see a point out of control. Investigation finds that the mounting of the tension-measuring device had slipped, resulting in readings that were too high. When the problem was corrected, Samples 14 through 20 are again in control.

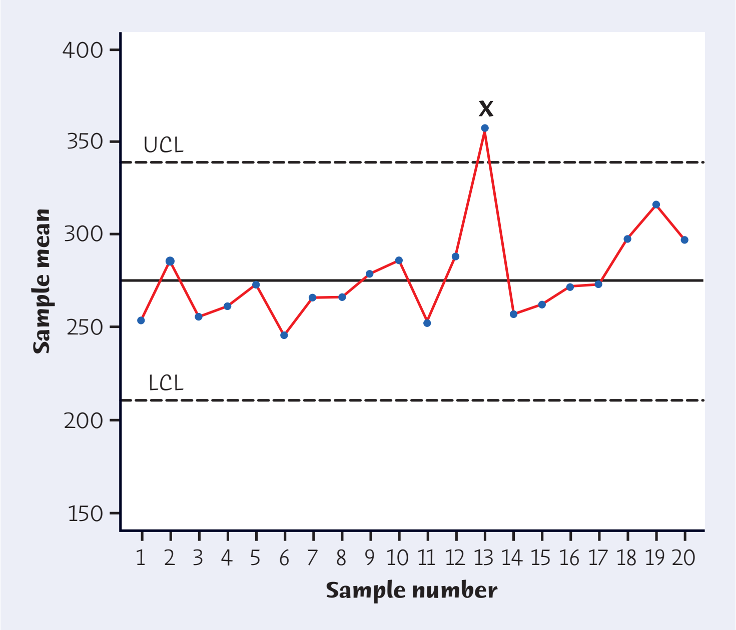



**FIGURE 31.6**
This $\bar{x}$ chart is identical to that in Figure 31.5(b), except that a special cause has driven $\bar{x}$ for Sample 13 above the upper control limit. The out-of-control point is marked with an x.

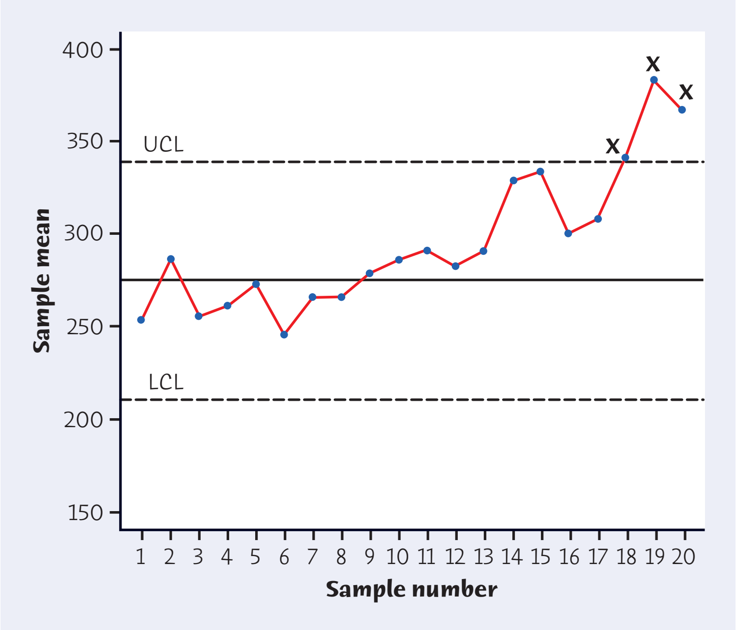



**FIGURE 31.7**
The first 10 points on this $\bar{x}$ chart are as in Figure 31.5(b). The process mean drifts upward after Sample 10, and the sample means $\bar{x}$ reflect this drift. The points for Samples 18, 19, and 20 are out of control.

Figure 31.7 shows the effect of a steady upward drift in the process center, starting at Sample 11. You see that some time elapses before the $\bar{x}$ for Sample 18 is out of control. Process drift results from gradual changes such as the wearing of a cutting tool or overheating. The one-point-out signal works better for detecting sudden large disturbances than for detecting slow drifts in a process.

## APPLY YOUR KNOWLEDGE

**31.10 Almond Flour.** The net weight (in ounces) of bags of almond flour is monitored by taking samples of five bags during each hour of production. The process mean should be $\mu = 32$ oz. When the process is properly adjusted, it varies with $\sigma = 0.5$ oz. The mean weight $\bar{x}$ for each hour's sample is plotted on an $\bar{x}$ control chart. Calculate the center line and control limits for this chart.

**31.11 Tablet Hardness.** A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. The hardness of a sample from each lot of tablets is measured to control the compression process. The process has been operating in control with mean at the target value $\mu = 12.3$ kilograms (kg) and estimated standard deviation $\sigma = 0.2$ kg. Table 31.2 gives three sets of data, each representing $\bar{x}$ for 20 successive samples of $n = 4$ tablets. One set remains in control at the target value. In a second set, the process mean $\mu$ shifts suddenly to a new value. In a third, the process mean drifts gradually. DATA HARD

(a) What are the center line and control limits for an $\bar{x}$ chart for this process?

(b) Draw a separate $\bar{x}$ chart for each of the three data sets. Mark any points that are beyond the control limits.

(c) Based on your work in part (b) and the appearance of the control charts, which set of data comes from a process that is in control? In which case does the process mean shift suddenly, and at about which sample do you think that the mean changed? Finally, in which case does the mean drift gradually?

**TABLE 31.2 Three sets of $\bar{x}$ values from 20 samples of size 4**

| Sample | Data Set A | Data Set B | Data Set C |
|---|---|---|---|
| 1 | 12.152 | 12.248 | 12.323 |
| 2 | 12.250 | 12.234 | 12.352 |
| 3 | 12.365 | 12.141 | 12.227 |
| 4 | 12.297 | 12.452 | 12.052 |
| 5 | 12.492 | 12.347 | 12.219 |
| 6 | 12.419 | 12.205 | 12.145 |
| 7 | 12.442 | 12.286 | 12.285 |
| 8 | 12.304 | 12.459 | 12.247 |
| 9 | 12.165 | 12.111 | 12.301 |
| 10 | 12.386 | 12.318 | 12.169 |
| 11 | 12.271 | 12.498 | 12.342 |
| 12 | 11.947 | 12.182 | 12.344 |
| 13 | 12.124 | 12.506 | 12.242 |
| 14 | 12.039 | 12.373 | 12.099 |
| 15 | 12.091 | 12.532 | 12.266 |
| 16 | 12.054 | 12.608 | 12.448 |
| 17 | 12.144 | 12.581 | 12.378 |
| 18 | 12.241 | 12.701 | 12.189 |
| 19 | 12.002 | 12.759 | 12.401 |
| 20 | 11.935 | 12.741 | 12.282 |

## 31.5 *s* Charts for Process Monitoring

The $\bar{x}$ charts in Figures 31.5(b), 31.6, and 31.7 were easy to interpret because the process standard deviation remained fixed at 43 mV. The effects of moving the process mean away from its in-control value (275 mV) are then clear to see. We know that even the simplest description of a distribution should give both a measure of center and a measure of variability. So it is with control charts. We must monitor both the process center, using an $\bar{x}$ chart, and the process variability, using a control chart for the sample standard deviation $s$.

The standard deviation $s$ does not have a Normal distribution, even approximately. Under the process-monitoring conditions, the sampling distribution of $s$ is skewed to the right. Nonetheless, control charts for any statistic are based on the "plus or minus three standard deviations" idea motivated by the 68–95–99.7 rule for Normal distributions. Control charts are intended to be practical tools that are easy to use. Standard practice in process control, therefore, ignores such details as the effect of non-Normal sampling distributions. Here is the general control chart setup for a sample statistic $Q$ (short for "quality characteristic").

### Three-Sigma Control Charts

To make a **three-sigma** ($3\sigma$) **control chart** for any statistic $Q$:

1. Take samples from the process at regular intervals and plot the values of the statistic $Q$ against the order in which the samples were taken.
2. Draw a **center line** on the chart at height $\mu_Q$, the mean of the statistic when the process is in control.
3. Draw upper and lower **control limits** on the chart three standard deviations of $Q$ above and below the mean. That is,

$$\text{UCL} = \mu_Q + 3\sigma_Q$$
$$\text{LCL} = \mu_Q - 3\sigma_Q$$

   Here $\sigma_Q$ is the standard deviation of the sampling distribution of the statistic $Q$ when the process is in control.
4. The chart produces an **out-of-control signal** when a plotted point lies outside the control limits.

We have applied this general idea to $\bar{x}$ charts. If $\mu$ and $\sigma$ are the process mean and standard deviation, the statistic $\bar{x}$ has mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. The center line and control limits for $\bar{x}$ charts follow from these facts.

What are the corresponding facts for the sample standard deviation $s$? Study of the sampling distribution of $s$ for samples from a Normally distributed process characteristic gives these facts:

1. The *mean* of $s$ is a constant times the process standard deviation $\sigma$, $\mu_s = c_4\sigma$.
2. The *standard deviation* of $s$ is also a constant times the process standard deviation, $\sigma_s = c_5\sigma$.

The constants are called $c_4$ and $c_5$ for historical reasons. Their values depend on the size of the samples. For large samples, $c_4$ is close to 1. That is, the sample standard deviation $s$ has little bias as an estimator of the process standard deviation $\sigma$.

Because statistical process control often uses small samples, we pay attention to the value of $c_4$. Following the general pattern for three-sigma control charts:

1. The *center line* of an $s$ chart is at $c_4\sigma$.
2. The *control limits* for an $s$ chart are at

$$\text{UCL} = \mu_s + 3\sigma_s = c_4\sigma + 3c_5\sigma = (c_4 + 3c_5)\sigma$$
$$\text{LCL} = \mu_s - 3\sigma_s = c_4\sigma - 3c_5\sigma = (c_4 - 3c_5)\sigma$$

That is, the control limits UCL and LCL are also constants times the process standard deviation. These constants are called (again for historical reasons) $B_6$ and $B_5$. We don't need to remember that $B_6 = c_4 + 3c_5$ and $B_5 = c_4 - 3c_5$ because tables give us the numerical values of $B_6$ and $B_5$.

### $\bar{x}$ and $s$ Control Charts for Process Monitoring[7]

Take regular samples of size $n$ from a process that has been in control with process mean $\mu$ and process standard deviation $\sigma$. The center line and control limits for an $\bar{x}$ **chart** are

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$
$$\text{CL} = \mu$$
$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

The center line and control limits for an $s$ **chart** are

$$\text{UCL} = B_6\sigma$$
$$\text{CL} = c_4\sigma$$
$$\text{LCL} = B_5\sigma$$

The **control chart constants** $c_4$, $B_5$, and $B_6$ depend on the sample size $n$.

Table 31.3 gives the values of the control chart constants $c_4$, $c_5$, $B_5$, and $B_6$ for samples of sizes 2 through 10. This table makes it easy to draw $s$ charts. The table has no $B_5$ entries for samples of size smaller than $n = 6$. The lower control limit for an $s$ chart is zero for samples of sizes 2 through 5. This is a consequence of the

**TABLE 31.3 Control chart constants**

| Sample Size $n$ | $c_4$ | $c_5$ | $B_5$ | $B_6$ |
|---|---|---|---|---|
| 2 | 0.7979 | 0.6028 | | 2.606 |
| 3 | 0.8862 | 0.4633 | | 2.276 |
| 4 | 0.9213 | 0.3889 | | 2.088 |
| 5 | 0.9400 | 0.3412 | | 1.964 |
| 6 | 0.9515 | 0.3076 | 0.029 | 1.874 |
| 7 | 0.9594 | 0.2820 | 0.113 | 1.806 |
| 8 | 0.9650 | 0.2622 | 0.179 | 1.751 |
| 9 | 0.9693 | 0.2459 | 0.232 | 1.707 |
| 10 | 0.9727 | 0.2321 | 0.276 | 1.669 |

fact that $s$ has a right-skewed distribution and takes only values greater than zero. Three standard deviations above the mean (UCL) lies on the long right side of the distribution. Three standard deviations below the mean (LCL) on the short left side is below zero, so we say that LCL = 0.

## EXAMPLE 31.5 $\bar{x}$ and $s$ Charts for Mesh Tension

Figure 31.8 is the $s$ chart for the computer monitor mesh tension data in Table 31.1. The samples are of size $n = 4$, and the process standard deviation in control is $\sigma = 43$ mV. The center line is, therefore,

$$\text{CL} = c_4\sigma = (0.9213)(43) = 39.6 \text{ mV}$$

The control limits are

$$\text{UCL} = B_6\sigma = (2.088)(43) = 89.8$$
$$\text{LCL} = B_5\sigma = (0)(43) = 0$$

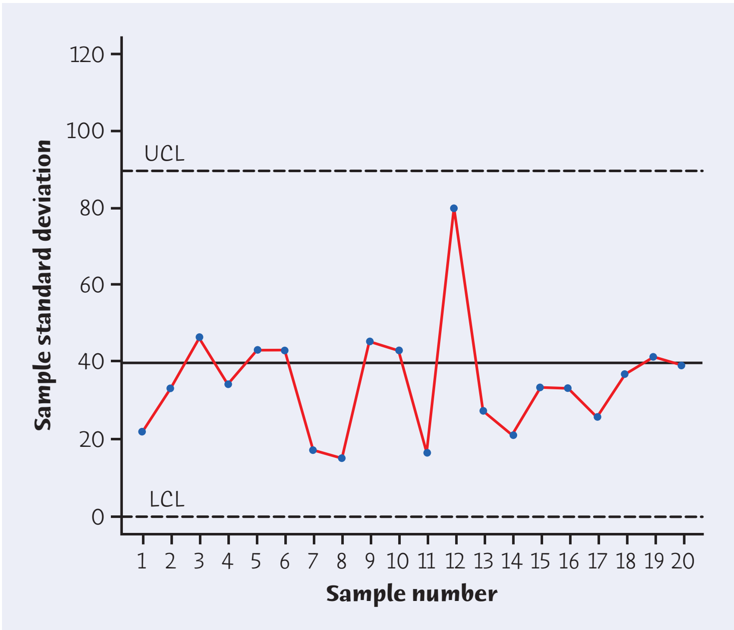



**FIGURE 31.8**
$s$ chart for the mesh tension data of Table 31.1. Both the $s$ chart and the $\bar{x}$ chart [Figure 31.5(b)] are in control.

