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CHAPTER 15

Statistics for Quality: Control and Capability

Introduction

For nearly 100 years, manufacturers have benefited from a variety of statistical tools for the monitoring and control of their critical processes. But in more recent years, companies have learned to integrate these tools into their corporate management systems dedicated to continual improvement of their processes.

- Health care organizations are increasingly using quality improvement methods to improve operations, outcomes, and patient satisfaction. The Mayo Clinic, Johns Hopkins Hospital, and New York-Presbyterian Hospital employ hundreds of quality professionals trained in Six Sigma techniques. As a result of having these focused quality professionals, these hospitals have achieved numerous improvements ranging from reduced blood waste due to better control of temperature variation to reduced waiting time for treatment of potential heart attack victims.
- Acushnet Company is the maker of Titleist golf balls, which is among the most popular brands used by professional and recreational golfers. To maintain consistency of the balls, Acushnet relies on statistical process control methods to control manufacturing processes.
- Cree Incorporated is a market-leading innovator of LED (light-emitting diode) lighting. Cree's light bulbs were used to glow several venues at the Beijing Olympics and are being used in the first U.S. LED-based highway lighting system in Minneapolis. Cree's mission is to continually improve upon its manufacturing processes so as to produce energy-efficient, defect-free, and environmentally

CHAPTER OUTLINE

friendly LEDs. To achieve high-quality processes and products, Cree generates a variety of control charts to display and understand process behaviors.

Quality overview

Moving into the twenty-first century, the marketplace signals were becoming clear: poor quality in products and services would not be tolerated by customers. Organizations increasingly recognized that what they didn't know about the quality of their products could have devastating results: customers often simply left when encountering poor quality rather than making complaints and hoping that the organization would make changes. To make matters worse, customers would voice their discontent to other customers, resulting in a spiraling negative effect on the organization in question. The competitive marketplace is pressuring organizations to leave no room for error in the delivery of products and services.

To meet these marketplace challenges, organizations have recognized that a shift to a different paradigm of management thought and action is necessary. The new paradigm calls for developing an organizational system dedicated to customer responsiveness and the quick development of products and services that at once combine exceptional quality, fast and on-time delivery, and lower prices to the customers. In the pursuit of developing such an organizational system, there has been an onslaught of recommended management approaches, including total quality management (TQM), continuous quality improvement (CQI), business process reengineering (BPR), business process improvement (BPI), and Six Sigma (6σ). In addition, the work of numerous individuals has helped shape contemporary quality thinking. These include W. Edwards Deming, Joseph Juran, Armand Feigenbaum, Kaoru Ishikawa, Walter Shewhart, and Genichi Taguchi.[1]

Because no approach or philosophy is one size fits all, organizations are learning to develop their own personalized versions of a quality management system that integrates the aspects of these approaches and philosophies that best suit the challenges of their competitive environments. However, in the end, it is universally accepted that any effective quality management approach must integrate certain basic themes. Four themes are particularly embraced:

- The modern approach to management views work as a process.
- The key to maintaining and improving quality is the systematic use of data in place of intuition or anecdotes.
- It is important to recognize that variation is present in all processes and the goal of an organization should be to understand and respond wisely to variation.
- The tools of process improvement—including the use of statistics and teams—are most effective if the organization's culture is supportive and oriented toward continuously pleasing customers.

process

The idea of work as a process is fundamental to modern approaches to quality, and even to management in general. A **process** can be simply defined as a collection of activities that are intended to achieve some result. Specific business examples of processes include manufacturing a part to a desired dimension, billing a customer, treating a patient, and delivering products to customers. Manufacturing and service organizations alike have processes. The challenge for organizations is to identify key processes to improve. Key processes are those that have significant impact on customers and, more generally, on organizational performance.

To know how a process is performing and whether attempts to improve the process have been successful requires data. Process improvement usually cannot be achieved by armchair reasoning or intuition. To emphasize the importance of data, quality professionals often state, "You can't improve what you can't measure." Examples of process data measures include

- Average number of days of sales outstanding (finance/accounting)
- Time needed to hire new employees (human resources)
- Number of on-the-job accidents (safety)
- Time needed to design a new product or service (product/service design)
- Dimensions of a manufactured part (manufacturing)
- Time to generate sales invoices (sales and marketing)
- Time to ship a product to a customer (shipping)
- Percent of abandoned calls (call center)
- Downtime of a network (information technology)
- Wait times for patients in a hospital clinic (customer service)

Our focus is on processes common within an organization. However, the notion of a process is universal. For instance, we can apply the ideas of a process to personal applications such as cooking, playing golf, or controlling one's weight. Or we may consider broader processes such as air pollution levels or crime rates. One of the great contributions of the quality revolution is the recognition that any process can be improved.

Systematic approach to process improvement

Management by intuition, slogans, or exhortation does not provide an environment or strategy conducive to process improvement. One of the key lessons of quality management is that process improvement should be based on an approach that is systematic, scientific, and fact (data) based.

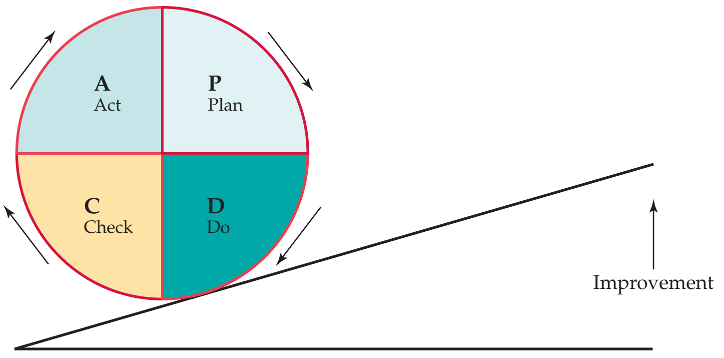
The systematic steps of process improvement involve identifying the key processes to improve, process understanding/description, root cause analysis, assessment of attempted improvement efforts, and implementation of successful improvements. The systematic steps for process improvement are captured in the Plan-Do-Check-Act (PDCA) cycle.

- The **Plan** step calls for identifying the process to improve, describing the current process, and coming up with solutions for improving the process.
- The **Do** step involves the implementation of the solution or change to the process; typically, improvements are first made on a small scale so as not to disrupt the routine activities of the organization.
- The **Check** step focuses on assessing postintervention process data to see if the improvement efforts have indeed been successful.
- The **Act** step involves the implementation of the process changes as part of the organization's routine activities if the process improvement efforts are successful.

Completion of these general steps represents one PDCA cycle. By continually initiating the PDCA cycle, continuous process improvement is accomplished, as depicted in Figure 15.1.

Advocates of the Six Sigma approach emphasize that the Six Sigma improvement model distinguishes itself from other process improvement

FIGURE 15.1 The PDCA Cycle.



models in that it calls for projects to be selected only if they are clearly aligned with business priorities. This means that projects not only must be linked to customers' needs, but they also must have a significant financial impact on the business's bottom line. Organizations pursuing process improvement as part of a Six Sigma effort use a tailored version of the generic PDCA improvement model, known as Define-Measure-Analyze-Improve-Control (DMAIC).

- In the **Define** phase, the goal is to clearly identify an improvement opportunity in measurable terms and establish project goals.
- In the **Measure** phase, data are gathered to establish the current process performance.
- In the **Analyze** phase, efforts are made to find the sources (root causes) of less-than-desirable process performance. In many applications, root cause analysis relies on performing appropriately designed experiments and analyzing the resulting data using statistical techniques such as analysis of variance (Chapter 9) and multiple regression (Chapter 13).
- In the **Improve** phase, solutions are developed and implemented to attack the root causes.
- In the **Control** phase, process improvements are institutionalized, and procedures and methods are put into place to hold the process in control so as to maintain the gains from the improvement efforts.

