6 Optional Section: Continuous Probability Distributions

6.5 The Normal Approximation to the Binomial Distribution

The normal distribution has many applications and is used extensively in statistical inference. In addition, many distributions, or populations, are approximately normal. The central limit theorem, which will be introduced in Section 7.2, provides some theoretical justification for this empirical evidence. In these cases, the normal distribution may be used to compute approximate probabilities.

Recall that the binomial distribution was defined in Section 5.4. Although it seems strange, under certain circumstances, a (continuous) normal distribution can be used to approximate a (discrete) binomial distribution.

For each rectangle, the width is 1 and the height is equal to the probability associated with a specific value. Therefore, the area of each rectangle (width $\times$ height) also represents probability.

Suppose X is a binomial random variable with $n = 10$ and $p = 0.5$. The probability histogram for this distribution is shown in Figure 6.83. Recall that in this case, the area of each rectangle represents the probability associated with a specific value of the random variable. For example, in Figure 6.84, the total area of the rectangular shaded regions is $P(4 \le X \le 7)$.

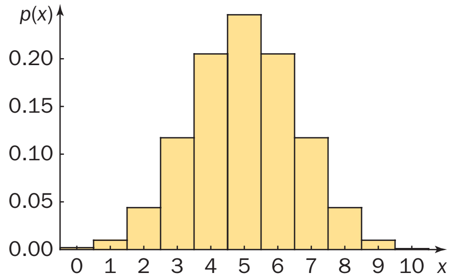

Figure 6.83 Probability histogram associated with a binomial random variable defined by $n = 10$ and $p = 0.5$.

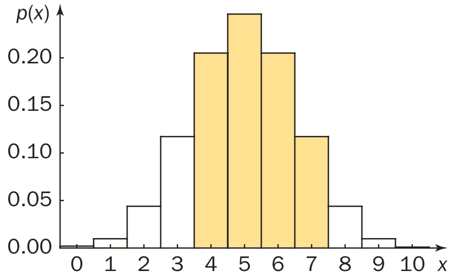

Figure 6.84 The area of the shaded rectangles is $P(4 \le X \le 7)$.

The probability histogram corresponding to the binomial distribution in Figure 6.83 is approximately bell-shaped. Consider a normal random variable with mean and variance from the corresponding binomial distribution. That is, $\mu = np = (10)(0.5) = 5$ and $\sigma^2 = np(1 - p) = (10)(0.5)(0.5) = 2.5$. The graph of the probability density function for this normal random variable, along with the probability histogram for the original binomial random variable, are shown in Figure 6.85.

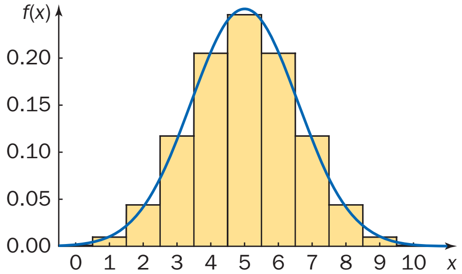

Figure 6.85 Probability histogram and probability density function.

The graph of the probability density function appears to pass through the middle top of each rectangle and is a smooth, continuous approximation to the probability histogram.

Remember that for a normal random variable, probability is the area under the graph of the density function. In this case, the total area under the probability density function seems to be pretty close to the total area of the probability histogram rectangles. Therefore, it seems reasonable to use the normal random variable to approximate probabilities associated with the binomial random variable. There is still one minor issue, but it is easily resolved. Here is an example to illustrate the approximation process.

Example 6.12 Suppose X is a binomial random variable with 10 trials and probability of success 0.5, that is, $X \sim \mathrm{B}(10, 0.5)$.

a. Find the exact probability, $\mathrm{P}(4 \le X \le 7)$.

b. Use the normal approximation to find $\mathrm{P}(4 \le X \le 7)$.