Figures 31.5(b) and 31.8 go together: they are $\bar{x}$ and $s$ charts for monitoring the mesh-tensioning process. Both charts are in control, showing only common cause variation within the bounds set by the control limits.

Figures 31.9 and 31.10 are $\bar{x}$ and $s$ charts for the mesh-tensioning process when a new and poorly trained operator takes over between Samples 10 and 11. The new operator introduces added variation into the process, increasing the process standard deviation from its in-control value of 43 mV to 60 mV. The $\bar{x}$ chart in Figure 31.9 shows one point out of control. Only on closer inspection do we see that the variability of the $\bar{x}$ values increases after Sample 10. In fact, the process mean has remained unchanged at 275 mV. The apparent lack of control in the $\bar{x}$ chart is entirely due to the larger process variation. There is a lesson here: *it is difficult to interpret an $\bar{x}$ chart unless s is in control. When you look at $\bar{x}$ and s charts, always start with the s chart.*

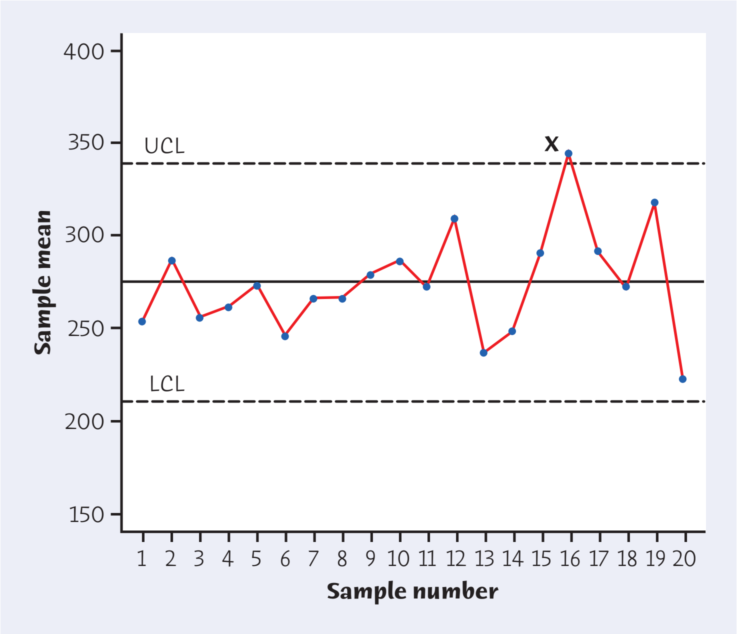



**FIGURE 31.9**
$\bar{x}$ chart for mesh tension when the process variability increases after Sample 10. The $\bar{x}$ chart does show the increased variability, but the *s* chart is clearer and should be read first.

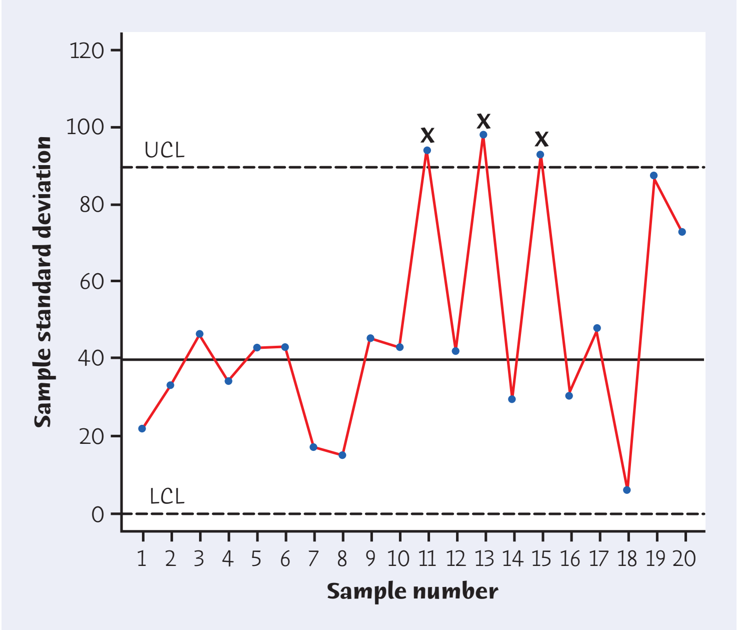



**FIGURE 31.10**
*s* chart for mesh tension when the process variability increases after Sample 10. Increased within-sample variability is clearly visible. Find and remove the *s*-type special cause before reading the $\bar{x}$ chart.

The *s* chart in Figure 31.10 shows lack of control starting at Sample 11. As usual, we mark the out-of-control points with an "x." The points for Samples 13 and 15 also lie above the UCL, and the overall variability of the sample points is much greater than for the first 10 samples. In practice, the *s* chart would call for action after Sample 11. We would ignore the $\bar{x}$ chart until the special cause (the new operator) for the lack of control in the *s* chart has been found and removed by training the operator.

Example 31.5 suggests a strategy for using $\bar{x}$ and *s* charts in practice. First examine the *s* chart. Lack of control on an *s* chart is due to special causes that affect the observations *within a sample* differently. New and nonuniform raw material, a new and poorly trained operator, poor metrology repeatability, and mixing results from several machines or several operators are typical "*s*-type" special causes.

Once the $s$ chart is in control, the stable value of the process standard deviation $\sigma$ means that the variation within samples serves as a benchmark for detecting variation in the level of the process over the longer time periods between samples. The $\bar{x}$ chart, with control limits that depend on $\sigma$, does this. The $\bar{x}$ chart, as we saw in Example 31.5, responds to $s$-type causes as well as to longer-range changes in the process, so it is important to eliminate $s$-type special causes first. Then the $\bar{x}$ chart will alert us to, for example, a change in process level caused by new raw material that differs from that used in the past or a gradual drift in the process level caused by wear in a cutting tool.

## EXAMPLE 31.6 *s*-Type and $\bar{x}$-Type Special Causes

A company that promotes and coordinates ride-sharing uses control charts to monitor how long it takes before a driver picks up a customer once a trip request is received. Each day, at random times within a given region, five random trip requests are selected and the waiting times are noted. On one day, one of the times was selected shortly after a major sporting event ended. This caused the waiting time for that customer to be much longer than the other four times that were collected that day. The sample has a large $s$ and signals that additional drivers are needed during peak times.

The same ride-sharing company observed that in this particular region, the waiting time for passengers was higher than in other regions. They kicked off an advertising campaign in this region to hire more drivers. The advertising was successful, and they noticed an overall shift downward in the mean waiting time. This is a desired special cause and a systematic change in the process. The company will have to establish new control limits that describe the new state of the process in this region, with smaller process mean $\mu$.

The second setting in Example 31.6 reminds us that a major change in the process returns us to the chart setup stage. In the absence of deliberate changes in the process, process monitoring uses the same values of $\mu$ and $\sigma$ for long periods of time. There is one important exception: careful monitoring and removal of special causes as they occur can permanently reduce the process $\sigma$. If the points on the $s$ chart remain near the center line for a long period, it is wise to update the value of $\sigma$ to the new, smaller value and compute new values of UCL and LCL for both $\bar{x}$ and $s$ charts.

## APPLY YOUR KNOWLEDGE

**31.12 Making Cappuccino.** A large chain of coffee shops records a number of performance measures. Among them is the time required to complete an order for a cappuccino, measured from the time the order is placed. Suggest some plausible examples of each of the following.

(a) Reasons for common cause variation in response time.

(b) $s$-type special causes.

(c) $\bar{x}$-type special causes.

**31.13 Almond Flour, continued.** In Exercise 31.10, you gave the center line and control limits for an $\bar{x}$ chart. What are the center line and control limits for an $s$ chart for this process?

**31.14 Tablet Hardness, continued.** Exercise 31.11 concerns process control data on the hardness of tablets (measured in kilograms) for a pharmaceutical product. Table 31.4 gives data for 20 new samples of size 4, with the $\bar{x}$ and $s$ for each sample. The process has been in control with mean at the target value $\mu = 12.3$ kg and standard deviation $\sigma = 0.2$ kg. HARD2

(a) Make both $\bar{x}$ and $s$ charts for these data based on the information given about the process.

(b) At some point, the within-sample process variation increased from $\sigma = 0.2$ kg to $\sigma = 0.4$ kg. About where in the 20 samples did this happen? What is the effect on the $s$ chart? On the $\bar{x}$ chart?

(c) At that same point, the process mean changed from $\mu = 12.3$ kg to $\mu = 12.5$ kg. What is the effect of this change on the $s$ chart? On the $\bar{x}$ chart?

**TABLE 31.4 Twenty samples of size 4, with $\bar{x}$ and $s$**

| Sample | Hardness (kilograms) | | | | $\bar{x}$ | $s$ |
|---|---|---|---|---|---|---|
| 1 | 12.087 | 12.492 | 12.145 | 12.077 | 12.200 | 0.1968 |
| 2 | 12.225 | 12.404 | 12.359 | 12.786 | 12.444 | 0.2407 |
| 3 | 12.183 | 12.449 | 12.117 | 12.279 | 12.257 | 0.1442 |
| 4 | 12.304 | 12.255 | 12.449 | 12.312 | 12.330 | 0.0832 |
| 5 | 12.480 | 12.221 | 12.314 | 12.262 | 12.319 | 0.1137 |
| 6 | 12.128 | 12.472 | 12.424 | 12.204 | 12.307 | 0.1669 |
| 7 | 12.136 | 12.196 | 12.169 | 12.391 | 12.223 | 0.1147 |
| 8 | 12.533 | 11.903 | 12.384 | 12.595 | 12.354 | 0.3133 |
| 9 | 12.351 | 11.983 | 13.048 | 12.187 | 12.392 | 0.4624 |
| 10 | 12.591 | 13.060 | 12.605 | 11.813 | 12.517 | 0.5176 |
| 11 | 11.638 | 12.412 | 11.745 | 12.300 | 12.024 | 0.3889 |
| 12 | 12.747 | 12.220 | 12.418 | 12.312 | 12.424 | 0.2299 |
| 13 | 12.878 | 12.335 | 12.931 | 12.240 | 12.596 | 0.3590 |
| 14 | 12.096 | 12.534 | 12.620 | 11.971 | 12.305 | 0.3198 |
| 15 | 12.049 | 12.909 | 12.370 | 12.869 | 12.549 | 0.4139 |
| 16 | 12.554 | 12.898 | 13.320 | 12.756 | 12.882 | 0.3243 |
| 17 | 12.868 | 12.308 | 12.237 | 11.894 | 12.327 | 0.4036 |
| 18 | 11.802 | 12.725 | 13.085 | 12.896 | 12.627 | 0.5693 |
| 19 | 12.636 | 12.414 | 12.882 | 12.331 | 12.566 | 0.2470 |
| 20 | 13.008 | 12.660 | 12.217 | 12.271 | 12.539 | 0.3697 |

**31.15 Dyeing Yarn.** The unique colors of the cashmere sweaters your firm makes result from heating undyed yarn in a kettle with a dye liquor. The pH (acidity) of the liquor is critical for regulating dye uptake and hence the final color. There are five kettles, all of which receive dye liquor from a common source. Twice each day, the pH of the liquor in each kettle is measured, giving samples of size 5. The process has been operating in control with $\mu = 5.21$ and $\sigma = 0.147$.

(a) Give the center line and control limits for the $s$ chart.

(b) Give the center line and control limits for the $\bar{x}$ chart.

**31.16 Mounting-Hole Distances.** Figure 31.11 reproduces a data sheet from the floor of a factory that makes electrical meters.[8] The sheet shows measurements on the distance between two mounting holes for 18 samples of size 5. The heading informs us that the measurements are in multiples of 0.0001 inch above 0.6000 inch. That is, the first measurement, 44, stands for 0.6044 inch. All the measurements end in 4. Although we don't know why this is true, it is clear that in effect the measurements were made to the nearest 0.001 inch, not to the nearest 0.0001 inch. MOUNT

VARIABLES CONTROL CHART ($\overline{X}$ & R)

| Part No. | Chart No. |
|---|---|
| 32506 | 1 |

| Part name (project) | Operation (process) | Specification limits |
|---|---|---|
| Metal frame | Distance between mounting holes | 0.6054" ± 0.0010" |

| Operator | Machine | Gage | Unit of measure | Zero equals |
|---|---|---|---|---|
| | R-5 | | 0.0001" | 0.6000" |

| Date | | 3/7 | | | | 3/8 | | | | | | 3/9 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time | | 8:30 | 10:30 | 11:45 | 1:30 | 8:15 | 10:15 | 11:45 | 2:00 | 3:00 | 4:00 | 8:30 | 10:00 | 11:45 | 1:30 | 2:30 | 3:30 | 4:30 | 5:30 | | |
| Sample measurements | 1 | 44 | 64 | 34 | 44 | 34 | 34 | 54 | 64 | 24 | 34 | 34 | 54 | 44 | 24 | 54 | 54 | 54 | 54 | | |
| | 2 | 44 | 44 | 44 | 54 | 14 | 64 | 64 | 34 | 54 | 44 | 44 | 44 | 24 | 24 | 24 | 34 | 34 | 24 | | |
| | 3 | 44 | 34 | 54 | 54 | 84 | 34 | 34 | 54 | 44 | 44 | 34 | 24 | 34 | 54 | 54 | 24 | 74 | 64 | | |
| | 4 | 44 | 34 | 44 | 34 | 54 | 44 | 44 | 44 | 34 | 34 | 64 | 54 | 34 | 44 | 44 | 44 | 44 | 34 | | |
| | 5 | 64 | 54 | 54 | 44 | 44 | 44 | 34 | 44 | 34 | 34 | 34 | 24 | 44 | 44 | 44 | 54 | 54 | 44 | | |
| | | | | | | | | | | | | | | | | | | | | | |
| Average, $\overline{X}$ | | | | | | | | | | | | | | | | | | | | | |
| Range, R | | 20 | 30 | 20 | 20 | 70 | 30 | 30 | 30 | 30 | 10 | 30 | 30 | 20 | 30 | 40 | 30 | 40 | 40 | | |
| | | | | | | | | | | | | | | | | | | | | | |

**FIGURE 31.11**

A process control record sheet kept by operators, for Exercise 31.16. This is typical of records kept by hand when measurements are not automated. We will see in the next section why such records mention $\bar{x}$ and $R$ control charts rather than $\bar{x}$ and $s$ charts.