One of the most common statistical tools used in the Control phase is the control chart, which is a focus of this chapter.

Process improvement toolkit

Each of the steps of the PDCA and DMAIC improvement models can potentially make use of a variety of tools. The quality literature is rich with examples of tools useful for process improvement. Indeed, a number of statistical tools that we have already introduced in earlier chapters frequently play a key role in process improvement efforts. Here are some basic tools (statistical and nonstatistical) frequently used for process improvement efforts:

- **Flowchart.** A flowchart is a picture of the stages of a process. Many organizations have formal standards for making flowcharts. A flowchart can often jump-start the process improvement effort by exposing unexpected complexities (e.g., unnecessary loops) or non-value-added activities (e.g., waiting points that increase overall cycle time). Figure 15.2(a) is a flowchart showing the steps of an order fulfillment process for an electronic order from a customer.

time plot, p. 20

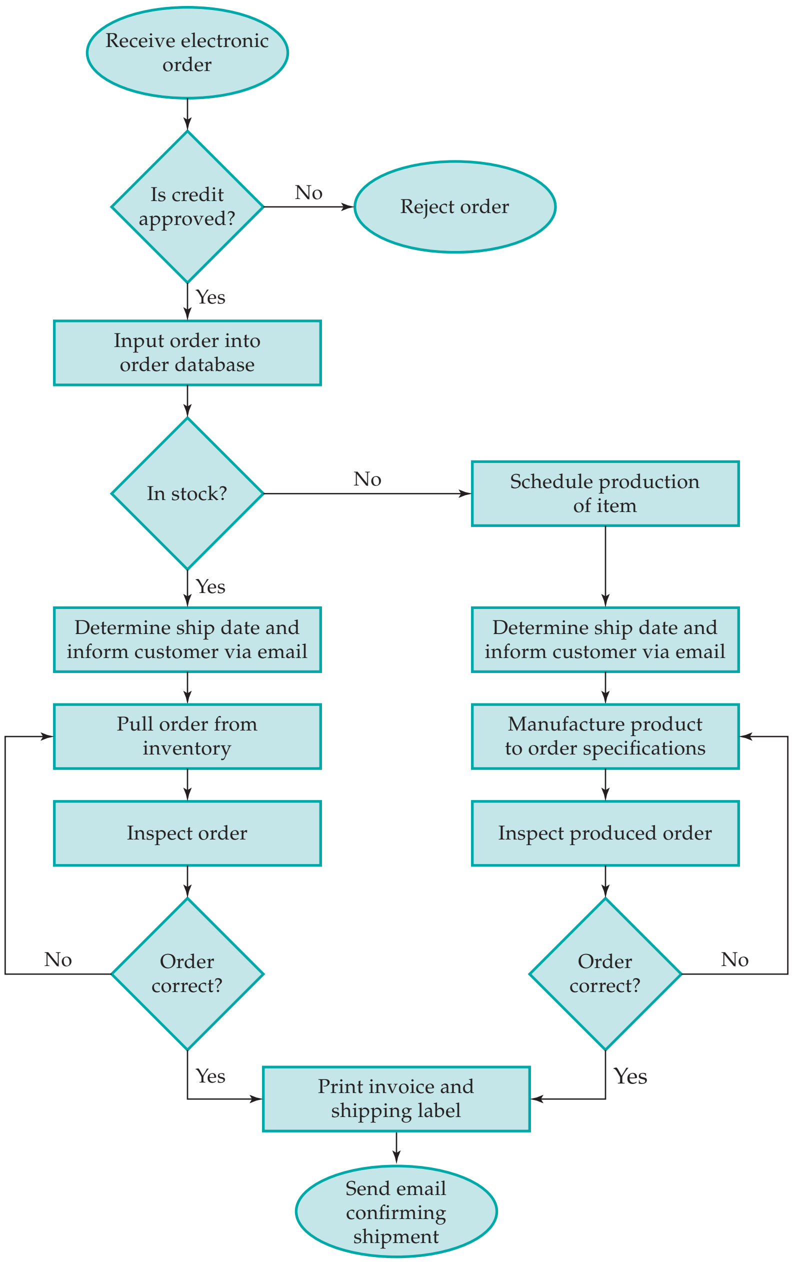
- **Run chart.** A run chart is what quality professionals call a time plot. A run chart allows one to observe the performance of a process over time. For example, Motorola's service centers calculate mean response times each month and depict overall performance with a run chart.

histogram, p. 13

- **Histogram.** Every process is subject to variability. The histogram is useful in process improvement efforts because it allows the practitioner to visualize the process behavior in terms of location, variability, and distribution. For example, in Section 15.2, we use histograms with superimposed product specification limits to display "process capability."

Pareto chart, p. 11

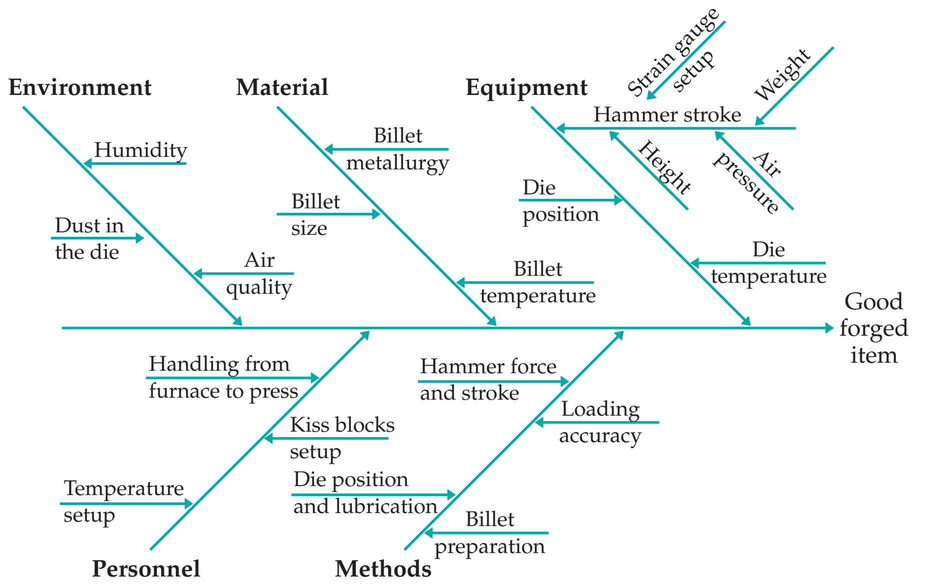
- **Pareto chart.** A Pareto chart is a bar graph with the bars ordered by height. Pareto charts help focus process improvement efforts on issues of greatest impact ("vital few") as opposed to less important issues ("trivial many").



(a)

FIGURE 15.2 Examples of nonstatistical process improvement tools. (a) Flowchart of an ordering process for an electronic order. (b) Cause-and-effect diagram of hypothesized causes related to the making of a good forged item.

FIGURE 15.2 Continued



(b)

- **Cause-and-effect diagram.** A cause-and-effect diagram is a simple visual tool used by quality improvement teams to show the possible causes of the quality problem under study. Figure 15.2(b) is a cause-and-effect diagram of the process of converting metal billets (ingots) into a forged item.[2] Here, the ultimate "effect" is a good forged item. Notice that the main branches (Environment, Material, Equipment, Personnel, Methods) organize the causes and serve as a skeleton for the detailed entries. The main branches shown in Figure 15.2(b) apply to many applications and can serve as a general template for organizing thinking about possible causes. Of course, you are not bound to these branch labels. Once a list of possible causes is generated, they can be organized into natural main groupings that represent the main branches of the diagram. Looking at Figure 15.2(b), you can see why cause-and-effect diagrams are sometimes called *fishbone diagrams*.

scatterplot, p. 66

- **Scatterplot.** The scatterplot can be used to investigate whether two variables are related, which might help in identifying potential root causes of problems.