SOLUTION

a. Use the techniques introduced in Section 5.4.

$$\begin{aligned} \mathrm{P}(4 \le X \le 7) &= \mathrm{P}(X \le 7) - \mathrm{P}(X \le 3) && \text{Convert to cumulative probability.} \\ &= 0.9453 - 0.1719 && \text{Use Table I in the Appendix.} \\ &= 0.7734 && \text{Simplify.} \end{aligned}$$

b. For $X \sim \mathrm{B}(10, 0.5)$, $\mu = 5$ and $\sigma^2 = 2.5$.

The symbol $\overset{\bullet}{\sim}$ means "is approximately distributed as."

Figure 6.85 suggests that $X \overset{\bullet}{\sim} \mathrm{N}(5, 2.5)$. Using this approximation,

$$\underset{\substack{\uparrow \\ \text{Binomial RV}}}{\mathrm{P}(4 \le X \le 7)} \approx \underset{\substack{\uparrow \\ \text{Normal RV}}}{\mathrm{P}(4 \le X \le 7)} \qquad \text{Use the normal approximation.}$$

$$\begin{aligned} &= \mathrm{P}\left(\frac{4-5}{\sqrt{2.5}} \le \frac{X-5}{\sqrt{2.5}} \le \frac{7-5}{\sqrt{2.5}}\right) && \text{Standardize.} \\ &= \mathrm{P}(-0.63 \le Z \le 1.26) && \text{Use Equation 6.8; simplify.} \\ &= \mathrm{P}(Z \le 1.26) - \mathrm{P}(Z \le -0.63) && \text{Use cumulative probability.} \\ &= 0.8962 - 0.2643 && \text{Use Table III in the Appendix.} \\ &= 0.6319 \end{aligned}$$

This approximation isn't very close to the exact probability, and Figure 6.85 suggests a reason. By finding the area under the normal probability density function from 4 to 7, we have left out half of the area of the rectangle above 4 and half of the area of the rectangle above 7.

The width of each rectangle is 1, so the edges of a rectangle above x are $x - 0.5$ and $x + 0.5$. For example, the rectangle above 4 has edges 3.5 and 4.5.

To capture the entire area of the rectangle above 4 and 7, we find the area under the normal probability density function from 3.5 to 7.5. This is called the *continuity correction*. We correct a continuous probability approximation associated with a discrete random variable (binomial) by adding or subtracting 0.5 for each endpoint, to capture all the area of the corresponding rectangles. Figures 6.86 and 6.87 illustrate the continuity correction in this example.

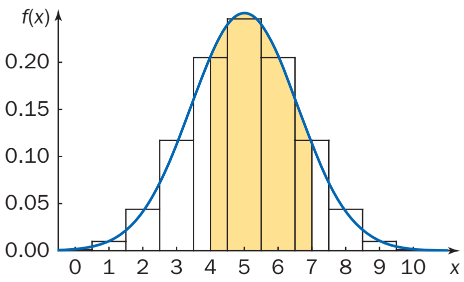

Figure 6.86 This approximation excludes half of the area of the rectangles above 4 and 7.

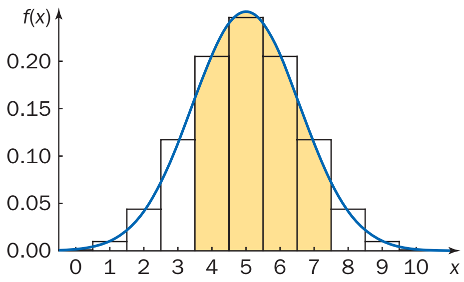

Figure 6.87 This approximation includes more of the area/probability associated with 4 and 7.

c. Using the continuity correction,

$$
\begin{aligned}
\underset{\substack{\uparrow \\ \text{Binomial RV}}}{P(4 \le X \le 7)} &\approx \underset{\substack{\uparrow \\ \text{Normal RV}}}{P(3.5 \le X \le 7.5)} && \text{Use the normal approximation with a continuity correction.}\\
&= P\left(\frac{3.5-5}{\sqrt{2.5}} \le \frac{X-5}{\sqrt{2.5}} \le \frac{7.5-5}{\sqrt{2.5}}\right) && \text{Standardize.}\\
&= P(-0.95 \le Z \le 1.58) && \text{Use Equation 6.8; simplify.}\\
&= P(Z \le 1.58) - P(Z \le -0.95) && \text{Use cumulative probability.}\\
&= 0.9429 - 0.1711 && \text{Use Table III in the Appendix.}\\
&= 0.7718 && \text{Simplify.}
\end{aligned}
$$

This probability found using the continuity correction, 0.7718, is much closer to the true probability than the computed value in part (b).