Calculate $\bar{x}$ and $s$ for the first two samples. The data file contains $\bar{x}$ and $s$ for all 18 samples. Based on long experience with this process, you are keeping control charts based on $\mu = 43$ and $\sigma = 12.74$. Make $s$ and $\bar{x}$ charts for the data in Figure 31.11 and describe the state of the process.

**31.17 Dyeing Yarn: Special Causes.** The process described in Exercise 31.15 goes out of control. Investigation finds that a new type of yarn was recently introduced. The pH in the kettles is influenced by both the dye liquor and the yarn. Moreover, on a few occasions a faulty valve on one of the kettles had allowed water to enter that kettle; as a result, the yarn in that kettle had to be discarded. Which of these special causes appears on the $s$ chart and which on the $\bar{x}$ chart? Explain your answer.

## 31.6 Using Control Charts

We are now familiar with the ideas that are common to all control charts and also with the details of making $\bar{x}$ and $s$ charts. This section discusses two topics related to using control charts in practice.

**$\bar{x}$ and $R$ charts.** We have seen that it is essential to monitor both the center and the variability of a process. Control charts were originally intended to be used by factory workers with limited knowledge of statistics in the era before even calculators, let alone software, were common. In that environment, it takes too long to calculate standard deviations. The $\bar{x}$ chart for center was, therefore, combined with a control chart for variability based on the **sample range** rather than the sample standard deviation. The range $R$ of a sample is just the difference between the largest and smallest observations. It is easy to find $R$ without a calculator. Using $R$ rather than $s$ to measure the variability of samples replaces the $s$ chart with an ***R* chart**. It also changes the $\bar{x}$ chart because the control limits for $\bar{x}$ use the estimated process variability. So $\bar{x}$ and $R$ charts differ in the details of both charts from $\bar{x}$ and $s$ charts.

***sample range***

***R chart***

Because the range $R$ uses only the largest and smallest observations in a sample, it is less informative than the standard deviation $s$ calculated from all the observations. For this reason, $\bar{x}$ and $s$ charts are now preferred to $\bar{x}$ and $R$ charts. $R$ charts remain common because tradition dies hard and also because it is easier for workers to understand $R$ than $s$. In this short introduction, we concentrate on the principles of control charts, so we won't give the details of constructing $\bar{x}$ and $R$ charts. These details appear in any text on quality control.[9] If you meet a set of $\bar{x}$ and $R$ charts, remember that the interpretation of these charts is just like the interpretation of $\bar{x}$ and $s$ charts.

**Additional out-of-control signals.** So far, we have used only the basic "one point beyond the control limits" criterion to signal that a process may have gone out of control. We would like a quick signal when the process moves out of control, but we also want to avoid "false alarms," or signals that occur just by chance when the process is really in control. The standard $3\sigma$ control limits are chosen to prevent too many false alarms because an out-of-control signal calls for an effort to find and remove a special cause. As a result, *$\bar{x}$ charts are often slow to respond to a gradual drift in the process center that continues for some time before finally forcing a reading outside the control limits.* We can speed the response of a control chart to lack of control—at the cost of also enduring more false alarms—by adding patterns other than "one-point-out" as signals. The most common step in this direction is to add a *runs signal* to the $\bar{x}$ chart.

### Out-of-Control Signals

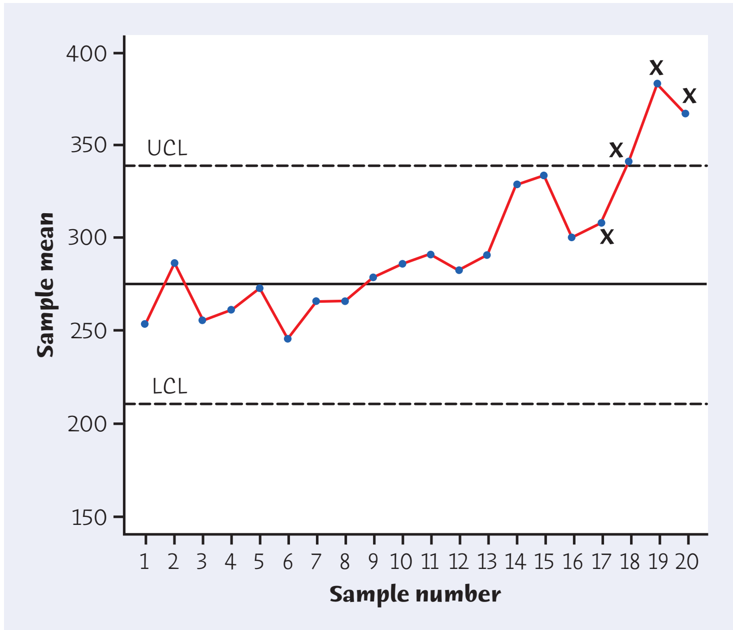
$\bar{x}$ and $s$ or $\bar{x}$ and $R$ control charts produce an out-of-control signal if:

- **One-point-out:** A single point lies outside the $3\sigma$ control limits of either chart.
- **Run:** The $\bar{x}$ chart shows nine consecutive points above the center line or nine consecutive points below the center line. The signal occurs when we see the ninth point of the run.

## EXAMPLE 31.7 Using the Runs Signal

Figure 31.12 reproduces the $\bar{x}$ chart from Figure 31.6. The process center began a gradual upward drift at Sample 11. The chart shows the effect of the drift—the sample means plotted on the chart move gradually upward, with some random variation. The one-point-out signal does not call for action until Sample 18 finally produces an $\bar{x}$ above the UCL. The runs signal reacts more quickly: Sample 17 is the ninth consecutive point above the center line.

**FIGURE 31.12**
$\bar{x}$ chart for mesh tension data when the process center drifts upward. The "run of nine" signal gives an out-of-control warning at Sample 17.




It is a mathematical fact that the runs signal responds to a gradual drift more quickly (on the average) than the one-point-out signal does. The motivation for a runs signal is that when a process is in control, the probability of a false alarm is about the same for the runs signal as for the one-point-out signal. There are many other signals that can be added to the rules for responding to $\bar{x}$ and $s$ or $\bar{x}$ and $R$ charts. *In our enthusiasm to detect various special kinds of loss of control, it is easy to forget that adding signals always increases the frequency of false alarms.* Frequent false alarms are so annoying that the people responsible for responding soon begin to ignore out-of-control signals. It is better to use only a few signals and to reserve signals other than one-point-out and runs for processes that are known to be prone to specific special causes for which there is a tailor-made signal.[10]

## APPLY YOUR KNOWLEDGE

**31.18 Special Causes.** Is each of the following examples of a special cause most likely to first result in (i) one-point-out on the $s$ or $R$ chart, (ii) one-point-out on the $\bar{x}$ chart, or (iii) a run on the $\bar{x}$ chart? In each case, briefly explain your reasoning.

(a) The time it takes a new coffee barista to complete your order at your favorite coffee shop.

(b) The precision of a measurement tool is affected by dirt getting on the sensors and needs to be cleaned when this happens.

(c) The accuracy of an inspector starts to degrade after the first six hours of his shift.

(d) A person who is training for a 5k race created a control chart for her running time on the same route each week. She started running at what she considered a slow pace and is now very happy with her running times.

**31.19 Mixtures.** A smartphone manufacturer uses three different battery suppliers. To ensure consistent battery life across all of its phones, the manufacturer of smartphones tests batteries from all suppliers and plots them on a single control chart. Unknown to the smartphone manufacturer, one of the suppliers is having quality issues, and the battery life on its units is on average lower than the other two suppliers. An engineer at the manufacturer notices an unusual pattern on the charts and determines that the battery life of one of the suppliers is different from the others. Sketch the $\bar{x}$ chart pattern that will result. Is there another way the manufacturer could have plotted this data so that the resulting quality issue was detected more rapidly?

## 31.7 Setting up Control Charts

When you first approach a process that has not been carefully studied, it is quite likely that the process is not in control. Your first goal is to discover and remove special causes and so bring the process into control. Control charts are an important tool. Control charts for *process monitoring* follow the process forward in time to keep it in control. Control charts at the *chart setup* stage, on the other hand, look back in an attempt to discover the present state of the process. An example will illustrate the method.

### EXAMPLE 31.8 Viscosity of an Elastomer

VISCOS

The viscosity of a material is its resistance to flow when under stress. Viscosity is a critical characteristic of rubber and rubber-like compounds called elastomers, which have many uses in consumer products. Viscosity is measured by placing specimens of the material above and below a slowly rotating roller, squeezing the assembly, and recording the drag on the roller. Measurements are in "Mooney units," named after the inventor of the instrument.

A specialty chemical company is beginning production of an elastomer that is supposed to have viscosity 45 ± 5 Mooneys. Each lot of the elastomer is produced by "cooking" raw material with catalysts in a reactor vessel. Table 31.5 records $\bar{x}$ and $s$ from samples of size $n = 4$ lots from the first 24 shifts as production begins.[11] An $s$ chart therefore monitors variation among lots produced during the same shift. If the $s$ chart is in control, an $\bar{x}$ chart looks for shift-to-shift variation.

**TABLE 31.5 $\bar{x}$ and $s$ for 24 samples of elastomer viscosity (in Mooneys)**

| Sample | $\bar{x}$ | $s$ | Sample | $\bar{x}$ | $s$ |
|---|---|---|---|---|---|
| 1 | 49.750 | 2.684 | 13 | 47.875 | 1.118 |
| 2 | 49.375 | 0.895 | 14 | 48.250 | 0.895 |
| 3 | 50.250 | 0.895 | 15 | 47.625 | 0.671 |
| 4 | 49.875 | 1.118 | 16 | 47.375 | 0.671 |
| 5 | 47.250 | 0.671 | 17 | 50.250 | 1.566 |
| 6 | 45.000 | 2.684 | 18 | 47.000 | 0.895 |
| 7 | 48.375 | 0.671 | 19 | 47.000 | 0.447 |
| 8 | 48.500 | 0.447 | 20 | 49.625 | 1.118 |
| 9 | 48.500 | 0.447 | 21 | 49.875 | 0.447 |
| 10 | 46.250 | 1.566 | 22 | 47.625 | 1.118 |
| 11 | 49.000 | 0.895 | 23 | 49.750 | 0.671 |
| 12 | 48.125 | 0.671 | 24 | 48.625 | 0.895 |

**Estimating $\mu$.** We do not know the process mean $\mu$ and standard deviation $\sigma$. What should we do? Sometimes we can easily adjust the center of a process by setting some control, such as the depth of a cutting tool in a machining operation or the temperature of a reactor vessel in a pharmaceutical plant. In such cases, it is usual to simply take the process mean $\mu$ to be the target value, the depth or temperature that the design of the process specifies as correct. The $\bar{x}$ chart then helps us keep the process mean at this target value.

There is less likely to be a "correct value" for the process mean $\mu$ if we are monitoring response times to customer calls or data entry errors. In Example 31.8, we have the target value 45 Mooneys, but there is no simple way to set viscosity at the desired level. In such cases, we want the $\mu$ we use in our $\bar{x}$ chart to describe the center of the process as it has actually been operating. To do this, just take the mean of all the individual measurements in the past samples. Because the samples are all the same size, this is just the mean of the sample $\bar{x}$ values. The overall "mean of the sample means" is, therefore, usually called $\bar{\bar{x}}$. For the 24 samples in Table 31.5,

$$\bar{\bar{x}} = \frac{1}{24}(49.750 + 49.375 + \cdots + 48.625)$$

$$= \frac{1161.125}{24} = 48.380$$

In the viscosity example, the target and center line are different. This is not unusual. The target reflects where the process is expected to run, and the center line is calculated using historical data and reflects where the process is actually running.

**Estimating $\sigma$.** It is almost never safe to use a "target value" for the process standard deviation $\sigma$ because it is almost never possible to directly adjust process variation. We must estimate $\sigma$ from past data. We want to combine the sample standard deviations $s$ from past samples rather than use the standard deviation of all the individual observations in those samples. That is, in Example 31.8, we want to combine the 24 sample standard deviations in Table 31.5 rather than calculate the standard deviation of the 96 observations in these samples. The reason is that it is the *within-sample* variation that is the benchmark against which we compare the longer-term process variation. Even if the process has been in control, we want only the variation over the short time period of a single sample to influence our value for $\sigma$.