- **Control chart.** A control chart is a time-sequenced plot used to study how a process changes over time. A control chart is more than a run chart in that control limits and a line denoting the average are superimposed on the plot. The control limits help practitioners determine if the process is consistent with past behavior or if there is evidence that the process has changed in some way. This chapter is largely devoted to the control chart technique.

Beyond the application of simple tools, there is an increasing use of more sophisticated statistical tools in the pursuit of quality. For example, the design of a new product as simple as a multivitamin tablet may involve interviewing samples of consumers to learn what vitamins and minerals they want included and using randomized comparative experiments to design the manufacturing process (Chapter 3). An experiment might discover, for example, what combination of moisture level in the raw vitamin powder and pressure in the tablet-forming press produces the right tablet hardness. In general, well-designed experiments reduce ambiguity about cause and effect and allow practitioners to determine what factors truly affect the quality of products and services. Let us now turn our attention to the area of *statistical process control* and its distinctive tool—the control chart.

APPLY YOUR KNOWLEDGE

15.1 Describe a process. Consider the process of going from curbside at an airport to sitting in your assigned airplane seat. Make a flowchart of the process. Do not forget to consider steps that involve Yes/No outcomes.

15.2 Operational definition and measurement. If asked to measure the percent of late departures of an airline, you are faced with an unclear task. Is late departure defined in terms of "leaving the gate" or "taking off from the runway"? What is required is an operational definition of the measurement—that is, an unambiguous definition of what is to be measured so that if you were to collect the data and someone else were to collect the data, both of you would come back with the same measurement values. Provide an example of an operational definition for the following:

(a) Reliable mobile provider.

(b) Clean desk.

(c) Effective teacher.

15.3 Causes of variation. Consider the process of uploading a video to an Instagram account from a cell phone. Brainstorm as least five possible causes for variation in upload time. Construct a cause-and-effect diagram based on your identified potential causes.

15.1 Statistical Process Control

When you complete this section, you will be able to:

- Explain what is meant by a process being in control by distinguishing common and special cause variation.
- Describe the basic purpose of a control chart.
- Explain the distinction between variable and attribute control charts.

The goal of statistical process control is to make a process stable and then keep it stable over time unless planned changes are made. You might want, for example, to keep your weight constant over time. A manufacturer of machine parts wants the critical dimensions to be the same for all parts. "Constant over time" and "the same for all" are not realistic requirements. They ignore the fact that *all processes have variation.* Your weight fluctuates from day to day; the critical dimension of a machined part varies a bit from item to item; the time to process a college admission application is not the same for all applications. Variation occurs in even the most precisely made product due to small changes in the raw material, the adjustment of the machine, the behavior of the operator, and even the temperature in the plant. Because variation is always present, the statistical description of stability over time requires that the *pattern of variation* remain stable, not that there be no variation in the variable measured.

STATISTICAL CONTROL

A process that continues to be described by the same distribution when observed over time is said to be in statistical control, or simply **in control.**

Control charts are statistical tools that monitor a process and alert us when the process has been changed so that it is now **out of control.** This is a signal to find and respond to the cause of the change.

common cause variation

In the language of statistical quality control, a process that is in control has only common cause variation. **Common cause variation** is the inherent variability of the system due to many small causes that are always present. Because it is assumed that these many underlying small causes result in small, *random* perturbations to which all process outcomes are exposed, their cumulative effect is, by definition, assumed to be random. Thus, an in-control process is a random process that generates random or independent process outcomes over time.

random process, p. 679

special cause variation

assignable cause

When the normal functioning of the process has changed, we say that **special cause variation** is added to the common cause variation. A special cause can be viewed as any factor impinging on the process and resulting in variation not consistent with common cause variation. In contrast to common causes, special causes can often be traced to some clear and identifiable event. As a result, some practitioners refer to a special cause as an **assignable cause.** Examples might include an operator error, a jammed machine, or a bad batch of raw material. These are classic manufacturing examples in which the special cause variation has negative implications on the process. In particular, when dealing with a manufacturing process in which the goal is to produce parts as close to specification as possible, any added variation is undesirable. In such situations, we hope to be able to discover what lies behind special cause variation and eliminate that cause to restore the stable functioning of the process.

Historically, statistical process control (SPC) methods were devised to monitor manufactured parts with the intention of detecting unwanted special cause variation. However, one of the great contributions of the quality revolution is the recognition that any process, not just classical manufacturing processes, has the potential to be improved. In the business arena, SPC methods are routinely used for monitoring services processes—for example, patient waiting time in a hospital clinic. These same methods, however, can be used to monitor the ratings of a television show, daily stock returns, the level of ozone in the atmosphere, or even golf scores. With this broader perspective, process change due to a special cause might be viewed favorably—for example, a decrease in waiting times or an increase in monthly customer satisfaction ratings. In such situations, our intention should not be to eliminate the special cause but, rather, to learn about the special cause and promote its effects.

EXAMPLE 15.1

Common Cause, Special Cause Imagine yourself doing the same task repeatedly, say, folding an advertising flyer, stuffing it into an envelope, and sealing the envelope. The time to complete the task will vary a bit, and it is hard to point to any one reason for the variation. Your completion time shows only common cause variation.

Now you receive a text. You engage in a text conversation, and though you continue folding and stuffing while texting, your completion time rises beyond the level expected from common causes alone. Texting adds special cause variation to the common cause variation that is always present. The process has been disturbed and is no longer in its normal and stable state.

If you are paying temporary employees to fold and stuff advertising flyers, you avoid this special cause by requiring your employees to turn off their cell phones while they are working. ■

The idea underlying control charts is simple but ingenious.[3] By setting limits on the natural variability of a process, control charts work by distinguishing the always-present common cause variation in a process from the additional variation that suggests that the process has been changed by a special cause. When a control chart indicates process change, it is a signal to respond, which often entails taking corrective action. On the flip side, when a control chart indicates that there has been no process change, the chart still serves a purpose: it restrains the user from taking unnecessary actions. All too often, time and resources are wasted by misinterpreting common cause variation as special cause variation. When a control chart is not signaling, the best management practice is one of no action.[4]

A wide variety of control charts are available to quality practitioners. Control charts can be broadly classified based on the type of data collection.

TYPES OF CONTROL CHARTS

Variable control charts are control charts devised for monitoring quantitative measurements, such as weights, time, temperature, or dimensions. Variable control charts include charts for monitoring the mean of the process and charts for monitoring the variability of the process. Section 15.2 discusses their construction and use.

Attribute control charts are control charts for monitoring counting data. Examples of counting data are number (or proportion) of defective items in a production run, number of invoice errors, or number of complaining customers per month. Section 15.4 discusses two of the most common attribute charts: the p chart and the c chart.

APPLY YOUR KNOWLEDGE

15.4 Special causes. Rachel participates in bicycle road races. She regularly rides 25 kilometers over the same course in training. Her time varies a bit from day to day but is generally stable. Give several examples of special causes that might unusually raise or lower Rachel's time on a particular day.

15.5 Common causes and special causes. In Exercise 15.1, you described the process of getting on an airplane. What are some sources of common cause variation in this process? What are some special causes that can result in out-of-control variation?