Figures 6.88 and 6.89 show technology solutions.

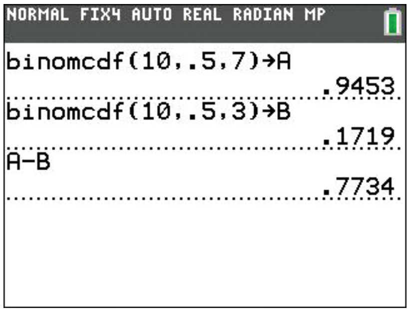

Figure 6.88 For $X \sim B(10, 0.5)$, $P(4 \le X \le 7)$ using cumulative probability.

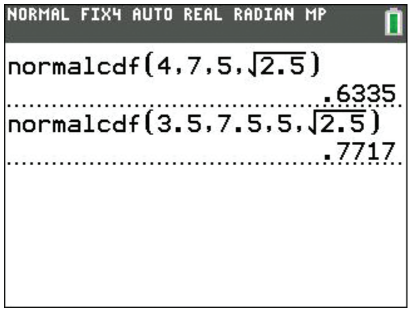

Figure 6.89 The normal approximation without and with the continuity correction.

Note: Remember that a round-off error is introduced in the above calculations. The calculator results shown in Figures 6.88 and 6.89 are more accurate and better illustrate the impact of the continuity correction.

The previous example suggests the following approximation procedure.

The Normal Approximation to the Binomial Distribution

Suppose X is a binomial random variable with n trials and probability of success p, $X \sim B(n, p)$. If n is large and both $np \ge 10$ and $n(1-p) \ge 10$, then the random variable X is approximately normal with mean $\mu = np$ and variance $\sigma^2 = np(1-p)$, $X \mathrel{\dot\sim} N(np, np(1-p))$.

A CLOSER LOOK

1. There is no threshold value for n. However, a *large* sample isn't enough for approximate normality. The two products np and $n(1-p)$ must *both* be greater than or equal to 10. This is called the *nonskewness criterion*. If both inequalities are satisfied, then it is reasonable to assume that the distribution of the approximate normal distribution is symmetric, centered far enough away from 0 or 1, and with the values $\mu \pm 3\sigma$ inside the interval $[0, 1]$.
2. When using the continuity correction, simply think carefully about whether to include or exclude the area of each rectangle associated with the binomial probability histogram. Consider a figure to illustrate the probability question. This will help you decide whether to add or subtract 0.5.

3. For n very large, for example, $n \geq 5000$, probability calculations involving the exact binomial distribution can be very long and inaccurate, even using technology. In this case, the normal approximation to the binomial distribution provides a good, quick alternative. ■

Example 6.13 The World Cup

The 2014 FIFA World Cup competition was held in Brazil. Despite being a soccer-enthusiastic nation, a poll prior to the competition showed that only 48% of Brazilians supported hosting the event.[33] The support actually dropped since 2008, perhaps because of cost overruns, and construction delays and accidents. Suppose 500 Brazilians are selected at random. Use the normal approximation to the binomial distribution to answer the following questions.

a. Find the probability that at least 255 of the Brazilians selected support hosting the World Cup.

b. Find the probability that exactly 260 of the Brazilians selected support hosting the World Cup.

c. Suppose 215 Brazilians selected favor hosting the World Cup. Is there any evidence to suggest that the proportion who favor hosting the World Cup has dropped (from 0.48)? Justify your answer.