There are several ways to estimate $\sigma$ from the sample standard deviations. In practice, software may use a somewhat sophisticated method and then calculate the control limits for you. We use a simple method that is traditional in quality control because it goes back to the era before software. If we are basing chart setup on $k$ past samples, we have $k$ sample standard deviations $s_1, s_2, \ldots, s_k$. Just average these to get

$$\bar{s} = \frac{1}{k}(s_1 + s_2 + \cdots + s_k)$$

For the viscosity example, we average the $s$-values for the 24 samples in Table 31.5:

$$\bar{s} = \frac{1}{24}(2.684 + 0.895 + \cdots + 0.895)$$

$$= \frac{24.156}{24} = 1.0065$$

Combining the sample $s$-values to estimate $\sigma$ introduces a complication: the samples used in process control are often small (size $n = 4$ in the viscosity example), so

$s$ has some bias as an estimator of $\sigma$. Recall that $\mu_s = c_4\sigma$. The mean $\bar{s}$ inherits this bias: its mean is also not $\sigma$ but $c_4\sigma$. The proper estimate of $\sigma$ corrects this bias. It is

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

We get control limits from past data by using the estimates $\bar{\bar{x}}$ and $\hat{\sigma}$ in place of the $\mu$ and $\sigma$ used in charts at the process-monitoring stage. Here are the results.[12]

### $\bar{x}$ and $s$ Control Charts Using Past Data

Take regular samples of size $n$ from a process. Estimate the process mean $\mu$ and the process standard deviation $\sigma$ from past samples by

$$\hat{\mu} = \bar{\bar{x}} \quad \text{(or use a target value)}$$

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

The center line and control limits for an **$\bar{x}$ chart** are

$$\text{UCL} = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}}$$
$$\text{CL} = \hat{\mu}$$
$$\text{LCL} = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}}$$

The center line and control limits for an **$s$ chart** are

$$\text{UCL} = B_6\hat{\sigma}$$
$$\text{CL} = c_4\hat{\sigma} = \bar{s}$$
$$\text{LCL} = B_5\hat{\sigma}$$

If the process was not in control when the samples were taken, these should be regarded as trial control limits.

We are now ready to outline the chart setup procedure for elastomer viscosity.

**Step 1** As usual, we look first at an $s$ chart. For chart setup, control limits are based on the same past data that we will plot on the chart. Calculate from Table 31.5 that

$$\bar{s} = 1.0065$$
$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{1.0065}{0.9213} = 1.0925$$

The center line and control limits for an $s$ chart based on past data are

$$\text{UCL} = B_6\hat{\sigma} = (2.088)(1.0925) = 2.281$$
$$\text{CL} = \bar{s} = 1.0065$$
$$\text{LCL} = B_5\hat{\sigma} = (0)(1.0925) = 0$$

Figure 31.13 is the $s$ chart. The points for Shifts 1 and 6 lie above the UCL. Both are near the beginning of production. Investigation finds that the reactor operator made an error on one lot in each of these samples. The error changed the viscosity

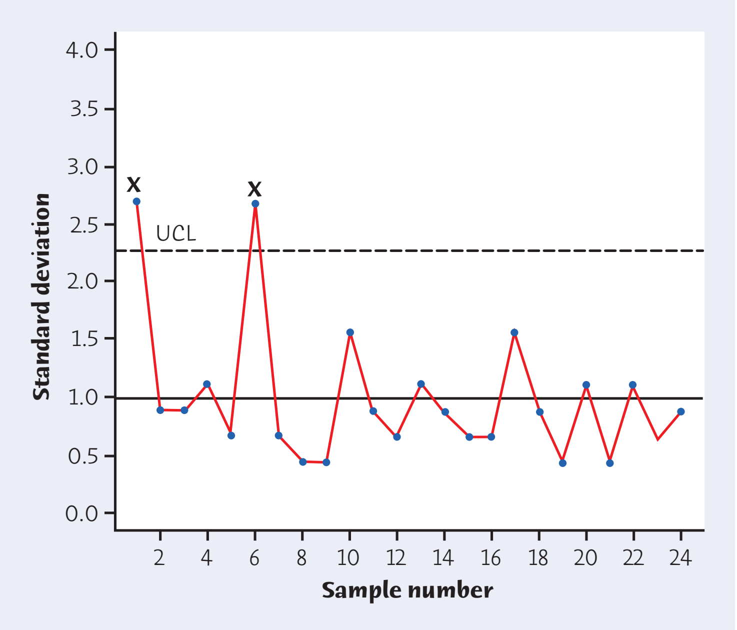



**FIGURE 31.13**
*s* chart based on past data for the viscosity data of Table 31.5. The control limits are based on the same *s*-values that are plotted on the chart. Points 1 and 6 are out of control.

of that lot and increased *s* for that one sample. The error will not be repeated now that the operators have gained experience. That is, this special cause has already been removed.

**Step 2** Remove the two values of *s* that were out of control. This is proper because the special cause responsible for these readings is no longer present. Recalculate from the remaining 22 shifts that $\bar{s} = 0.854$ and $\hat{\sigma} = 0.854/0.9213 = 0.927$. Make a new *s* chart with

$$\begin{aligned} \text{UCL} &= B_6\hat{\sigma} = (2.088)(0.927) = 1.936 \\ \text{CL} &= \bar{s} = 0.854 \\ \text{LCL} &= B_5\hat{\sigma} = (0)(0.927) = 0 \end{aligned}$$

We don't show the chart, but you can see from Table 31.5 that none of the remaining *s*-values lie above the new, lower, UCL; the largest remaining *s* is 1.566. If additional points were now out of control, we would repeat the process of finding and eliminating *s*-type causes until the *s* chart for the remaining shifts was in control. In practice, of course, this is often a challenging task.

**Step 3** Once *s*-type causes have been eliminated, make an $\bar{x}$ chart *using only the samples that remain* after dropping those that had out-of-control *s*-values. For the 22 remaining samples, we know that $\hat{\sigma} = 0.927$, and we calculate that $\bar{x} = 48.4716$. The center line and control limits for the $\bar{x}$ chart are

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = 48.4716 + 3\frac{0.927}{\sqrt{4}} = 49.862 \\ \text{CL} &= \bar{\bar{x}} = 48.4716 \\ \text{LCL} &= \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = 48.4716 - 3\frac{0.927}{\sqrt{4}} = 47.081 \end{aligned}$$

Figure 31.14 is the $\bar{x}$ chart. Shifts 1 and 6 have been dropped. Seven of the 22 points are beyond the $3\sigma$ limits, four high and three low. Although within-shift variation is now stable, there is excessive variation from shift to shift. To find the

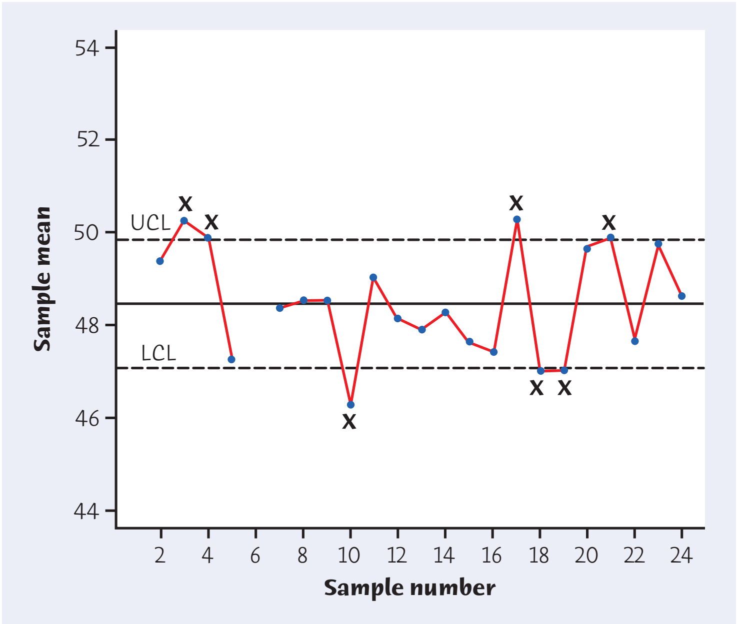



**FIGURE 31.14**
$\bar{x}$ chart based on past data for the viscosity data of Table 31.5. The samples for Shifts 1 and 6 have been removed because $s$-type special causes active in those samples are no longer active. The $\bar{x}$ chart shows poor control.

cause, we must understand the details of the process, but knowing that the special cause or causes operate between shifts is a big help. If the reactor is set up anew at the beginning of each shift, that's one place to look more closely.

**Step 4** Once the $\bar{x}$ and $s$ charts are both in control (looking backward), use the estimates $\hat{\mu}$ and $\hat{\sigma}$ from the points in control to set tentative control limits to monitor the process going forward. If it remains in control, we can update the charts and move to the process-monitoring stage.

## APPLY YOUR KNOWLEDGE

**31.20 From Setup to Monitoring.** Suppose that when the chart setup project of Example 31.8 is complete, the points remaining after removing special causes have $\bar{\bar{x}} = 48.7$ and $\bar{s} = 0.92$. What are the center line and control limits for the $\bar{x}$ and $s$ charts you would use to monitor the process going forward?

**31.21 Estimating Process Parameters.** The $\bar{x}$ and $s$ control charts for the mesh-tensioning example (Figures 31.5(b) and 31.8) were based on $\mu = 275$ mV and $\sigma = 43$ mV. Table 31.1 gives the 20 most recent samples from this process. DATA MONITRS

(a) Estimate the process $\mu$ and $\sigma$ based on these 20 samples.

(b) Your calculations suggest that the process $\sigma$ may now be less than 43 mV. Explain why the $s$ chart in Figure 31.7 suggests the same conclusion. (If this pattern continues, we would eventually update the value of $\sigma$ used for control limits.)

**31.22 Hospital Losses.** Table 31.6 gives data on the losses (in dollars) incurred by a hospital in treating major joint replacement (DRG 209) patients.[13] The hospital has taken from its records a random sample of eight such patients each month for 15 months. DATA DRG2

(a) Make an $s$ control chart using center lines and limits calculated from these past data. There are no points out of control.

(b) Because the $s$ chart is in control, base the $\bar{x}$ chart on all 15 samples. Make this chart. Is it also in control?

**TABLE 31.6 Hospital losses (dollars) for 15 samples of DRG 209 patients**

| Sample | Loss | | | | | | | | Sample Mean | Standard Deviation |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 7994 | 6005 | 8049 | 7565 | 8011 | 6988 | 6204 | 9637 | 7556.6 | 1166.7 |
| 2 | 7574 | 6650 | 7962 | 8295 | 7396 | 6970 | 4707 | 7205 | 7094.9 | 1098.1 |
| 3 | 6382 | 5363 | 8726 | 7485 | 7469 | 8573 | 6519 | 6086 | 7075.4 | 1194.7 |
| 4 | 5714 | 8353 | 7765 | 8774 | 9232 | 8047 | 7271 | 7132 | 7786.0 | 1098.3 |
| 5 | 7231 | 9272 | 7600 | 8964 | 6007 | 7944 | 8117 | 7730 | 7858.1 | 1014.1 |
| 6 | 8688 | 6548 | 7644 | 6898 | 8324 | 6261 | 7983 | 7900 | 7530.8 | 870.2 |
| 7 | 7909 | 8210 | 6965 | 4913 | 7967 | 7491 | 8362 | 7628 | 7430.6 | 1107.7 |
| 8 | 7223 | 7481 | 7516 | 8283 | 5470 | 5415 | 7290 | 8850 | 7191.0 | 1210.7 |
| 9 | 7534 | 7892 | 6841 | 7293 | 6949 | 8439 | 7424 | 7955 | 7540.9 | 536.8 |
| 10 | 8312 | 6166 | 6955 | 6777 | 7178 | 8598 | 8266 | 8210 | 7557.8 | 897.0 |
| 11 | 8617 | 8542 | 5675 | 6685 | 7424 | 7717 | 6004 | 7828 | 7311.5 | 1098.2 |
| 12 | 5901 | 7698 | 7649 | 6645 | 9018 | 8504 | 7022 | 6875 | 7414.0 | 1015.7 |
| 13 | 8073 | 7122 | 6633 | 6521 | 6928 | 6444 | 6576 | 8872 | 7146.1 | 874.9 |
| 14 | 7165 | 8312 | 8343 | 8033 | 8632 | 7983 | 7311 | 6160 | 7742.4 | 814.4 |
| 15 | 8640 | 6593 | 7908 | 7388 | 6435 | 7390 | 6898 | 6547 | 7224.9 | 766.5 |

**31.23 Critical Dimensions.** When manufacturing semiconductor devices that are used in everyday electronic equipment, an important part of the process is in the formation of metal lines that connect devices to form the desired circuit. At a critical step in the manufacturing process, the width of a metal line is measured four times at the end of each hour of production. Table 31.7 gives $\bar{x}$ and $s$ for the first 21 samples, coded in units of nanometers meters ($10^{-9}$ meters) from the center of the specifications. The specifications allow a range of $\pm 4.5$ nanometers about the center. CRITDIM

**TABLE 31.7 $\bar{x}$ and $s$ for 21 samples critical dimension**

| Sample | $\bar{x}$ | $s$ | Sample | $\bar{x}$ | $s$ |
|---|---|---|---|---|---|
| 1 | −1.85 | 2.23 | 12 | 2.36 | 0.47 |
| 2 | −0.45 | 1.21 | 13 | 2.05 | 1.18 |
| 3 | 0.04 | 1.12 | 14 | 1.26 | 2.11 |
| 4 | −0.51 | 1.77 | 15 | 3.21 | 0.99 |
| 5 | −2.04 | 2.33 | 16 | 1.87 | 1.58 |
| 6 | 1.20 | 1.21 | 17 | 2.42 | 1.95 |
| 7 | 0.83 | 1.83 | 18 | 3.76 | 0.37 |
| 8 | 0.16 | 1.46 | 19 | 3.58 | 1.49 |
| 9 | 1.93 | 0.62 | 20 | 1.99 | 1.09 |
| 10 | 0.77 | 0.62 | 21 | 3.82 | 0.56 |
| 11 | 1.93 | 1.18 | | | |

(a) Make an $s$ chart based on past data, and comment on control of short-term process variation.