SECTION 15.1 SUMMARY

- Work is organized in **processes,** or chains of activities that lead to some result. We use **flowcharts** and **cause-and-effect diagrams** to describe processes. **Pareto charts** and **scatterplots** can be useful in isolating primary root causes for quality problems.

- All processes have variation. **Common cause** variation reflects the natural variation inherent in every process. A process exhibiting only common cause variation is said to be **in control. Special cause** variation is variation inconsistent with common cause variation. Processes influenced by special cause variation are **out of control.**

- **Control charts** are statistical devices indicating when the process is in control or when it is affected by special cause variation. **Variable control charts** are used for monitoring measurements taken on some continuous scale. **Attribute control charts** are used for monitoring count data.

SECTION 15.1 EXERCISES

For Exercises 15.1 to 15.3, see page 15-7; and for 15.4 and 15.5, see page 15-9.

15.6 Which type of control chart? For each of the following process outcomes, indicate if a variable control chart or an attribute control chart is most applicable:

(a) Number of lost-baggage claims per day.

(b) Time to respond to a field service call.

(c) Thickness (in millimeters) of cold-rolled steel plates.

(d) Percent of late shipments per week.

15.7 Describe a process. Each weekday morning, you must get to work or to your first class on time. Make a flowchart of your daily process for doing this, starting when you wake. Be sure to include the time at which you plan to start each step.

15.8 Common cause, special cause. Each weekday morning, you must get to work or to your first class on time. The time at which you reach work or class varies from day to day, and your planning must allow for this variation. List several common causes of variation in your arrival time. Then list several special causes that might result in unusual variation, such as being late to work or class.

15.9 Pareto charts. Continue the study of the process of getting to work or class on time. If you kept good records, you could make a Pareto chart of the reasons (special causes) for late arrivals at work or class. Make a Pareto chart that you think roughly describes your own reasons for lateness. That is, list the reasons from your experience, and chart your estimates of the percent of late arrivals each reason explains.

15.10 Pareto charts. Painting new auto bodies is a multistep process. There is an "electrocoat" that resists corrosion, a primer, a color coat, and a gloss coat. A quality study for one paint shop produced this breakdown of the primary problem type for those autos whose paint did not meet the manufacturer's standards:

Problem	Percent
Electrocoat uneven—redone	4
Poor adherence of color to primer	5
Lack of clarity in color	2
"Orange peel" texture in color	32
"Orange peel" texture in gloss	1
Ripples in color coat	28
Ripples in gloss coat	4
Uneven color thickness	19
Uneven gloss thickness	5
Total	100

Make a Pareto chart. Which stage of the painting process should we look at first?

15.2 Variable Control Charts

When you complete this section, you will be able to:

- Explain the basic sampling scheme issues that need to be addressed prior to setting up a control chart.
- Explain the goals of retrospective and prospective phases of a control chart.
- Contrast the mean ($\bar{x}$) chart against range (R) and standard deviation (s) charts in terms of what they monitor and which should be interpreted first.
- Compute the center line and control limits for an $\bar{x}$ chart, R chart, and an s chart.
- Compute the center line and control limits for an individuals (I) chart and moving-range (MR) chart.
- Identify issues that affect the application of control charts.

This section considers the scenario in which regular samples on measurement data are obtained to monitor process behavior. In the quality area, samples of observations are often referred to as *subgroups*. For each subgroup, pertinent statistics are computed and then charted over time. For example, the subgroup means can be plotted over time to control the overall mean level of the process, while process variability might be controlled by plotting subgroup standard deviations or a more simplistic statistic known as the range statistic.

The effectiveness of a control chart depends on how the subgroups were collected. Three basic issues need to be addressed:

1. **Rational subgrouping.** Walter Shewhart, the founder of statistical process control, conceptualized a basis for forming subgroups. He suggested that subgroups should be chosen in such a way that the

individual observations within the subgroups have been measured under similar process conditions. Subgroups formed on this principle are known as *rational subgroups*. The idea is that if the individual observations within the subgroups are as homogeneous as possible, then any special causes disrupting the process will be reflected by greater variability between the subgroups. Thus, when special causes are present, rational subgrouping attempts to maximize the likelihood that subgroup statistics will signal that the process is out of control. In manufacturing settings, the most common way to create rational subgroups is to take individual measurements over a short period of time; often, this means measuring consecutive items produced.

2. **Subgroup size.** Subgroup sizes are usually small. Sampling cost is one important consideration. Another is that large samples may span too much time, making it possible for the process to change while the sample is being taken. When sample sizes are large, the subgroups are at risk of not being rational subgroups. Subgroup sizes (n) for variable control charts nearly always range from 1 to 25. For various historical reasons, sample sizes of 4 or 5 are among the most common choices. As we will see, control charts rely on approximate Normality of the subgroup statistics. It is fortunate that for certain statistics, like the mean, the central limit theorem effect will provide approximate Normality for sample sizes as small as 4 or 5.

central limit theorem, p. 313

3. **Sampling frequency.** A final subgroup design issue is the frequency of sampling, that is, the timing between subgroups. Cost factors obviously come to bear. There are not only the costs of sampling (e.g., costs of testing and measuring), but also the costs associated with missing significant process changes. If sampling is done infrequently, then there is a risk that the process is out of control between subgroups, resulting in a variety of costs ranging from higher rework to customer dissatisfaction for receiving unacceptable product or service. Process stability (or the lack thereof) is another consideration. A process that is erratic needs frequent surveillance, but a process that has achieved stability can be less frequently sampled. A common strategy is to start with frequent sampling of the process and then to relax the frequency of sampling as one gains confidence about the stability of the process.

Once the sampling scheme has been designed in terms of establishing rational subgroups, sample size, and sampling frequency, data are collected. When first applying control charts to a process, the process behavior is not fully understood. By using a control chart to analyze a given set of initial data, we are looking back *retrospectively* on the performance of the process. In this phase, control charts are said to be used *as a judgment*.

If the process is found to be in control, then the control chart can be used *prospectively* to monitor future process performance. In this phase, control charts are said to be used *as an ongoing operation*. If retrospective analysis shows the process to be not in control, then any numerical descriptions of the data used to construct the control chart will not serve as reliable guides for monitoring the process into the future. The priority is then to bring the process into control. This may mean uncovering and removing the unwanted effects of special causes. Or it may mean incorporating the favorable effects of the special causes and stabilizing the process at a more favorable position. Later, when the process has been operating in control for some time and you understand its usual behavior, control charts can be used prospectively.

$\bar{x}$ and R charts

We begin with a quantitative variable x that is an important measure of quality. The variable might be the diameter of a part or the time to respond to a customer call. Given this quality characteristic is subject to variation, there is a distribution underlying the process. As discussed earlier, an in-control process is a stable process whose underlying distribution remains the same over time. Associated with this distribution is a process mean μ and a process standard deviation σ.

To make subgroup control charts, we begin by taking samples of size n from the process at regular intervals. For instance, we might measure four consecutive parts or time the responses to five consecutive customer calls. For each subgroup, we compute the sample mean $\bar{x}$. From Chapter 6 (page 311), we learned that $\bar{x}$ is a random variable with mean μ and standard deviation $\sigma/\sqrt{n}$. Furthermore, the central limit theorem tells us that the sampling distribution of $\bar{x}$ is approximately Normal.