SOLUTION

a. Let X be the number of Brazilians selected who favor hosting the World Cup. X is a binomial random variable with $n = 500$ and $p = 0.48$: $X \sim \mathrm{B}(500, 0.48)$. Use Equation 5.8, page 218, to find the mean and variance.

$$\mu = np = (500)(0.48) = 240$$
$$\sigma^2 = np(1 - p) = (500)(0.48)(0.52) = 124.8$$

For $n = 500$ and $p = 0.48$, check the nonskewness criterion.

$$np = (500)(0.48) = 240 \geq 10 \qquad \text{and} \qquad n(1 - p) = (500)(0.52) = 260 \geq 10$$

Both inequalities are satisfied. The distribution of X is approximately normal with $\mu = 240$ and $\sigma^2 = 124.8$: $X \overset{\bullet}{\sim} \mathrm{N}(240, 124.8)$.

The probability that at least 255 Brazilians favor hosting the World Cup is

$$\underset{\text{Binomial RV}}{\underset{\uparrow}{\mathrm{P}(X \geq 255)}} \approx \underset{\text{Normal RV}}{\underset{\uparrow}{\mathrm{P}(X \geq 254.5)}}$$ Use the normal approximation with a continuity correction.

$$= \mathrm{P}\left(\frac{X - 240}{\sqrt{124.8}} \geq \frac{254.5 - 240}{\sqrt{124.8}}\right)$$ Standardize.

$$= 1 - \mathrm{P}(Z \leq 1.30)$$ Simplify, use cumulative probability.

$$= 1 - 0.9032$$ Use Table III in the Appendix.

$$= 0.0968$$ Simplify.

The probability that at least 255 Brazilians selected favor hosting the World Cup is approximately 0.0968. Figure 6.90 shows part of the graph of the binomial probability histogram and approximate normal probability density function. We need to include

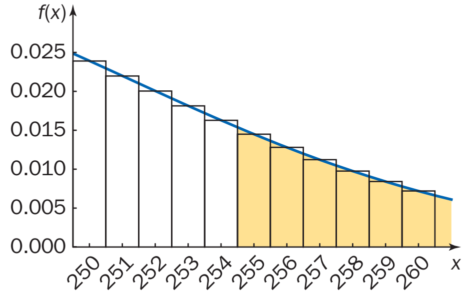

Figure 6.90 Using the normal approximation, start at 254.5.

the area of the rectangle corresponding to 255. Using the continuity correction with the normal approximation, start at $255 - 0.5 = 254.5$.

Recall that for any continuous random variable X, the probability X equals a single value is 0. Therefore, we must use a continuity correction to approximate the area (probability) of a rectangle corresponding to a single value.

b. The probability that exactly 260 Brazilians favor hosting the World Cup is

$$\underbrace{P(X = 260)}_{\text{Binomial RV}} \approx \underbrace{P(259.5 \le X \le 260.5)}_{\text{Normal RV}}$$
Use the normal approximation with a continuity correction.

$$= P\left(\frac{259.5 - 240}{\sqrt{124.8}} \le \frac{X - 240}{\sqrt{124.8}} \le \frac{260.5 - 240}{\sqrt{124.8}}\right)$$
Standardize.

$$= P(1.75 \le Z \le 1.84)$$
Use Equation 6.8, simplify.

$$= P(Z \le 1.84) - P(Z \le 1.75)$$
Use cumulative probability.

$$= 0.9671 - 0.9599$$
Use Table III in the Appendix.

$$= 0.0072$$
Simplify.

The probability that exactly 260 Brazilians selected favor hosting the World Cup is approximately 0.0072. Figure 6.91 shows part of the graph of the binomial probability histogram and approximate normal probability density function. To approximate the area of the single rectangle corresponding to 260, use the endpoints of the rectangle.