(b) Because the data are coded about the center of the specs, we have a given target $\mu = 0$ (as coded) for the process mean. Make an $\bar{x}$ chart and comment on control of long-term process variation. What special $\bar{x}$-type cause might explain the lack of control of $\bar{x}$?

**31.24 The Boston Marathon.** The Boston Marathon has been run each year since 1897. Winning times were highly variable in the early years, but control improved as the best runners became more professional. A clear downward trend continued until the 1980s. Rick plans to make a control chart for the winning times from 1950 to the present. The first few times are 153, 148, 152, 139, 141, and 138 minutes. Calculation from the winning times from 1950 to 2013 gives

$$\bar{x} = 133.954 \text{ minutes} \quad \text{and} \quad s = 6.366 \text{ minutes}$$

Rick draws a center line at $\bar{x}$ and control limits at $\bar{x} \pm 3s$ for a plot of individual winning times. Explain carefully why these control limits are too wide to effectively signal unusually fast or slow times.

## 31.8 Comments on Statistical Control

Having seen how $\bar{x}$ and $s$ (or $\bar{x}$ and $R$) charts work, we can turn to some important comments and cautions about statistical control in practice.

**Focus on the process rather than on the products.** This is a fundamental idea in statistical process control. We might attempt to attain high quality by careful inspection of the finished product, measuring every completed forging and reviewing every outgoing invoice and expense account payment. Inspection of finished products can ensure good quality, but it is expensive. Perhaps more important, final inspection comes too late: when something goes wrong early in a process, much bad product may be produced before final inspection discovers the problem. This adds to the expense because the bad product must then be scrapped or reworked.

The small samples that are the basis of control charts are intended to monitor the process at key points, not to ensure the quality of the particular items in the samples. If the process is kept in control, we know what to expect in the finished product. We want to do it right the first time, not inspect and fix finished product.

**Rational subgroups.** The interpretation of control charts depends on the distinction between $\bar{x}$-type special causes and $s$-type special causes. This distinction in turn depends on how we choose the samples from which we calculate $s$ (or $R$). We want the variation *within* a sample to reflect only the item-to-item chance variation that (when in control) results from many small common causes. Walter Shewhart, the founder of statistical process control, used the term **rational subgroup** to emphasize that we should think about the process when deciding how to choose samples. *rational subgroup*

### EXAMPLE 31.9 Random Sampling versus Rational Subgroups

A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. To monitor the compression process, we will measure the hardness of a sample from each 10 minutes' production of tablets. Should we choose a random sample of tablets from the several thousand produced in a 10-minute period?

A random sample would contain tablets spread across the entire 10 minutes. It fairly represents the 10-minute period, but that isn't what we want for process control. If the setting of the press drifts or a new lot of filler arrives during the 10 minutes, the variability of the sample will be increased. That is, a random sample contains both the short-term variation among tablets produced in quick succession and the longer-term variation among tablets produced minutes apart. We prefer to measure a rational subgroup of five consecutive tablets every 10 minutes. We expect the process to be stable during this very short time period so that variation within the subgroups is a benchmark against which we can see special cause variation.

Samples of consecutive items are rational subgroups when we are monitoring the output of a single activity that does the same thing over and over again. Several consecutive items is the most common type of sample for process control. There is no formula for choosing samples that are rational subgroups. You must think about causes of variation in your process and decide which you are willing to think of as common causes that you will not try to eliminate. Rational subgroups are samples chosen to express variation due to these causes and no others. Because the choice requires detailed process knowledge, we will usually accept samples of consecutive items as being rational subgroups.

**Why statistical control is desirable.** To repeat, if the process is kept in control, we know what to expect in the finished product. The process mean $\mu$ and standard deviation $\sigma$ remain stable over time, so (assuming Normal variation) the 99.7 part of the 68–95–99.7 rule tells us that almost all measurements on individual products will lie in the range $\mu \pm 3\sigma$. These are sometimes called the **natural tolerances** for the product. *Be careful to distinguish $\mu \pm 3\sigma$, the range we expect for individual measurements, from the $\bar{x}$ chart control limits $\mu \pm 3\sigma/\sqrt{n}$, which mark off the expected range of sample means.*

***natural tolerances***

CAUTION

## EXAMPLE 31.10 Natural Tolerances for Mesh Tension

The process of setting the mesh tension on computer monitors has been operating in control. The $\bar{x}$ and $s$ charts were based on $\mu = 275$ mV and $\sigma = 43$ mV. The $s$ chart in Figure 31.8 and your calculation in Exercise 31.21 suggest that the process $\sigma$ is now less than 43 mV. We may prefer to calculate the natural tolerances from the recent data on 20 samples (80 monitors) in Table 31.1. The estimate of the mean is $\bar{\bar{x}} = 275.07$, very close to the target value.

Now a subtle point arises. The estimate $\hat{\sigma} = \bar{s}/c_4$ used for past-data control charts is based entirely on variation *within the samples*. That's what we want for control charts because within-sample variation is likely to be "pure common cause" variation. Even when the process is in control, there is some additional variation from sample to sample, just by chance. So the variation in the process output will be greater than the variation within samples. *To estimate the natural tolerances, we should estimate $\sigma$ from all 80 individual monitors rather than by averaging the 20 within-sample standard deviations.* The standard deviation for all 80 mesh tensions is

$$s = 38.38$$

(For a sample of size 80, $c_4$ is very close to 1, so we can ignore it.)

We are therefore confident that almost all individual monitors will have mesh tension

$$\bar{\bar{x}} \pm 3s = 275.07 \pm (3)(38.38) = 275 \pm 115$$

We expect mesh tension measurements to vary between 160 and 390 mV. You see that the variability of individual measurements is wider than the variability of sample means used for the control limits of the $\bar{x}$ chart.

The natural tolerances in Example 31.10 depend on the fact that the mesh tensions of individual monitors follow a Normal distribution. We know that the process was in control when the 80 measurements in Table 31.1 were made, so we can graph them to assess Normality.

## APPLY YOUR KNOWLEDGE

**31.25 No Incoming Inspection.** The computer makers who buy monitors require that the monitor manufacturer practice statistical process control and submit control charts for verification. This allows the computer makers to eliminate inspection of monitors as they arrive, a considerable cost saving. Explain carefully why incoming inspection can safely be eliminated.

**31.26 Natural Tolerances.** Table 31.6 gives data on hospital losses for samples of DRG 209 patients. The distribution of losses has been stable over time. What are the natural tolerances within which you expect losses on nearly all such patients to fall? DATA DRG2

**31.27 Normality?** Do the losses on the 120 individual patients in Table 31.6 appear to come from a single Normal distribution? Make a graph and discuss what it shows. Are the natural tolerances you found in the previous exercise trustworthy? DATA DRG2

## 31.9 Don't Confuse Control with Capability

A process in control is stable over time. We know how much variation the finished product will show. Control charts are, so to speak, the voice of the process telling us what state it is in. *There is no guarantee that a process in control produces products of satisfactory quality.* "Satisfactory quality" is measured by comparing the product to some standard outside the process, set by technical specifications, customer expectations, or the goals of the organization. These external standards are unrelated to the internal state of the process, which is all that statistical control pays attention to.

CAUTION

### Capability

**Capability** refers to the ability of a process to meet or exceed the requirements placed on it.

Capability has nothing to do with control—except for the very important point that if a process is not in control, it is hard to tell if it is capable or not. There are many ways in which the capability of a process can be quantified.[14]

### EXAMPLE 31.11 Capability

The primary customer for our monitors is a large maker of computers. The customer informed us that adequate image quality requires that the mesh tension lie between 100 mV and 400 mV. Because the mesh-tensioning process is in control, we know that almost all monitors will have mesh tension between 160 mV and 390 mV (Example 31.10). The process is capable of meeting the customer's requirement.

Figure 31.15 compares the distribution of mesh tension for individual monitors with the customer's specifications. The distribution of tension is approximately

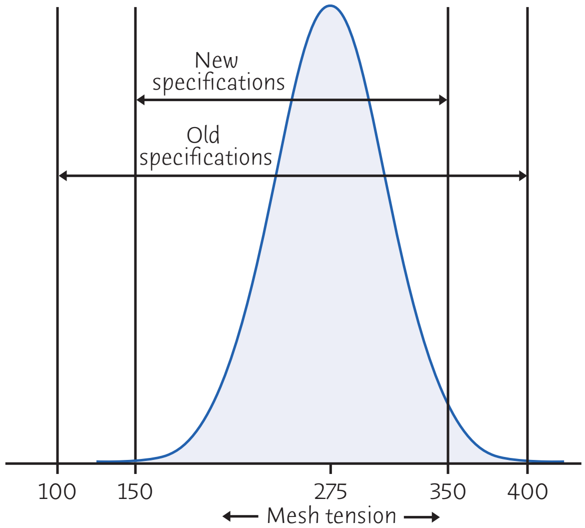



**FIGURE 31.15** Comparison of the distribution of mesh tension (Normal curve) with original and tightened specifications, for Example 31.11. The process in its current state is not capable of meeting the new specifications.

Normal, and we estimate its mean to be very close to 275 mV and the standard deviation to be about 38.4 mV. The distribution is safely within the specifications.

Times change, however. As computer buyers demand better screen quality, the computer maker restudies the effect of mesh tension and decides to require that tension lie between 150 and 350 mV. These new specification limits also appear in Figure 31.15. The process is not capable of meeting the new requirements. The process remains in control. The change in its capability is entirely due to a change in external requirements.

Because the mesh-tensioning process is in control, we know that it is not capable of meeting the new specifications. That's an advantage of control, but the fact remains that control does not guarantee capability. *If a process that is in control does not have adequate capability, fundamental changes in the process are needed.* The process is doing as well as it can and displays only the chance variation that is natural to its present state. Better training for workers, new equipment, or more uniform material may improve capability, depending on the findings of a careful investigation.

## APPLY YOUR KNOWLEDGE

**31.28 Describing Capability.** If the mesh tension of individual monitors follows a Normal distribution, we can describe capability by giving the percent of monitors that meet specifications. The old specifications for mesh tension are 100–400 mV. The new specifications are 150–350 mV. Because the process is in control, we can estimate that tension has mean 275 mV and standard deviation 38.4 mV.

(a) What percent of monitors meet the old specifications?

(b) What percent meet the new specifications?

**31.29 Improving Capability.** The center of the specifications for mesh tension in the previous exercise is 250 mV, but the center of our process is 275 mV. We can improve capability by adjusting the process to have center 250 mV. This is an easy adjustment that does not change the process variation. What percent of monitors now meet the new specifications?

**31.30 Mounting-Hole Distances.** Figure 31.11 displays a record sheet for 18 samples of distances between mounting holes in an electrical meter. The data file adds $\bar{x}$ and $s$ for each sample. In Exercise 31.16, you found that Sample 5 was out of control on the process-monitoring $s$ chart. The special cause responsible was found and removed. Based on the 17 samples that were in control, what are the natural tolerances for the distance between the holes? DATA MOUNT

**31.31 Mounting-Hole Distances, continued.** The record sheet in Figure 31.11 gives the specifications as $0.6054 \pm 0.0010$ inch. That is $54 \pm 10$ as the data are coded on the record sheet. Assuming that the distance varies Normally from meter to meter, about what percent of meters meet the specifications?

## 31.10 Control Charts for Sample Proportions

We have considered control charts for just one kind of data: measurements of a quantitative variable in some meaningful scale of units. We describe the distribution of measurements by its center and variability and use $\bar{x}$ and $s$ or $\bar{x}$ and $R$ charts for process control. There are control charts for other statistics that are appropriate for other kinds of data. The most common of these is the $p$ chart for use when the data are proportions.

### *p* Chart

A ***p* chart** is a control chart based on plotting sample proportions $\hat{p}$ from regular samples from a process against the order in which the samples were taken.

### EXAMPLE 31.12 *p* Chart Settings

Here are three examples of the usefulness of $p$ charts:

- Measure two dimensions of a manufactured part and also grade its surface finish by eye. The part conforms if both dimensions lie within their specifications and the finish is judged acceptable. Otherwise, it is nonconforming. Plot the proportion of nonconforming parts in samples of parts from each shift.
- An urban school system records the percent of its eighth-grade students who are absent three or more days each month. Because students with high absenteeism in eighth grade often fail to complete high school, the school system has launched programs to reduce absenteeism. These programs include calls to parents of absent students, public-service messages to change community expectations, and measures to ensure that the schools are safe and attractive. A $p$ chart will show if the programs are having an effect.
- Each day a popular shopping website monitors the percent of customers who complete a purchase once items are added to their shopping cart. Because they are seeing a large percentage of customers who leave their selections in the shopping cart without finishing the buying process, they are going to start offering different types of incentives (free shipping, discounted prices, and free gift wrap) to try to increase the number of customers that complete the purchase. Each incentive that is offered will be tracked separately to see which of the incentives has the most improvement in enticing people to complete the purchase. The company is hoping that at least one of these incentives increases the percent of customers who complete their purchase.