68–95–99.7 rule, p. 45

The 99.7 part of the 68–95–99.7 rule for Normal distributions says that as long as the process remains in control, 99.7% of the values of $\bar{x}$ will fall between:

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$

$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

three-sigma control limits

where UCL and LCL stand for "upper control limit" and "lower control limit," respectively. These limits are called **three-sigma control limits** and serve as the basis for most control charts. Along with the control limits, it is standard practice to draw a center line (CL) at the mean.

false alarm rate

More precisely, for a Normal distribution, the probability is 0.9973 that an observation will fall within three standard deviations of the mean. Thus, with three-sigma limits, if the process remains in control, we will rarely observe an $\bar{x}$ outside the control limits: the probability is 0.0027 that a sample mean will randomly fall outside the limits. The probability of 0.0027 is referred to as the **false alarm rate.** Given that false alarms occur so rarely for an in-control process, if we do observe an $\bar{x}$ outside the control limits, it serves as a strong signal that the underlying conditions of the process may have changed.

grand mean

In practice, μ and σ are typically not known and must be estimated. One obvious estimate for the mean is the average of all the individual observations taken from all the preliminary subgroups. If the sample sizes are all equal, then the average of all the individual observations can be computed as the average of the subgroup means. In particular, if we are basing the construction of the control chart on k subgroups, then the overall average (referred to as the **grand mean**) is computed as

$$\bar{\bar{x}} = \frac{1}{k}(\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k)$$

range statistic R

We now need an estimate of σ. One possibility is to use the subgroup sample standard deviations. However, in practice, it is more common to use a more simplistic estimate of variation based on the **range statistic** $\boldsymbol{R}$**.** The range statistic is simply the sample range, which is the difference between the largest and the smallest observations in a sample. With k preliminary subgroups, the average range $\bar{R}$ can be computed as

$$\bar{R} = \frac{1}{k}(R_1 + R_2 + \cdots + R_k)$$

Based on statistical theory, it can shown that if $\bar{R}$ is multiplied by a constant (A_2) that is a function of the subgroup size n, we then have a reasonable estimate for $3\sigma/\sqrt{n}$. Table 15.1 provides values for A_2 for various subgroup sizes. The estimated center line and control limits for the mean are then given by

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$
$$\text{CL} = \bar{\bar{x}}$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

$\bar{x}$ chart

The control chart for the mean, called an ***$\bar{x}$ chart,*** is dedicated to detecting changes in the process level. It is also important to monitor the variability of the process. Because we have the subgroup ranges in hand, it is sensible to develop a control chart for ranges. Such a chart is known as an ***R* chart.** The center line and control limits of the R chart are given by

R chart

$$\text{UCL} = D_4\bar{R}$$
$$\text{CL} = \bar{R}$$
$$\text{LCL} = D_3\bar{R}$$

The control chart constants D_3 and D_4 are provided in Table 15.1.

On the surface, the R chart does not appear to have the same format as the $\bar{x}$ chart in the sense of establishing control limits a certain amount above and

TABLE 15.1 Control chart constants

Sample size n	D_3	D_4	B_3	B_4	A_2	A_3	d_2
2	0.000	3.267	0.000	3.267	1.881	2.659	1.128
3	0.000	2.574	0.000	2.568	1.023	1.954	1.693
4	0.000	2.282	0.000	2.266	0.729	1.628	2.059
5	0.000	2.114	0.000	2.089	0.577	1.427	2.326
6	0.000	2.004	0.030	1.970	0.483	1.287	2.534
7	0.076	1.924	0.118	1.882	0.419	1.182	2.704
8	0.136	1.864	0.185	1.815	0.373	1.099	2.847
9	0.184	1.816	0.239	1.761	0.337	1.032	2.970
10	0.223	1.777	0.284	1.716	0.308	0.975	3.078
11	0.256	1.744	0.321	1.679	0.285	0.927	3.173
12	0.283	1.717	0.354	1.646	0.266	0.886	3.258
13	0.307	1.693	0.382	1.618	0.249	0.850	3.336
14	0.328	1.672	0.406	1.594	0.235	0.817	3.407
15	0.347	1.653	0.428	1.572	0.223	0.789	3.472
16	0.363	1.637	0.448	1.552	0.212	0.763	3.532
17	0.378	1.622	0.466	1.534	0.203	0.739	3.588
18	0.391	1.609	0.482	1.518	0.194	0.718	3.640
19	0.404	1.597	0.497	1.503	0.187	0.698	3.689
20	0.415	1.585	0.510	1.490	0.180	0.680	3.735
21	0.425	1.575	0.523	1.477	0.173	0.663	3.778
22	0.435	1.566	0.534	1.466	0.168	0.647	3.819
23	0.443	1.557	0.545	1.455	0.162	0.633	3.858
24	0.452	1.548	0.555	1.445	0.157	0.619	3.895
25	0.459	1.541	0.565	1.435	0.153	0.606	3.931

below the center line. However, underlying the development of the control chart constants D_3 and D_4 is a three-sigma structure for the range statistic. You will notice that for subgroup sizes of two to six, the D_3 factor is 0, which implies a lower control limit of 0. For small subgroup sizes, it can be shown that the theoretical control limits placed plus/minus three standard deviations of the range statistic above and below the range mean will result in a negative lower control limit. Because the range statistic can never be negative, the lower control limit is accordingly set to 0. Here is a summary of our discussion.

CONSTRUCTION OF $\bar{x}$ AND R CHARTS

Take regular samples of size n from a process. The center line and control limits for an **$\bar{x}$ chart** are

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$
$$\text{CL} = \bar{\bar{x}}$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

where $\bar{\bar{x}}$ is the average of the subgroup means and $\bar{R}$ is the average of the subgroup ranges. The center line and control limits for an ***R* chart** are

$$\text{UCL} = D_4\bar{R}$$
$$\text{CL} = \bar{R}$$
$$\text{LCL} = D_3\bar{R}$$

The **control chart constants** A_2, D_3, and D_4 depend on the sample size n.


monkeybusiness/Deposit Photos

CASE 15.1 **Turnaround Time for Lab Results** With escalating health care costs and continual demand for better patient service and patient outcomes, the health care industry is rapidly embracing the use of quality management techniques. Improving the timeliness and accuracy of lab results within a hospital is a common focus of health care improvement teams.

Consider the case of a hospital looking at the time from request to receipt of blood tests for the emergency room (ER). One of the most commonly requested tests from the ER is a complete blood count (CBC). A CBC provides doctors with red and white cell counts, as well as blood clotting measures, all of which can be crucial in an emergency situation. Because of the importance of the turnaround time for CBC requests, hospital management appointed a quality improvement team to study the process. The team selected random samples of five CBC requests per shift (day, evening, late night) over the course of 10 days. Thus, the team had $3 \times 10 = 30$ preliminary subgroups. The sampling within shifts associates the individual observations with similar conditions (e.g., staffing) and thus abides by the rational subgrouping principle. For each of the CBC requests sampled, the turnaround time (minutes) was recorded. Table 15.2 provides the observation values along with the subgroup means and ranges. ■

Before we proceed to the construction of control charts for Case 15.1, we must mention a subtle implementation issue with respect to the $\bar{x}$ and R charts. The $\bar{x}$ chart limits rely on the average range $\bar{R}$. If process variability is not stable and is affected by special causes, then $\bar{R}$ is not a reliable estimate of variability, and thus, the $\bar{x}$ chart limits are less meaningful. There is a lesson here: *it is difficult to interpret an $\bar{x}$ chart unless the ranges are in control.* When you look at $\bar{x}$ and R charts, always start with the R chart.