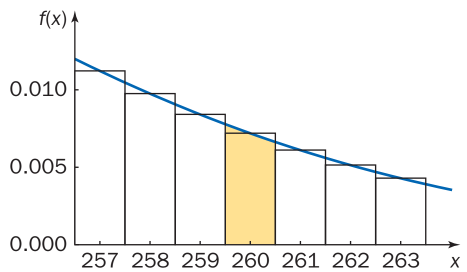

Figure 6.91 Use the continuity correction to approximate the area of a single rectangle.

c. The claim made following the poll is that $p = 0.48$. This implies that the random variable X has a binomial distribution with $n = 500$ and $p = 0.48$. Using the normal approximation to the binomial distribution, X has approximately a normal distribution with $\mu = (500)(0.48) = 240$ and $\sigma^2 = (500)(0.48)(0.52) = 124.8$.

Claim: $p = 0.48 \quad \Rightarrow \quad \mu = 240 \quad \Rightarrow \quad X \stackrel{\bullet}{\sim} N(240, 124.8)$

The experimental outcome is that 215 Brazilians selected favor hosting the World Cup.

Experiment: $x = 215$

Because $x = 215$ is to the left of the mean, and because we are searching for evidence that the proportion of Brazilians who favor hosting the World Cup has dropped, we will consider a left-tail probability.

Likelihood:

$$\underbrace{P(X \le 215)}_{\text{Binomial RV}} \approx \underbrace{P(X \le 215.5)}_{\text{Normal RV}}$$
Use the normal approximation with a continuity correction.

$$= P\left(\frac{X - 240}{\sqrt{124.8}} \le \frac{215.5 - 240}{\sqrt{124.8}}\right)$$
Standardize.

$$= P(Z \le -2.19)$$
Use Equation 6.8; simplify.

$$= 0.0143$$
Use Table III in the Appendix.

Conclusion: Because the tail probability is so small (less than 0.05), $x = 215$ is a very unusual observation (if $p = 0.48$). Therefore, there is evidence to suggest that the claim is wrong, that $p < 0.48$, that is, that the proportion of Brazilians who favor hosting the World Cup has dropped.

Figure 6.92 shows a technology solution.

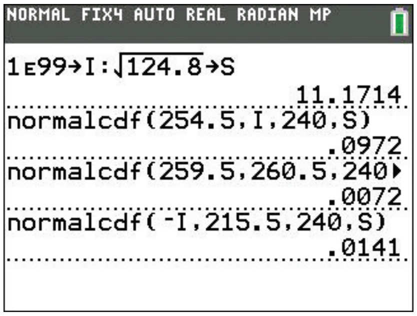

Figure 6.92 Approximate probabilities using the normal distribution.

SECTION 6.5 EXERCISES

Concept Check

6.127 True/False The normal approximation to the binomial distribution can be used for any values of n and p.

6.128 True/False Using the normal approximation to a binomial distribution, the probability of a single value is always 0.

6.129 True/False To use the normal approximation to the binomial distribution, n must be at least 30.

6.130 Short Answer If $X \sim \mathrm{B}(n, p)$, find the mean and variance of the approximate normal random variable.

6.131 Short Answer Explain why using the continuity correction provides a better approximation than not using the continuity correction.

Practice

6.132 Suppose X is a binomial random variable with n trials and probability of success p. In each problem below, check the nonskewness criterion and find the distribution of the corresponding approximate normal random variable.

a. $n = 30, p = 0.40$ **b.** $n = 85, p = 0.55$
c. $n = 340, p = 0.38$ **d.** $n = 605, p = 0.75$

6.133 Suppose X is a binomial random variable with $n = 25$ trials and probability of success $p = 0.45$.

a. Check the nonskewness criterion.
b. Find the distribution of the corresponding approximate normal random variable.
c. Carefully sketch the probability histogram for the binomial random variable and the probability density function for the normal random variable on the same coordinate axes.

6.134 Suppose X is a binomial random variable with $n = 100$ trials and probability of success $p = 0.03$.

a. Check the nonskewness criterion.
b. Compute the values $\mu \pm 3\sigma$.
c. Find the distribution of the corresponding approximate normal random variable. Using the results from parts (a) and (b), is the normal distribution a good approximation to the binomial distribution? Justify your answer.
d. Carefully sketch a portion of the probability histogram for the binomial random variable and the probability density function for the normal random variable near 0 on the same coordinate axes. Explain geometrically why the approximation is or is not accurate.