The manufacturing example illustrates an advantage of $p$ charts: they can combine several specifications in a single chart. Nonetheless, $p$ charts have been rendered outdated in many manufacturing applications by improvements in typical levels of quality. For example, Delphi, the largest North American auto electronics manufacturer, said that it reduced its proportion of problem parts from 200 per million in 1997 to 20 per million in 2001.[15] At either of these levels, even large samples of parts will rarely contain any bad parts. The sample proportions will almost all be zero, so that plotting them is uninformative. It is better to choose important measured characteristics—voltage at a critical circuit point, for example—and keep $\bar{x}$ and $s$ charts. Even if the voltage is satisfactory, quality can be improved by moving it yet closer to the exact voltage specified in the design of the part.

The school absenteeism example is a management application of $p$ charts. More than 20% of all American eighth-graders miss three or more days of school per month, and this proportion is higher in large cities. A $p$ chart will be useful. Proportions of "things going wrong" are often higher in business processes than in manufacturing, so that $p$ charts are an important tool in business.

The website shopping example is a business application of $p$ charts. A $p$ chart will be useful to determine if any of the incentives show an increase or shift in the percent of customers who complete a purchase.

## 31.11 Control Limits for *p* Charts

We studied the sampling distribution of a sample proportion $\hat{p}$ in Chapter 22. The center line and control limits for a $3\sigma$ control chart follow directly from the facts stated there, in the box on page 508. We ought to call such charts "$\hat{p}$ charts" because they plot sample proportions. Unfortunately, they have always been called $p$ charts in quality control circles. We will keep the traditional name but also keep our usual notation: $p$ is a *process* proportion and $\hat{p}$ is a *sample* proportion.

### *p* Chart Using Past Data

Take regular samples from a process that has been in control. Estimate the process proportion $p$ of "successes" by

$$\bar{p} = \frac{\text{total number of successes in past samples}}{\text{total number of individuals in these samples}}$$

The center line and control limits for a ***p* chart** for future samples of size $n$ are

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{CL} = \bar{p}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Common **out-of-control signals** are one sample proportion $\hat{p}$ outside the control limits or a run of nine sample proportions on the same side of the center line.

If we have $k$ past samples of the *same* size $n$, then $\bar{p}$ is just the average of the $k$ sample proportions. In some settings, you may meet samples of unequal size—differing numbers of students enrolled in a month or differing numbers of parts inspected in a shift. The average $\bar{p}$ estimates the process proportion $p$ even when the sample sizes vary. Note that the control limits use the actual size $n$ of a sample.

## EXAMPLE 31.13 Reducing Absenteeism

ABSENT

Unscheduled absences by clerical and production workers are an important cost in many companies. You have been asked to improve absenteeism in a production facility where 12% of the workers are now absent on a typical day.

Start with data: the Pareto chart in Figure 31.16 shows that there are major differences among supervisors in the absenteeism rate of their workers. You retrain all the supervisors in human relations skills, using B, E, and H as discussion leaders. In addition, a trainer works individually with supervisors I and D. You also improve lighting and other work conditions.

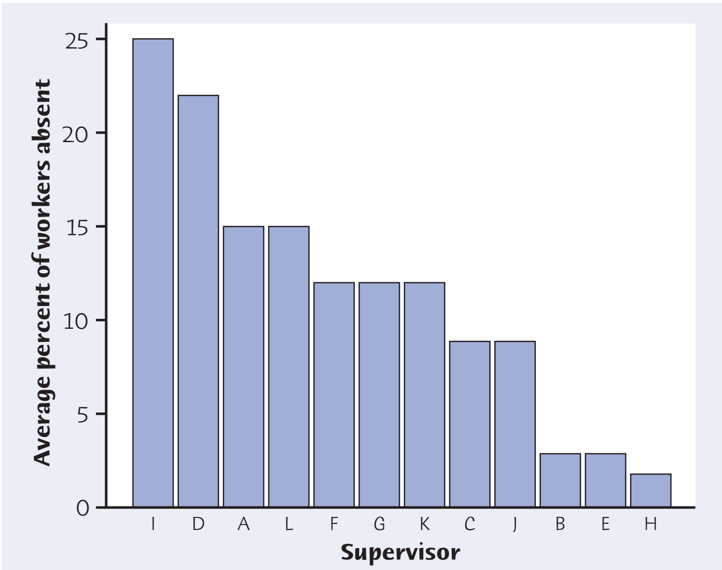



**FIGURE 31.16** Pareto chart of the average absenteeism rate for workers reporting to each of 12 supervisors, for Example 31.13.

Are your actions effective? You hope to see a reduction in absenteeism. To view progress (or lack of progress), you will keep a $p$ chart of the proportion of absentees. The plant has 987 production workers. For simplicity, you just record the number who are absent from work each day. Only unscheduled absences count, not planned time off such as vacations.

Each day you will plot

$$\hat{p} = \frac{\text{number of workers absent}}{987}$$

You first look back at data for the past three months. There were 64 workdays in these months. The total of workdays available for the workers was

$$(64)(987) = 63{,}168 \text{ person-days}$$

Absences among all workers totaled 7580 person-days. The average daily proportion absent was, therefore,

$$\bar{p} = \frac{\text{total days absent}}{\text{total days available for work}}$$
$$= \frac{7580}{63{,}168} = 0.120$$

The daily rate has been in control at this level.

These past data allow you to set up a $p$ chart to monitor future proportions absent:

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.120 + 3\sqrt{\frac{(0.120)(0.880)}{987}}$$

$$= 0.120 + 0.031 = 0.151$$

$$\text{CL} = \bar{p} = 0.120$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.120 - 3\sqrt{\frac{(0.120)(0.880)}{987}}$$

$$= 0.120 - 0.031 = 0.089$$

Table 31.8 gives the data for the next four weeks. Figure 31.17 is the $p$ chart.

**TABLE 31.8 Proportions of workers absent during four weeks**

| | M | T | W | Th | F | M | T | W | Th | F |
|---|---|---|---|---|---|---|---|---|---|---|
| Workers absent | 129 | 121 | 117 | 109 | 122 | 119 | 103 | 103 | 89 | 105 |
| Proportion $\hat{p}$ | 0.131 | 0.123 | 0.119 | 0.110 | 0.124 | 0.121 | 0.104 | 0.104 | 0.090 | 0.106 |
| | M | T | W | Th | F | M | T | W | Th | F |
| Workers absent | 99 | 92 | 83 | 92 | 92 | 115 | 101 | 106 | 83 | 98 |
| Proportion $\hat{p}$ | 0.100 | 0.093 | 0.084 | 0.093 | 0.093 | 0.117 | 0.102 | 0.107 | 0.084 | 0.099 |

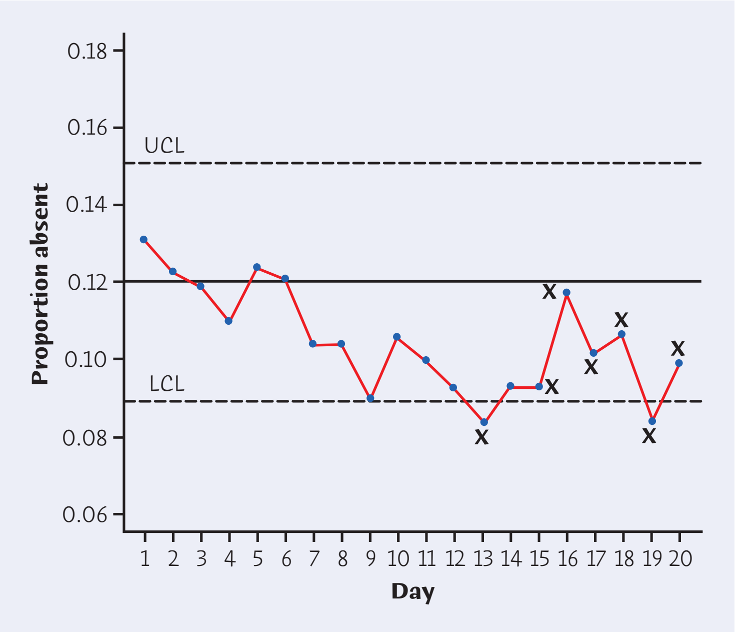



**FIGURE 31.17**
$p$ chart for daily proportion of workers absent over a four-week period, for Example 31.13. The lack of control shows an improvement (decrease) in absenteeism. Update the chart to continue monitoring the process.

Figure 31.17 shows a clear downward trend in the daily proportion of workers who are absent. Days 13 and 19 lie below LCL, and a run of nine days below the center line is achieved at Day 15 and continues. The points marked "x" are, therefore, all out of control. It appears that a special cause (the various actions you took) has reduced the absenteeism rate from around 12% to around 10%. The data for the last two weeks suggest that the rate has stabilized at this level. You will update the chart based on the new data. If the rate does not decline further (or even rises again as the effect of your actions wears off), you will consider further changes.

Example 31.13 is a bit oversimplified. The number of workers available did not remain fixed at 987 each day. Hirings, resignations, and planned vacations change the number a bit from day to day. The control limits for a day's $\hat{p}$ depend on $n$, the number of workers that day. If $n$ varies, the control limits will move in and out from day to day. Software will do the extra arithmetic needed for a different $n$ each day, but as long as the count of workers remains close to 987, the greater detail will not change your conclusion.

A single $p$ chart for all workers is not the only, or even the best, choice in this setting. Because of the important role of supervisors in absenteeism, it would be wise to also keep separate $p$ charts for the workers under each supervisor. These charts may show that you must reassign some supervisors.

## APPLY YOUR KNOWLEDGE

**31.32 Setting Up a *p* Chart.** After inspecting Figure 31.17, you decide to monitor the next four weeks' absenteeism rates using a center line and control limits calculated from the last two weeks of data recorded in Table 31.8. Find $\bar{p}$ for these 10 days and give the new values of CL, LCL, and UCL. (Until you have more data, these are trial control limits. As long as you are taking steps to improve absenteeism, you have not reached the process-monitoring stage.) ABSENT

**31.33 Ride-Share.** A luxury sports car dealership offers its clients a complimentary shuttle service to and from the dealership when they are having their car serviced. Currently, the dealership has a driver to shuttle clients to and from locations. However, using its own driver has drawbacks, because it is a single driver and clients sometimes have to wait an extended period of time in order to get to their destinations. In hopes of improving service and pleasing clients, the dealership decides to change from an in-house shuttle service to using a ride-share service that is still free to the client. The dealership wants to monitor the impact of this change to see if the percentage of clients who take advantage of their transportation service changes. The first thing it does is look at historical data to determine the percentage of clients who have been using the shuttle service. It looked at records for the past 12 months. The average number of clients who visit the dealership each month is 215, with relatively little month-to-month variation. During the past 12 months, a total of 724 clients have requested rides.

(a) What is the estimated total number of clients during these 12 months? What is $\bar{p}$?

(b) Give the center line and control limits for a $p$ chart on which to plot the future monthly proportions of clients requesting rides.

**31.34 Lost Baggage.** The Department of Transportation reports that about one of every 208 passengers on domestic flights of the 18 largest U.S. airlines files a report of mishandled baggage. Starting with this information, you plan to sample records for 1000 passengers per day at a large airport to monitor the effects of efforts to reduce mishandled baggage. What are the initial center line and control limits for a chart of the daily proportion of mishandled-baggage reports? (You will find that LCL $< 0$. Because proportions $\hat{p}$ are always zero or positive, take LCL $= 0$.)

**31.35 Smartphones.** The manufacturer of a smartphone does rigorous testing to ensure its phones can perform under adverse conditions. This includes inducing electrical shocks, dropping, bending, getting it wet and dirty, and other ways in which it might be treated. In all, there are thousands of tests a phone will see before the manufacturing process is deemed fit to mass produce a product. In the past three months, testing has shown a total of 180 failures from all of the phones tested. On average, 3460 phones are tested per month. What are the initial center line and control limits for a chart of the monthly proportion of failures for this type of phone? With this percentage of failures, would you purchase a phone from this manufacturer?

**31.36 School Absenteeism.** Here are data from an urban school district on the number of eighth-grade students with three or more unexcused absences from school during each month of a school year. Because the total number of eighth-graders changes a bit from month to month, these totals are also given for each month. ABSENT2

| | Sept. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Apr. | May | June |
|---|---|---|---|---|---|---|---|---|---|---|
| Students | 811 | 847 | 839 | 842 | 818 | 820 | 831 | 825 | 802 | 783 |
| Absent | 251 | 309 | 324 | 295 | 261 | 282 | 304 | 284 | 263 | 304 |

(a) Find $\bar{p}$. Because the number of students varies from month to month, also find $\bar{n}$, the average per month.

(b) Make a $p$ chart using control limits based on $\bar{n}$ students each month. Comment on control.

(c) The exact control limits are different each month because the number of students $n$ is different each month. This situation is common in using $p$ charts. What are the exact limits for October and June, the months with the largest and smallest $n$? Add these limits to your $p$ chart, using short lines spanning a single month. Do exact limits affect your conclusions?

## CHAPTER 31 SUMMARY

### Chapter Specifics

- Work is organized in **processes**, chains of activities that lead to some result. Use **flowcharts** and **cause-and-effect diagrams** to describe processes. Other graphs such as **Pareto charts** are often useful.
- All processes have variation. If the pattern of variation is stable over time, the process is **in statistical control**. **Control charts** are statistical plots intended to warn when a process is **out of control**.
- Standard $3\sigma$ **control charts** plot the values of some statistic $Q$ for regular samples from the process against the time order of the samples. The **center line** is at the mean of $Q$. The **control limits** lie three standard deviations of $Q$ above and below the center line. A point outside the control limits is an **out-of-control signal**. For **process monitoring** of a process that has been in control, the mean and standard deviation are based on past data from the process and are updated regularly.
- When we measure some quantitative characteristic of the process, we use **$\bar{x}$ and $s$ charts** for process control. The $s$ chart monitors variation within individual samples. If the $s$ chart is in control, the $\bar{x}$ chart monitors variation from sample to sample. To interpret the charts, always look first at the $s$ chart.
- An **$R$ chart** based on the **range** of observations in a sample is often used in place of an $s$ chart. Interpret $\bar{x}$ and $R$ charts exactly as you would interpret $\bar{x}$ and $s$ charts.
- It is common to use **out-of-control signals** in addition to "one point outside the control limits." In particular, a **runs signal** for the $\bar{x}$ chart allows the chart to respond more quickly to a gradual drift in the process center.