TABLE 15.2 Thirty control chart subgroups of lab testing turnaround times (in minutes)

Subgroup	Turnaround observations					Subgroup mean	Range
1	39	33	65	50	41	45.6	32
2	46	36	34	53	37	41.2	19
3	37	35	28	37	41	35.6	13
4	50	38	35	60	39	44.4	25
5	29	27	22	43	50	34.2	28
6	32	35	40	27	42	35.2	15
7	42	43	37	44	39	41.0	7
8	40	45	50	43	24	40.4	26
9	34	47	54	39	51	45.0	20
10	43	65	25	45	25	40.6	40
11	35	48	44	45	34	41.2	14
12	55	54	44	36	55	48.8	19
13	29	39	47	42	47	40.8	18
14	41	31	29	37	27	33.0	14
15	41	40	32	33	52	39.6	20
16	32	32	41	47	43	39.0	15
17	24	54	34	53	56	44.2	32
18	36	45	53	31	31	39.2	22
19	48	57	36	31	30	40.4	27
20	38	27	39	35	27	33.2	12
21	53	33	51	50	42	45.8	20
22	53	45	37	44	33	42.4	20
23	27	50	35	29	47	37.6	23
24	39	39	51	49	44	44.4	12
25	33	29	38	68	34	40.4	39
26	34	43	48	49	56	46.0	22
27	43	20	51	49	50	42.6	31
28	33	42	51	58	26	42.0	32
29	25	46	25	43	42	36.2	21
30	30	34	42	36	51	38.6	21

EXAMPLE 15.2

CASE 15.1

LAB

Constructing $\bar{x}$ and R Charts We begin the analysis of the turnaround times for the lab-testing process by focusing on process variability. We use the 30 ranges shown in Table 15.2 to find the average range:

$$\bar{R} = \frac{1}{30}(32 + 19 + \cdots + 21)$$

$$= \frac{659}{30} = 21.967$$

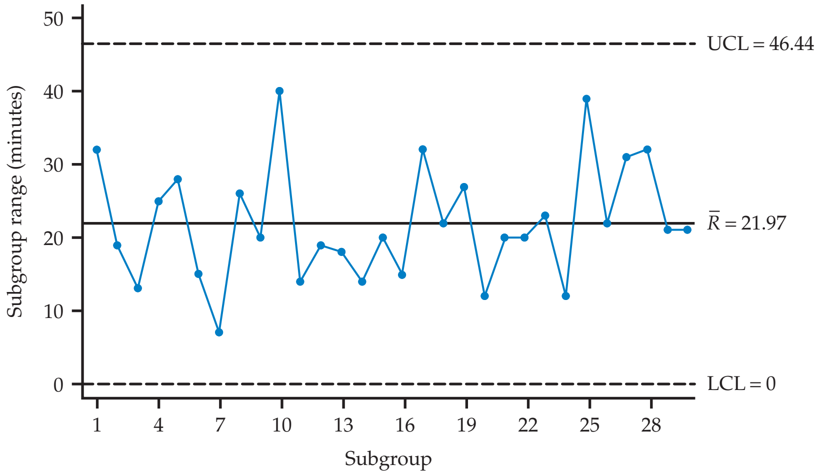


FIGURE 15.3 *R* chart for the lab-testing data of Table 15.2.

From Table 15.1 (page 15-13), for subgroup size $n = 5$, the values of D_3 and D_4 are 0 and 2.114, respectively. Accordingly, the center line and control limits for the R chart are

$$\begin{aligned} \text{UCL} &= D_4\bar{R} = 2.114(21.967) = 46.438 \\ \text{CL} &= \bar{R} = 21.967 \\ \text{LCL} &= D_3\bar{R} = 0(21.967) = 0 \end{aligned}$$

Figure 15.3 shows the R chart for the lab testing process. The R chart shows no points outside the upper control limit. Furthermore, the ranges plotted over time show no unusual pattern. We can say that from the perspective of process variation, the process is in control.

We now construct the $\bar{x}$ chart. Because the R chart exhibited in-control behavior, we can safely use the value of 21.967 computed earlier. Referring to Table 15.2, the grand mean is:

$$\begin{aligned} \bar{\bar{x}} &= \frac{1}{30}(45.6 + 41.2 + \cdots + 38.6) \\ &= \frac{1218.6}{30} = 40.62 \end{aligned}$$

From Table 15.1 (page 15-13), we find $A_2 = 0.577$. The center line and control limits for the $\bar{x}$ chart are then as follows:

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2\bar{R} = 40.62 + 0.577(21.967) = 53.29 \\ \text{CL} &= \bar{\bar{x}} = 40.62 \\ \text{LCL} &= \bar{\bar{x}} - A_2\bar{R} = 40.62 - 0.577(21.967) = 27.95 \end{aligned}$$

Figure 15.4 shows the $\bar{x}$ chart. The subgroup means of the 30 samples do vary, but all lie within the range of variation marked out by the control limits. We are seeing the common cause variation of a stable process with no indications of special causes. ■

Example 15.2 shows that the lab-testing process for the ER is stable and in control. Does this mean the process is *acceptable*? This is not a statistical question but rather a managerial question for the ER and the hospital administration. Currently, the mean turnaround time is estimated to be 40.62 minutes. The process is stable and in control around that estimated mean. If this process performance is considered acceptable, then the control chart limits for both the $\bar{x}$ chart and the R chart can be projected out into the future to monitor the process so as to maintain performance. If, however, the mean time of around 41 minutes is viewed as unacceptably high, then efforts need

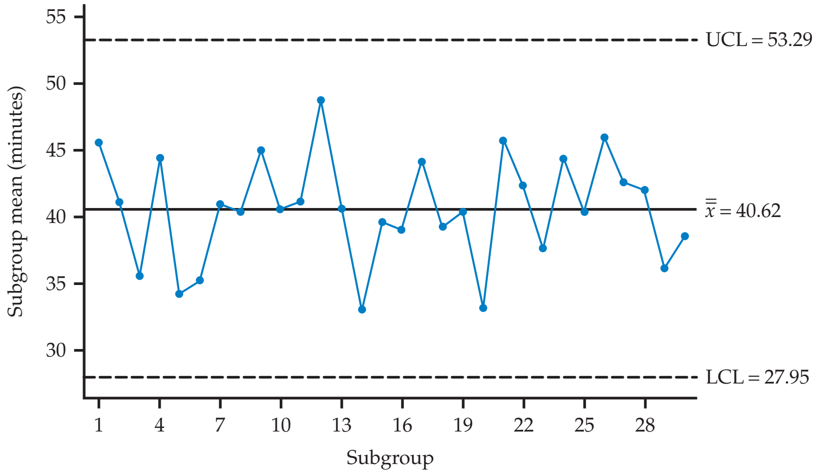


FIGURE 15.4 $\bar{x}$ chart for the lab-testing data of Table 15.2.

to be initiated to find ways to improve the process. Here is where the basic tools of flowcharts, Pareto charts, and cause-and-effect diagrams can be used by quality improvement teams to better understand the lab turnaround process and to search out the underlying causes for delay.

When efforts are made to improve a process, the control chart can play an important role. Prospectively, control charts can be used to judge whether an attempt to improve a process has resulted in a successful change. Checking for successful change is a critical aspect of the Check phase of the PDCA cycle (page 15-3) and the Improve phase of the DMAIC model (page 15-4).