6.135 Suppose X is a binomial random variable with $n = 25$ trials and probability of success $p = 0.60$. Find each of the following probabilities using the binomial random variable, the approximate normal random variable without the continuity correction, and the approximate normal random variable with the continuity correction.

a. $P(X \leq 14)$ **b.** $P(X < 14)$
c. $P(16 \leq X \leq 19)$ **d.** $P(X > 10)$

6.136 Suppose X is a binomial random variable with $n = 600$ trials and probability of success $p = 0.35$. Find each of the following probabilities using the normal approximation to the binomial distribution with a continuity correction.

a. $P(X > 220)$ **b.** $P(X \leq 198)$
c. $P(190 < X < 200)$ **d.** $P(X = 212)$

Applications

6.137 Fuel Consumption and Cars Digital Radio UK recently reported that 55% of all new cars sold in the United Kingdom are equipped with a digital radio.[34] Suppose a random sample of 100 UK new car purchases is obtained. Answer each question using the normal approximation to the binomial distribution.

a. Find the approximate probability that at least 60 new cars are equipped with a digital radio.
b. Find the approximate probability that fewer than 57 new cars are equipped with a digital radio.
c. Find the approximate probability that between 45 and 55 (inclusive) new cars are equipped with a digital radio.

6.138 Technology and the Internet Many companies are researching ways in which drones could help improve their business. For example, Amazon could use drones to deliver packages more quickly and pizza could be delivered to your door faster and hotter. Despite the many possibilities, results from a Pew Research Center poll indicated that 63% of Americans believe that personal and commercial drones should not be allowed in U.S. airspace.[35] Suppose 160 Americans are selected at random. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Find the approximate probability that fewer than 95 Americans believe drones should not be allowed in U.S. airspace.
- **b.** Find the probability that at least 110 Americans believe drones should not be allowed in U.S. airspace.
- **c.** Find the approximate probability that exactly 97 Americans believe drones should not be allowed in U.S. airspace.

6.139 Education and Child Development In May 2014, the National Center for Education Statistics (NCES) released results from the National Assessment of Educational Progress (NAEP). Somewhat discouraging, they suggested that less than 40% of all 12th graders in the United States are academically prepared for college.[36] Suppose 200 U.S. 12th graders are selected at random. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Find the approximate probability that at least 80 12th graders are prepared for college.
- **b.** Find the approximate probability that between 90 and 100 (inclusive) 12th graders are prepared for college.
- **c.** Suppose the NAEP results for each student are used to find that 68 (of the 200) students are prepared for college. Is there any evidence to suggest that fewer than 40% of 12th graders are prepared for college? Justify your answer.

6.140 Biology and Environmental Science There have been many science conferences and research papers concerning global warming. Several climate model predictions include higher surface temperatures, rising sea levels, and larger subtropical deserts. Global leaders continue to discuss possible actions to stop or slow these trends. Despite the ominous warnings, a recent survey indicated that only 44% of Americans said that global warming should be a high priority for political leaders and governments.[37] Suppose 250 Americans are selected at random and each is asked if global warming should be a priority issue. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Find the approximate probability that at most 110 Americans believe global warming should be a priority issue.
- **b.** Find the approximate probability that fewer than 100 Americans believe global warming should be a priority issue.
- **c.** Find the approximate probability that between 115 and 125 (inclusive) believe global warming should be a priority issue.

6.141 Medicine and Clinical Studies When filling a prescription, there is often a generic option instead of a brand-name drug. The generic drug is usually equally effective and less expensive. In Canada, 37% of prescriptions are filled using brand-name drugs.[38] Suppose 300 prescriptions filled in Canada are randomly selected. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Let X be the number of prescriptions filled using a brand-name drug. Find the approximate distribution of X.
- **b.** Find the approximate probability that at least 120 prescriptions were filled using a brand-name drug.
- **c.** Suppose 95 of the prescriptions were filled using brand-name drugs. Is there any evidence to suggest that the claim is wrong, that the true proportion of prescriptions filled using brand-name drugs is less than 0.37? Justify your answer.