- **Control charts based on past data** are used at the **chart setup** stage for a process that may not be in control. Start with control limits calculated from the same past data that you are plotting. Beginning with the $s$ chart, narrow the limits as you find special causes, and remove the points influenced by these causes. When the remaining points are in control, use the resulting limits to monitor the process.
- Statistical process control maintains quality more economically than inspecting the final output of a process. Samples that are **rational subgroups** are important to effective control charts. A process in control is stable, so that we can predict its behavior. If individual measurements have a Normal distribution, we can give the **natural tolerances**.
- A process is **capable** if it can meet the requirements placed on it. Control (stability over time) does not in itself improve capability. Remember that control describes the internal state of the process, whereas capability relates the state of the process to external specifications.
- There are control charts for several different types of process measurements. One important type is the **$p$ chart** for sample proportions $\hat{p}$.
- The interpretation of $p$ charts is very similar to that of $\bar{x}$ charts. The out-of-control signals used are also the same.

## Statistics in Summary

Here are the most important skills you should have acquired from reading this chapter.

**A. Processes**

1. Describe the process leading to some desired output using flowcharts and cause-and-effect diagrams.
2. Choose promising targets for process improvement, combining the process description with data collection and tools such as Pareto charts.
3. Demonstrate understanding of statistical control, common causes, and special causes by applying these ideas to specific processes.
4. Choose rational subgroups for control charting based on an understanding of the process.

**B. Control Charts**

1. Make $\bar{x}$ and $s$ charts using given values of the process $\mu$ and $\sigma$ (usually from large amounts of past data) for monitoring a process that has been in control.
2. Demonstrate understanding of the distinction between short-term (within sample) and longer-term (across samples) variation by identifying possible $\bar{x}$-type and $s$-type special causes for a specific process.
3. Interpret $\bar{x}$ and $s$ charts, starting with the $s$ chart. Use both one-point-out and runs signals.
4. Estimate the process $\mu$ and $\sigma$ from recent samples.
5. Set up initial control charts using recent process data, removing special causes, and basing an initial chart on the remaining data.
6. Decide when a $p$ chart is appropriate. Make a $p$ chart based on past data.

**C. Process Capability**

1. Know the distinction between control and capability and apply this distinction in discussing specific processes.
2. Give the natural tolerances for a process in control, after verifying Normality of individual measurements on the process.

## Link It

In this chapter, we apply and extend many of the ideas discussed previously to understanding processes. Flowcharts, cause-and-effect diagrams, Pareto charts, and control charts are graphical displays for understanding processes, extending the tools discussed in Part I. Choosing rational subgroups reminds us of sample design, discussed in Part II. Evaluating capability of a process to meet or exceed the requirement placed on it reminds us of Normal probability calculations, discussed in Part III. The use of control limits to determine whether a process is in control reminds us of hypothesis testing for means and proportions, discussed in Part IV. Thus, we see many of the topics discussed previously applied in new ways to help us evaluate processes.

## Macmillan Learning Online Resources

If you are having difficulty with any of the sections of this chapter, this online resource should help prepare you to solve the exercises at the end of this chapter:

- LearningCurve provides you with a series of questions about the chapter that adjust to your level of understanding.

## CHECK YOUR SKILLS

**31.37** A manufacturer of ultrasonic parking sensors samples four sensors during each production shift. The expectation is that the sensor will initially alarm if there is an object within 48 inches of the sensor. The sensors are put on a rack and an object is moved toward the sensors at a 90° angle until it alarms. The distance from the object to the sensor is recorded. The process mean should be $\mu = 48$ inches. Past experience indicates that the response varies with $\sigma = 0.8$ inch. The mean response distance is plotted on an $\bar{x}$ control chart. The center line for this chart is

(a) 0.8 inch. (b) 48 inches. (c) 4 inches.

**31.38** In Exercise 31.37, the UCL for the chart is

(a) 50.4 inches. (b) 49.2 inches. (c) 51 inches.

**31.39** In Exercise 31.37, suppose the standard deviation $s$ for each hour's sample is plotted on an $s$ control chart. The center line for this chart is

(a) 0.8 inch. (b) 0.74 inch. (c) 0.31 inch.

**31.40** In Exercise 31.37, suppose the standard deviation $s$ for each hour's sample is plotted on an $s$ control chart. The LCL for this chart is

(a) 1.67 inches. (b) 0.8 inch. (c) 0 inches.

**31.41** Samples of size 4 are taken from a manufacturing process every hour. A quality characteristic is measured, and $\bar{x}$ and $s$ are measured for each sample. The process is in control, and after 25 samples, we compute $\bar{\bar{x}} = 48.4$ and $\bar{s} = 5.1$. The LCL for an $\bar{x}$ control chart based on these data is

(a) 40.1. (b) 33.1. (c) 31.8.

**31.42** A process produces rubber fan belts for automobiles. The process is in control, and 100 belts are inspected each day for a period of 20 days. The proportion of nonconforming belts found over this 20-day period is $\bar{p} = 0.12$. Based on these data, a $p$ chart for future samples of size 100 would have center line

(a) 0.12. (b) 12.0. (c) 240.

**31.43** Referring to the previous exercise, the UCL for a $p$ chart for future samples of size 100 would be

(a) 0.22. (b) 0.15. (c) 0.12.

## CHAPTER 31 EXERCISES

**31.44 Enlighten management.** A manager who knows no statistics asks you, "What does it mean to say that a process is in control? Is being in control a guarantee that the quality of the product is good?" Answer these questions in plain language that the manager can understand.

**31.45 Special causes.** Is each of the following examples of a special cause most likely to result first in (i) a sudden change in level on the $s$ or $R$ chart, (ii) a sudden change in level on the $\bar{x}$ chart, or (iii) a gradual drift up or down on the $\bar{x}$ chart? In each case, briefly explain your reasoning.

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(a) An airline pilots' union puts pressure on management during labor negotiations by asking its members to "work to rule" in doing the detailed checks required before a plane can leave the gate.

(b) Measurements of part dimensions that were formerly made by hand are now made by a very accurate laser system. (The process producing the parts does not change—measurement methods can also affect control charts.)

(c) Inadequate air conditioning on a hot day allows the temperature to rise during the afternoon in an office that prepares a company's invoices.

**31.46 Deming speaks.** The quality guru W. Edwards Deming (1900–1993) taught (among much else) that[16]

(a) "People work in the system. Management creates the system."

(b) "Putting out fires is not improvement. Finding a point out of control, finding the special cause and removing it, is only putting the process back to where it was in the first place. It is not improvement of the process."

(c) "Eliminate slogans, exhortations and targets for the workforce asking for zero defects and new levels of productivity."

(d) "No one can guess the future loss of business from a dissatisfied customer. The cost to replace a defective item on the production line is fairly easy to estimate, but the cost of a defective item that goes out to a customer defies measure."

Choose one of these sayings. Explain carefully what facts about improving quality the saying attempts to summarize.

**31.47 Pareto charts.** You manage the customer service operation for a maker of electronic equipment sold to business customers. Traditionally, the most common complaint is that equipment does not operate properly when installed, but attention to manufacturing and installation quality will reduce these complaints. You hire an outside firm to conduct a sample survey of your customers. Here are the percents of customers with each of several kinds of complaints: DATA COMPLNT

| Category | Percent |
|---|---|
| Accuracy of invoices | 27 |
| Clarity of operating manual | 6 |
| Complete invoice | 25 |
| Complete shipment | 16 |
| Correct equipment shipped | 15 |
| Ease of obtaining invoice adjustments/credits | 34 |
| Equipment operates when installed | 5 |
| Meeting promised delivery date | 11 |
| Sales rep returns calls | 3 |
| Technical competence of sales rep | 12 |

(a) Why do the percents not add to 100%?

(b) Make a Pareto chart. What area would you choose as a target for improvement?

**31.48 What type of chart?** What type of control chart or charts would you use as part of efforts to improve each of the following performance measures in a college admissions office? Explain your choices.

(a) Time to acknowledge receipt of an application

(b) Percent of admission offers accepted

(c) Student participation in a healthy meal plan

**31.49 What type of chart?** What type of control chart or charts would you use as part of efforts to improve each of the following performance measures in an online business information systems department? Explain your choices.

(a) Website availability

(b) Time to respond to requests for help

(c) Percent of website changes not properly documented

**31.50 Online purchases.** An online shopping website ships on average 11,000 orders each day. At the present time, about 5 out of 1000 orders that are shipped are shipped with the incorrect merchandise. As part of an effort to reduce errors in the fulfillment of orders, robots have been inserted into the process of filling orders. You will monitor the proportion of incorrect orders discovered each day and determine if the robots improve the process. What type of control chart will you use? What are the initial center line and control limits? If there has been an improvement, what do you expect to see on the control chart?

**31.51 Pareto charts.** A pizza delivery company is working to improve customer satisfaction. On each receipt, a survey link is included so that customers can provide feedback. To encourage the feedback, a discount is taken off their next delivery order if the survey is completed. The company received both positive and not so positive feedback. The top complaints are listed in the table: DATA PIZZA

| Problem | Percent |
|---|---|
| Charged too much for the order | 4 |
| Pizza looked bad | 10 |
| Other reason | 7 |
| Quality and quantity of ingredients | 3 |
| Rudeness of employee who took order | 2 |
| The order took too long | 45 |
| The pizza was cold upon arrival | 22 |
| Wrong order | 7 |
| Total | 100 |

Make a Pareto chart. What is the first item that should be considered to improve customer satisfaction?

**31.52 Assemble parts.** When parts are machined, it is important that they are created with enough precision so that they can be assembled with other parts. No machine can hold dimensions exactly, so it is important that there is an agreed upon level of variation. A company that creates nuts and bolts makes their parts with specific tolerances that follow rules established by an international standard. The nut (or hole) has a slightly larger tolerance than the bolt (or shaft) so that the nuts and bolts will work together. This company uses process control, with samples taken five times during each hour, to ensure the processes are stable and running on target. For the nuts, the process is running with $\bar{\bar{x}} = 10.004$ mm and a sigma estimate of all measurements $s = 0.002$ mm. For the bolts, $\bar{\bar{x}} = 10.000$ mm with a sigma estimate of all measurements $s = 0.001$ mm. Compute the natural tolerances for both the nuts and bolts. What issue do you see with where the process is currently running?

**31.53 Piston rings.** The inside diameter of automobile engine piston rings is important to the proper functioning of the engine. The manufacturer checks the control of the piston ring forging process by measuring a sample of five consecutive items during each hour's production. The target diameter for a ring is $\mu = 74.000$ millimeters. The process has been operating in control with center close to the target and $\sigma = 0.015$ millimeter.


*Oliver Zivkovic/Getty Images*

(a) What center line and control limits should be drawn on the $s$ chart? On the $\bar{x}$ chart?

(b) A different manufacturer creates the pistons in which the rings will be fit. This manufacturer has a target value of 73.945 mm for the piston diameter. The manufacturer checks control of the piston diameter four times each hour. Recently, the process has been running high with $\mu = 74.000$ millimeters and a $\sigma = 0.005$ millimeter. Do you see any issues that might arise for the manufacturer of the engine when the two parts from the different manufacturers are assembled?

**31.54 $p$ charts are out of date.** A manufacturer of consumer electronic equipment makes full use not only of statistical process control, but also of automated testing equipment that efficiently tests all completed products. Data from the testing equipment show that finished products have only 3.0 defects per million opportunities.

(a) What is $\bar{p}$ for the manufacturing process? If the process turns out 4000 pieces per day, how many defects do you expect to see per day? In a typical month of 24 working days, how many defects do you expect to see?

(b) What are the center line and control limits for a $p$ chart for plotting daily defect proportions?

(c) Explain why a $p$ chart is of no use at such high levels of quality.

**31.55 Manufacturing isn't everything.** Because the manufacturing quality in Exercise 31.54 is so high, the process of writing up orders is the major source of quality problems: the defect rate there is 9000 per million opportunities. The manufacturer processes about 600 orders per month.

(a) What is $\bar{p}$ for the order-writing process? How many defective orders do you expect to see in a month?

(b) What are the center line and control limits for a $p$ chart for plotting monthly proportions of defective orders? What is the smallest number of bad orders in a month that will result in a point above the upper control limit?