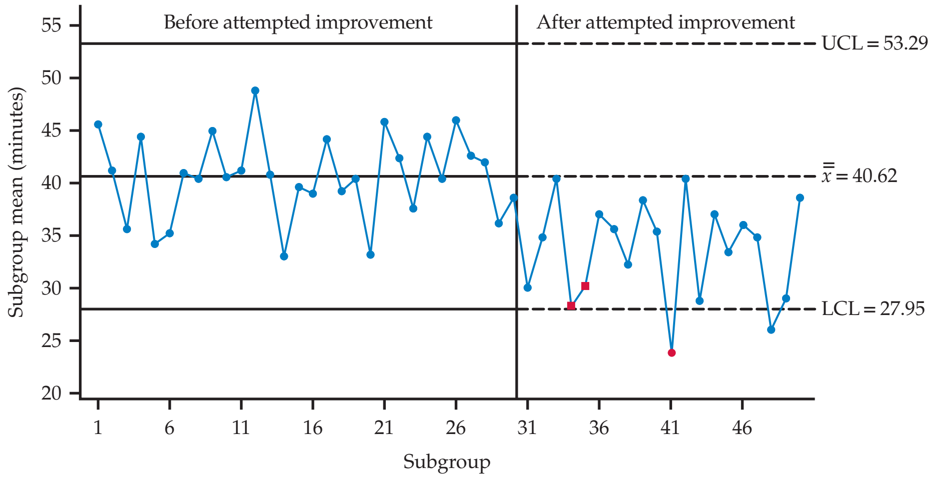
Figure 15.5(a) shows a case where the control chart demonstrates a successful attempt to decrease the turnaround time. However, notice that a three-sigma signal doesn't occur until subgroup 41. So far, we have considered only the basic "one point beyond the control limits" criterion to signal that a process may have gone out of control. If the shift in the process is small to moderate, it may take some time before a subgroup falls outside the control limits. We can speed the response of a control chart to process change by adding more patterns to detect other than "one point out" as signals. The most common step in this direction is to add a **runs rule** to the control chart.

runs rule

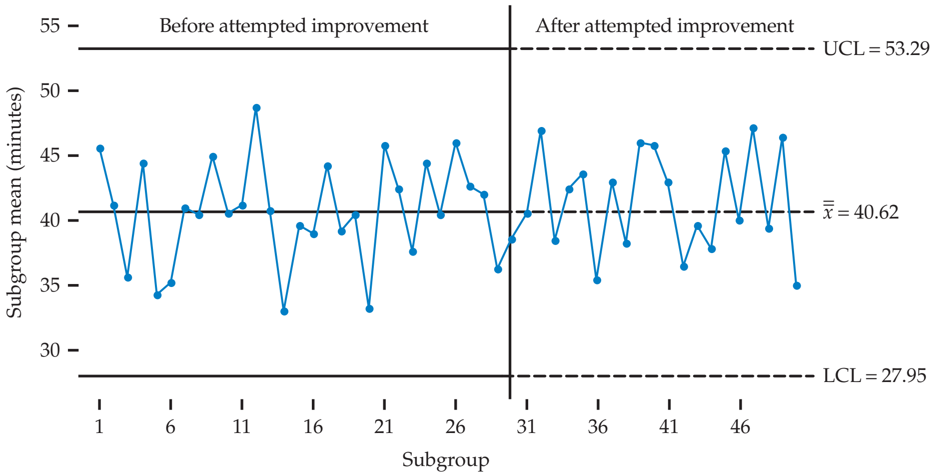
For example, Minitab offers a runs rule that signals "out of control" if two out of three consecutive observations are more than two standard deviations from the center line. JMP signals if two consecutive observations are more than two standard deviations from the center line. From Figure 15.5(a), we find that subgroups 34 and 35 have been highlighted. Both of these subgroups are beyond two standard deviations from the center line. As such, we have a signal of a possible change in the process earlier than the three-sigma signal at subgroup 41.

Another runs rule is to signal when nine consecutive observations are all above the center line or nine consecutive observations are all below the center line. (*Note:* JMP uses 10 consecutive observations.) From Figure 15.5(a), we indeed find that after the attempted improvement, many more than nine observations all fall below the center line. Software packages offer many more runs rules than we have illustrated here.

In our enthusiasm to detect various special kinds of out-of-control states, it is easy to forget that adding rules always increases the false alarm rate. For example, by supplementing all of Minitab's runs rules to the standard one-point-out rule, the overall false alarm rate is about 0.02, which is more than 7 times the false alarm rate of 0.0027 associated with the one-point-out rule. Frequent false alarms are so annoying that the people responsible for responding soon begin to ignore out-of-control signals. *It is better to use only a few out-of-control rules and to reserve tailor-made rules other than one-point-out rule for processes that are known to be prone to specific special causes.*



(a)



(b)

FIGURE 15.5 Using control charts to assess improvement efforts. (a) The improvement effort was successful. (b) The improvement effort was unsuccessful.

Assuming we are convinced that there is evidence of a changed process with Figure 15.5(a), control chart limits should be revised, and the new process should be monitored to maintain the gains, as called for in the Control phase of the DMAIC model. In contrast to Figure 15.5(a), the control chart in Figure 15.5(b) provides no evidence of an impact from the attempted process improvement; the organization should seek alternative improvement ideas.

The preliminary samples of Example 15.2 appear to come from an in-control process. Let us now consider control chart implementation issues when there is evidence of special cause effects in the preliminary samples.

CASE 15.2 **O-Ring Diameters** A manufacturer of synthetic-rubber O-rings must monitor and control their dimensions. O-rings are used in numerous industries, including medical, aerospace, oil refining, automotive, and chemical processing. O-rings are doughnut-shaped gaskets used to seal joints against high pressure from gases or fluids. The two primary dimensions that need to be controlled are the cross-sectional width of the ring and the inside diameter of the doughnut. Within the O-ring product family, the manufacturer produces an aerospace industry class of O-rings known as AS568A. Table 15.3 gives the observations for 25 preliminary subgroups of size 4 for the inside diameter along with subgroup means and ranges. This O-ring is specified to have an inside diameter of 2.612 inches. The tolerances are set at ±0.02 inch around this specification. ■

TABLE 15.3 Twenty-five control chart subgroups of O-ring measurements (in inches)

Subgroup	O-ring measurements				Sample mean	Range
1	2.6088	2.6120	2.6167	2.6059	2.61085	0.0108
2	2.5993	2.6120	2.6089	2.6046	2.60620	0.0127
3	2.6117	2.6074	2.6118	2.6101	2.61025	0.0044
4	2.6063	2.6055	2.6119	2.6076	2.60783	0.0064
5	2.6139	2.6030	2.6038	2.6097	2.60760	0.0109
6	2.6019	2.6075	2.6086	2.6076	2.60640	0.0067
7	2.6045	2.6005	2.5980	2.5964	2.59985	0.0081
8	2.6114	2.6050	2.6063	2.6086	2.60783	0.0064
9	2.6091	2.6100	2.6146	2.6100	2.61093	0.0055
10	2.6078	2.6067	2.6111	2.6044	2.60750	0.0067
11	2.6055	2.6089	2.6010	2.6093	2.60618	0.0083
12	2.6107	2.6098	2.6043	2.6095	2.60858	0.0064
13	2.6155	2.6050	2.6094	2.6050	2.60873	0.0105
14	2.6068	2.6067	2.6075	2.5975	2.60463	0.0100
15	2.6054	2.6021	2.6103	2.6054	2.60580	0.0082
16	2.6068	2.6084	2.6103	2.6004	2.60648	0.0099
17	2.6061	2.6185	2.5953	2.6075	2.60685	0.0232
18	2.6185	2.6096	2.6077	2.6050	2.61020	0.0135
19	2.6072	2.6067	2.6121	2.6017	2.60693	0.0104
20	2.6091	2.6113	2.6037	2.6092	2.60833	0.0076
21	2.6054	2.6149	2.6114	2.6020	2.60843	0.0129
22	2.6074	2.6092	2.6113	2.5992	2.60678	0.0121
23	2.6034	2.5972	2.6124	2.6070	2.60500	0.0152
24	2.6107	2.6101	2.6079	2.6072	2.60898	0.0035
25	2.6021	2.6073	2.6044	2.5995	2.60333	0.0078