6.142 Manufacturing and Product Development There is an ongoing debate about whether to label genetically engineered (GE) foods. In the United States, many processed foods include at least one GE ingredient, for example, GE soybeans, corn, or canola. Suppose 70% of all Americans believe GE foods should be labeled and a random sample of 500 Americans is selected. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Find the approximate probability that exactly 360 Americans believe GE foods should be labeled.
- **b.** Find the approximate probability that at least 340 Americans believe GE foods should be labeled.
- **c.** Find the approximate probability that more than 340 Americans believe GE foods should be labeled.

6.143 Marketing and Consumer Behavior "Black Friday" is the day after Thanksgiving and the traditional first day of the Christmas shopping season. Suppose a recent poll suggested that 66% of Black Friday shoppers are actually buying for themselves. A random sample of 130 Black Friday shoppers is obtained. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Find the approximate probability that fewer than 76 Black Friday shoppers are buying for themselves.
- **b.** Find the approximate probability that between 77 and 87 (inclusive) Black Friday shoppers are buying for themselves.
- **c.** Suppose 90 Black Friday shoppers are buying for themselves. Is there any evidence to suggest that the claim is wrong, that the true proportion of Black Friday shoppers buying for themselves is greater than 0.66? Justify your answer.

6.144 Psychology and Human Behavior We all need to call a customer service representative at some time. It is often difficult to speak with a live person, and the waiting time can be aggravating. Consequently, 36% of Americans admit that they have yelled at a customer service representative during the past year.[39] Suppose a random sample of 275 Americans who recently talked with a customer service representative is obtained. Answer each problem using the normal approximation to the binomial distribution.

- **a.** Find the approximate probability that at least 105 Americans yelled at a customer service representative.
- **b.** Find the approximate probability that between 90 and 100 (inclusive) Americans yelled at a customer service representative.

c. Suppose 85 Americans actually yelled at a customer service representative. Is there any evidence to suggest that the claim is wrong, that the true proportion of Americans who have yelled at a customer service representative is less than 0.36? Justify your answer.

6.145 Stuck on Band-Aids Despite the fact that a majority of accidents happen in one's home, only 44% of Americans have a first-aid kit in their homes. Suppose a random sample of 300 American homes is obtained. Answer each problem using the normal approximation to the binomial distribution.

a. Find the probability that fewer than 125 homes have first-aid kits.

b. Find the probability that more than 145 homes have first-aid kits.

c. Find the probability that exactly 130 homes have first-aid kits.

6.146 Counting Sheep There are many reasons why some people have a difficult time falling asleep or staying asleep, for example, too much caffeine, climate, lack of physical activity, or even a lumpy mattress. In a recent poll, it was reported that for those people who suffer from insomnia, 51% blame stress. Suppose a random sample of 80 adults who suffer from insomnia was obtained and asked if they blame stress.

a. The sample size $(n = 80)$ is relatively small in this example. Why is the normal approximation to the binomial distribution appropriate?

b. Find the approximate probability that more than 42 adults blame stress for their insomnia.

c. Find the approximate probability that at least 38 adults blame stress for their insomnia.

d. Find the approximate probability that fewer than 35 or more than 45 adults blame stress for their insomnia.

Extended Applications

6.147 Where's the Remote? We all misplace the remote control to our television. Although some lost remotes are found in the refrigerator, outside, or in a car, approximately 50% are stuck between sofa cushions.[40] Suppose a random sample of 150 lost (and found) remotes is obtained. Answer each problem using the normal approximation to the binomial distribution.

a. Find the probability that fewer than 70 lost remotes were found stuck between sofa cushions.

b. Suppose at least 80 lost remotes were found stuck between sofa cushions. Find the approximate probability that at least 85 were found stuck between sofa cushions.

c. Suppose 90 lost remotes were found stuck between sofa cushions. Is there any evidence to suggest that the claim is wrong, that the true proportion of lost remotes found between sofa cushions is greater than 0.50? Justify your answer.