*Table 31.9 gives process control samples for a study of response times to customer calls arriving at a corporate call center. A sample of six calls is recorded each shift for quality improvement purposes. The time from the first ring until a representative answers the call is recorded. Table 31.9 gives data for 50 shifts, with 300 calls total.[17] Exercises 31.56 through 31.58 make use of this setting.*

**TABLE 31.9 Fifty control chart samples of call center response times (seconds)**

| Sample | Time | | | | | | Sample Mean | Standard Deviation |
|---|---|---|---|---|---|---|---|---|
| 1 | 59 | 13 | 2 | 24 | 11 | 18 | 21.2 | 19.93 |
| 2 | 38 | 12 | 46 | 17 | 77 | 12 | 33.7 | 25.56 |
| 3 | 46 | 44 | 4 | 74 | 41 | 22 | 38.5 | 23.73 |
| 4 | 25 | 7 | 10 | 46 | 78 | 14 | 30.0 | 27.46 |
| 5 | 6 | 9 | 122 | 8 | 16 | 15 | 29.3 | 45.57 |
| 6 | 17 | 17 | 9 | 15 | 24 | 70 | 25.3 | 22.40 |
| 7 | 9 | 9 | 10 | 32 | 9 | 68 | 22.8 | 23.93 |
| 8 | 8 | 10 | 41 | 13 | 17 | 50 | 23.2 | 17.79 |
| 9 | 12 | 82 | 97 | 33 | 76 | 56 | 59.3 | 32.11 |
| 10 | 42 | 19 | 14 | 21 | 12 | 44 | 25.3 | 14.08 |
| 11 | 63 | 5 | 21 | 11 | 47 | 8 | 25.8 | 23.77 |

*continued*

| Sample | Time | | | | | | Sample Mean | Standard Deviation |
|---|---|---|---|---|---|---|---|---|
| 12 | 12 | 4 | 111 | 37 | 12 | 24 | 33.3 | 39.76 |
| 13 | 43 | 37 | 27 | 65 | 32 | 3 | 34.5 | 20.32 |
| 14 | 9 | 26 | 5 | 10 | 30 | 27 | 17.8 | 10.98 |
| 15 | 21 | 14 | 19 | 44 | 49 | 10 | 26.2 | 16.29 |
| 16 | 24 | 11 | 10 | 22 | 43 | 70 | 30.0 | 22.93 |
| 17 | 27 | 10 | 32 | 96 | 11 | 29 | 34.2 | 31.71 |
| 18 | 7 | 28 | 22 | 17 | 9 | 24 | 17.8 | 8.42 |
| 19 | 15 | 14 | 34 | 5 | 38 | 29 | 22.5 | 13.03 |
| 20 | 16 | 65 | 6 | 5 | 58 | 17 | 27.8 | 26.63 |
| 21 | 7 | 44 | 14 | 16 | 4 | 46 | 21.8 | 18.49 |
| 22 | 32 | 52 | 75 | 11 | 11 | 17 | 33.0 | 25.88 |
| 23 | 31 | 8 | 36 | 25 | 14 | 85 | 33.2 | 27.45 |
| 24 | 4 | 46 | 23 | 58 | 5 | 54 | 31.7 | 24.29 |
| 25 | 28 | 6 | 46 | 4 | 28 | 11 | 20.5 | 16.34 |
| 26 | 111 | 6 | 3 | 83 | 27 | 6 | 39.3 | 46.34 |
| 27 | 83 | 27 | 2 | 56 | 26 | 21 | 35.8 | 28.88 |
| 28 | 276 | 14 | 30 | 8 | 7 | 12 | 57.8 | 107.20 |
| 29 | 4 | 29 | 21 | 23 | 4 | 14 | 15.8 | 10.34 |
| 30 | 23 | 22 | 19 | 66 | 51 | 60 | 40.2 | 21.22 |
| 31 | 14 | 111 | 20 | 7 | 7 | 87 | 41.0 | 45.82 |
| 32 | 22 | 11 | 53 | 20 | 14 | 41 | 26.8 | 16.56 |
| 33 | 30 | 7 | 10 | 11 | 9 | 9 | 12.7 | 8.59 |
| 34 | 101 | 55 | 18 | 20 | 77 | 14 | 47.5 | 36.16 |
| 35 | 13 | 11 | 22 | 15 | 2 | 14 | 12.8 | 6.49 |
| 36 | 20 | 83 | 25 | 10 | 34 | 23 | 32.5 | 25.93 |
| 37 | 21 | 5 | 14 | 22 | 10 | 68 | 23.3 | 22.82 |
| 38 | 8 | 70 | 56 | 8 | 26 | 7 | 29.2 | 27.51 |
| 39 | 15 | 7 | 9 | 144 | 11 | 109 | 49.2 | 60.97 |
| 40 | 20 | 4 | 16 | 20 | 124 | 16 | 33.3 | 44.80 |
| 41 | 16 | 47 | 97 | 27 | 61 | 35 | 47.2 | 28.99 |
| 42 | 18 | 22 | 244 | 19 | 10 | 6 | 53.2 | 93.68 |
| 43 | 43 | 20 | 77 | 22 | 7 | 33 | 33.7 | 24.49 |
| 44 | 67 | 20 | 4 | 28 | 5 | 7 | 21.8 | 24.09 |
| 45 | 118 | 18 | 1 | 35 | 78 | 35 | 47.5 | 43.00 |
| 46 | 71 | 85 | 24 | 333 | 50 | 11 | 95.7 | 119.53 |
| 47 | 12 | 11 | 13 | 19 | 16 | 91 | 27.0 | 31.49 |
| 48 | 4 | 63 | 14 | 22 | 43 | 25 | 28.5 | 21.29 |
| 49 | 18 | 55 | 13 | 11 | 6 | 13 | 19.3 | 17.90 |
| 50 | 4 | 3 | 17 | 11 | 6 | 17 | 9.7 | 6.31 |

**31.56 Rational subgroups?** The six calls each shift are chosen at random from all calls received during the shift. Discuss the reasons behind this choice and those behind a choice to time six consecutive calls. DATA RTIMES

**31.57 Chart setup.** Table 31.9 also gives $\bar{x}$ and $s$ for each of the 50 samples. DATA RTIMES2

(a) Make an $s$ chart, and check for points out of control.

(b) If the $s$-type cause responsible is found and removed, what would be the new control limits for the $s$ chart? Verify that no points $s$ are now out of control.

(c) Use the remaining 46 samples to find the center line and control limits for an $\bar{x}$ chart. Comment on the control (or lack of control) of $\bar{x}$. (Because the distribution of response times is strongly skewed, $\bar{s}$ is large and the control limits for $\bar{x}$ are wide. Control charts based on Normal distributions often work poorly when measurements are strongly skewed.)

**31.58 Using process knowledge.** Three of the out-of-control values of $s$ in part (a) of Exercise 31.56 are explained by a single outlier, a very long response time to one call in the sample. What are the values of these outliers, and what are the $s$-values for the three samples when the outliers are omitted? (The interpretation of the data is, unfortunately, now clear. Few customers will wait five minutes for a call to be answered, as the customer whose call took 333 seconds to answer did. We suspect that other customers hung up before their calls were answered. If so, response time data for the calls that were answered don't adequately picture the quality of service. We should now look at data on calls lost before being answered to see a fuller picture.) DATA RTIMES2

**31.59 Web searches.** There are several key metrics that are used to monitor e-commerce sites. One of these metrics is the click-through rate (CTR). This is defined as the percentage of clicks the website receives based on a search using any of the common search engines. This metric is an indication of how well your indexed search result is enticing users to click. It might indicate that your indexed results need some work. A company noted that, historically, its CTR is 1.2% and the industry average CTR is 2.4%. The company hires an expert to improve keyword targeting for user searches and to update its website to include more compelling descriptions. A $p$ chart will monitor the CTR. Use information about past historical CTR values to set initial center line and control limits on a day when there are 60,000 searches. What about the center line and control limits on a day when there are only 40,000 searches?

*You have just installed a new system that uses an interferometer to measure the thickness of polystyrene film. To control the thickness, you plan to measure three film specimens every 10 minutes and keep $\bar{x}$ and $s$ charts. To establish control, you measure 22 samples of three films each at 10-minute intervals. Table 31.10 gives $\bar{x}$ and $s$ for these samples. The units are ten-thousandths of a millimeter. Exercises 31.60 through 31.62 are based on this chart setup setting.*

**31.60 $s$ chart.** Calculate control limits for $s$, make an $s$ chart, and comment on control of short-term process variation. DATA THICK

**31.61 $\bar{x}$ chart.** Interviews with the operators reveal that, in Samples 2 and 17, mistakes in operating the interferometer resulted in one high outlier thickness reading that was clearly incorrect. Recalculate $\bar{x}$ and $s$ after removing

**TABLE 31.10** $\bar{x}$ **and** $s$ **for 22 samples of film thickness (in ten-thousandths of a millimeter)**

| Sample | $\bar{x}$ | $s$ | Sample | $\bar{x}$ | $s$ |
|---|---|---|---|---|---|
| 1 | 669 | 6.9 | 12 | 666 | 3.9 |
| 2 | 698 | 22.1 | 13 | 672 | 6.1 |
| 3 | 679 | 10.6 | 14 | 659 | 7.9 |
| 4 | 683 | 12.0 | 15 | 680 | 8.4 |
| 5 | 676 | 5.3 | 16 | 663 | 6.2 |
| 6 | 663 | 4.5 | 17 | 699 | 23.5 |
| 7 | 683 | 11.0 | 18 | 676 | 8.3 |
| 8 | 685 | 6.0 | 19 | 659 | 11.6 |
| 9 | 661 | 11.3 | 20 | 670 | 5.7 |
| 10 | 675 | 8.3 | 21 | 682 | 5.8 |
| 11 | 671 | 4.1 | 22 | 687 | 10.7 |

Samples 2 and 17. Recalculate UCL for the $s$ chart and add the new UCL to your $s$ chart from the previous exercise. Control for the remaining samples is excellent. Now find the appropriate center line and control limits for an $\bar{x}$ chart, make the $\bar{x}$ chart, and comment on control.

**31.62 Categorizing the output.** Previously, control of the process was based on categorizing the thickness of each film inspected as satisfactory or not. Steady improvement in process quality has occurred so that just 15 of the last 5000 films inspected were unsatisfactory.

(a) What type of control chart would be used in this setting, and what would be the control limits for a sample of 100 films?

(b) The chart in part (a) is of little practical value at current quality levels. Explain why.

## EXPLORING THE WEB

**31.63 Spotting a mass murderer.** The *Chance* website discusses an application of statistical process control methods for spotting a mass murderer. Read the article at `www.causeweb.org/wiki/chance/index.php/Chance_News_(September-October_2005)` and the information found at the links in this article. Write a paragraph summarizing how statistical process control methods might have been used to identify a mass murderer.

**31.64 Six Sigma and Lean Manufacturing.** Six Sigma is a methodology used in many companies. Search the web to learn more about Six Sigma. Write a paragraph explaining what Six Sigma is and how it is related to material discussed in this chapter. Give a list of some companies that use Six Sigma.

Lean Manufacturing is often used in conjunction with Six Sigma. Search the web to learn more about Lean Manufacturing. Write a paragraph explaining what Lean Manufacturing is and how it is related to material discussed in this chapter. You will often hear the phrase "Lean Six Sigma," which is when a company implements both systems.

## Notes and Data Sources

1. CNNMoney, "My Golden Rule," at `money.cnn.com`, November 2005.
2. Texts on quality management give more detail about these and other simple graphical methods for quality problems. The classic reference is Kaoru Ishikawa, *Guide to Quality Control*, Tokyo: Asian Productivity Organization, 1986.
3. The flowchart and a more elaborate version of the cause-and-effect diagram for Example 31.1 were prepared by S. K. Bhat of the General Motors Technical Center as part of a course assignment at Purdue University.
4. For more information and references on DRGs, see the Wikipedia entry "diagnosis-related group." Search for this term at `en.wikipedia.org`.
5. Data courtesy of the Census Bureau `https://www.census.gov/`.
6. The terms "chart setup" and "process monitoring" are adopted from Andrew C. Palm's discussion of William H. Woodall, "Controversies and contradictions in statistical process control," *Journal of Quality Technology*, 32 (2000), pp. 341–350. Palm's discussion appears in the same issue, pp. 356–360. We have combined Palm's stages B ("process improvement") and C ("process monitoring") when writing for beginners because the distinction between them is one of degree.
7. It is common to call these "standards given" $\bar{x}$ and $s$ charts. We avoid this term because it easily leads to the common and serious error of confusing control limits (based on the process itself) with standards or specifications imposed from outside.
8. Provided by Charles Hicks, Purdue University.
9. See, for example, Chapter 3 of Stephen B. Vardeman and J. Marcus Jobe, *Statistical Quality Assurance Methods for Engineers*, New York: Wiley, 1999.

10. The classic discussion of out-of-control signals and the types of special causes that may lie behind special control chart patterns is the *AT&T Statistical Quality Control Handbook*, New York: Western Electric, 1956.
11. The data in Table 31.5 are adapted from data on viscosity of rubber samples appearing in Table P3.3 of Irving W. Burr, *Statistical Quality Control Methods*, New York: Marcel Dekker, 1976.
12. The control limits for the *s* chart based on past data are commonly given as $B_4\bar{s}$ and $B_3\bar{s}$. That is, $B_4 = B_6/c_4$ and $B_3 = B_5/c_4$. This is convenient for users, but avoiding this notation minimizes the number of control chart constants students must keep straight and emphasizes that process-monitoring and past-data charts are exactly the same except for the source of $\mu$ and $\sigma$.
13. Simulated data based on information appearing in Arvind Salvekar, "Application of six sigma to DRG 209," found at the Smarter Solutions website, `www.smartersolutions.com`.
14. The book by Samual Kotz and Cynthia R. Lovelace, *Process Capability Indices in Theory and Practice*, London: Arnold, 1998, provides details on computing various capability metrics.
15. Micheline Maynard, "Building success from parts," *New York Times*, March 17, 2002.
16. The first two Deming quotes are from *Public Sector Quality Report*, December 1993, p. 5. They were found online at `deming.eng.clemson.edu/pub/den/files/demqtes.txt`. The third quote is part of the tenth of Deming's "14 points of quality management," from his book *Out of the Crisis*, Cambridge, MA: MIT Press, 1986. The fourth quote is also from his book *Out of the Crisis*.
17. The data in Table 31.9 are simulated from a probability model for call pickup times. That pickup times for large financial institutions have median 20 seconds and mean 32 seconds is reported by Jon Anton, "A case study in benchmarking call centers," Purdue University Center for Customer-Driven Quality, no date.