6.148 Economics and Finance Online banking is quick and convenient, but it does present some security risks. For those people who have an online bank account, 47% use their online bank password for at least one other online site. Suppose a random sample of 100 people who have an online bank account is selected and asked if they use their bank password for at least one other online site. Answer each problem using the normal approximation to the binomial distribution.

a. Find the approximate probability that fewer than 40 people use their online bank password for at least one other online site.

b. Find the approximate probability that at least 50 people use their online bank password for at least one other online site.

c. Suppose a second random sample of 100 people who have an online bank account is selected and also asked if they use their bank password for at least one other online site. Find the approximate probability that between 42 and 52 (inclusive) people use their online bank password for at least one other online site in both random samples.

6.149 Business and Management In January 2014, Sam's Club announced plans to cut 2% of its workforce.[41] The cuts were expected to affect both managers and hourly employees. Suppose a random sample of 150 Sam's Club employees is obtained.

a. Use the binomial distribution to find the exact probability that at most 2 employees will lose their jobs.

b. Use the normal approximation to the binomial distribution to find the approximate probability that at most 2 employees will lose their jobs.

c. Explain why in this case the normal approximation to the binomial distribution is not very accurate.

d. Carefully sketch a graph of the binomial probability histogram and the approximate normal probability density function near the mean. Use this graph to justify your answer to part (c).

e. Suppose 7 of the employees actually lose their job. Is there any evidence to suggest that the Sam's Club claim is wrong, that the true proportion of employees who lose their job is greater than 0.02? Justify your answer using both the binomial distribution and the normal approximation.

6.150 Bazinga Despite scientific evidence (and the popularity of Sheldon Leonard), 51% of Americans do not believe in the Big Bang.[42] Suppose 1000 Americans are selected at random and asked if they believe in the Big Bang.

a. Use the binomial distribution to find the exact probability that 510 Americans do not believe in the Big Bang.

b. Use the normal approximation to the binomial distribution to find the approximate probability that exactly 510 Americans do not believe in the Big Bang.

c. Explain why in this case the normal approximation to the binomial distribution is very accurate.

d. Suppose 525 Americans do not believe in the Big Bang. Is there any evidence to suggest that the proportion of Americans who do not believe in the Big Bang has increased? Justify your answer using the normal approximation to the binomial distribution.

Challenge

6.151 Biology and Environmental Science Recall that a Poisson random variable is often used to model rare events and is a (discrete) count of the number of times a specific event occurs during a given interval. Suppose X is a Poisson random variable with mean λ. Then $\mu = \lambda$ and $\sigma^2 = \lambda$.

If $\lambda > 10$, then the random variable X is approximately normal with mean $\mu = \lambda$ and variance $\sigma^2 = \lambda$: $X \stackrel{\bullet}{\sim} N(\lambda, \lambda)$. As λ increases, the approximation becomes more accurate. An appropriate continuity correction should be used whenever this normal approximation to a Poisson distribution is applied.

Hops are used in brewing beer and make up about 5% of the total volume but contribute about 50% of the taste. Suppose farmers spray their hops 14 times per year using a wide variety of pesticides and fungicides. Suppose a hops farmer is selected at random.

a. Use the Poisson distribution to find the probability that the farmer sprays the hops at least 17 times during the year.

b. Use the Poisson distribution to find the probability that the farmer sprays the hops fewer than 10 times during the year.

c. Suppose the farmer sprays the hops between 8 and 20 times (inclusive) during the year. Use the Poisson distribution to find the probability that the farmer sprays the hops at most 12 times during the year.

d. Use the normal approximation to the Poisson distribution with the appropriate continuity correction to answer parts (a), (b), and (c).

e. Suppose the farmer sprays the hops 22 times during the year. In addition to avoiding beer made with these hops, is there any evidence to suggest that the mean number of times the hops are sprayed in a year is greater than 14? Justify your answer.