EXAMPLE 15.3

CASE 15.2

ORING

$\bar{x}$ and R Charts and Out-of-Control Signals Our initial step is to study the variability of the O-ring process. Using the 25 ranges shown in Table 15.3, we find the average range to be

$$\bar{R} = \frac{1}{25}(0.0108 + 0.0127 + \cdots + 0.0078)$$

$$= \frac{0.2381}{25} = 0.009524$$

From Table 15.1 (page 15-13), for subgroups of size $n = 4$, the values of D_3 and D_4 are 0 and 2.282, respectively. Accordingly, the center line and control limits for the R chart are

$$\text{UCL} = D_4\bar{R} = 2.282(0.009524) = 0.021734$$

$$\text{CL} = \bar{R} = 0.009524$$

$$\text{LCL} = D_3\bar{R} = 0(0.009524) = 0$$

Figure 15.6 is the R chart for the O-ring process. Subgroup 17 lies outside the upper control limit. Had we constructed an $\bar{x}$ chart at this time, we would have found that Subgroup 17 would not have signaled out of control. It is not unusual for the R chart to signal out of control while the $\bar{x}$ chart does not,

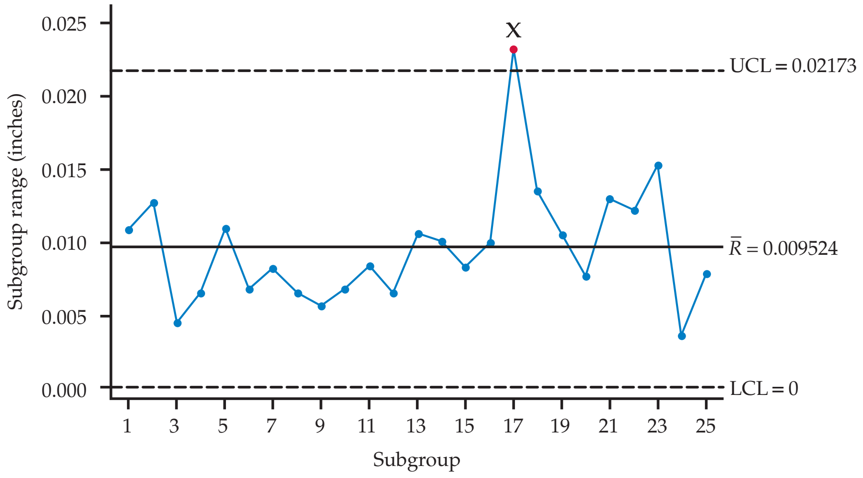


FIGURE 15.6 *R* chart for the O-ring data of Table 15.3. Subgroup 17 signals out of control.

or vice versa. Each chart is looking for different departures. The R chart is looking for changes in variability, and the $\bar{x}$ chart is looking for changes in the process level. It is, of course, possible for both process variation and level to go out of control together, resulting in signals on both charts. At this stage, an explanation should be sought for the out-of-control signal.

Suppose that an investigation reveals a machine problem at the time of the out-of-control signal. Because a special cause was discovered, the associated subgroup should be set aside and a new R chart constructed. By deleting Subgroup 17, the revised range estimate based on the remaining 24 subgroups is:

$$\bar{R} = \frac{0.2149}{24} = 0.008954$$

Figure 15.7 shows the updated R chart applied to the 24 subgroups. Now, all the subgroup ranges are found to be in control. We can now turn our attention to the construction of the $\bar{x}$ chart limits. For the 24 samples in Table 15.3, the grand mean is

$$\bar{\bar{x}} = \frac{1}{24}(2.61085 + 2.60620 + \cdots + 2.60333)$$
$$= \frac{62.5735}{24} = 2.60723$$

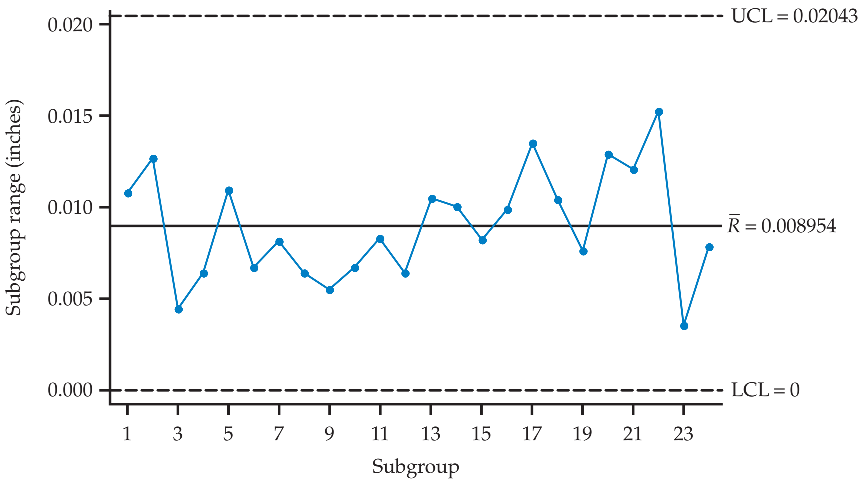


FIGURE 15.7 Updated *R* chart for the O-ring data of Table 15.3 with Subgroup 17 removed.

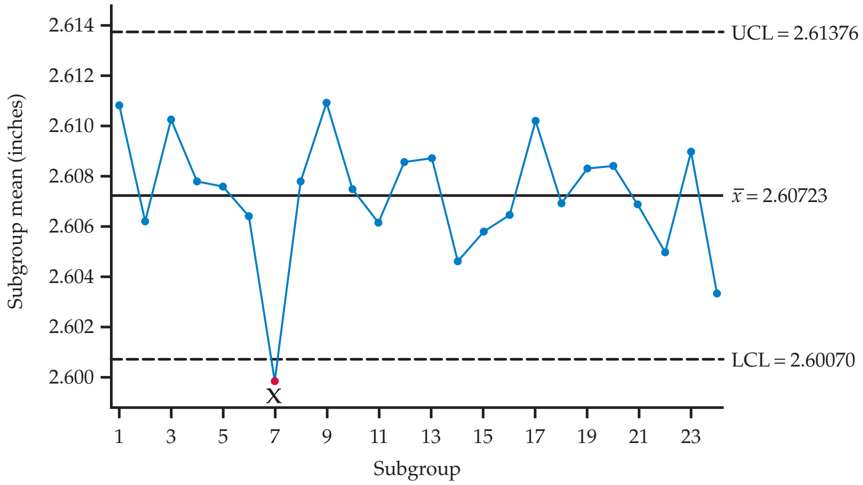


FIGURE 15.8 $\bar{x}$ chart for the O-ring data of Table 15.3 with Subgroup 17 removed.

From Table 15.1 (page 15-13), we find $A_2 = 0.729$. The center line and control limits for the $\bar{x}$ chart are then

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 2.60723 + 0.729(0.008954) = 2.61376$$
$$\text{CL} = \bar{\bar{x}} = 2.60723$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 2.60723 - 0.729(0.008954) = 2.60070$$

Figure 15.8 shows the $\bar{x}$ chart. The $\bar{x}$ chart shows an out-of-control signal at Subgroup 7. A special cause investigation reveals that the abnormally smaller diameters associated with this subgroup were caused by a problem in the postcuring stage that resulted in too much shrinkage of the rubberized material. With an explanation in hand, Subgroup 7 needs to be discarded and the R chart limits need to be recomputed. Figure 15.9 displays both the $\bar{x}$ chart and the R chart based on the remaining 23 samples. The data for both charts appear well behaved. These two sets of control limits can be used for prospective control. ■

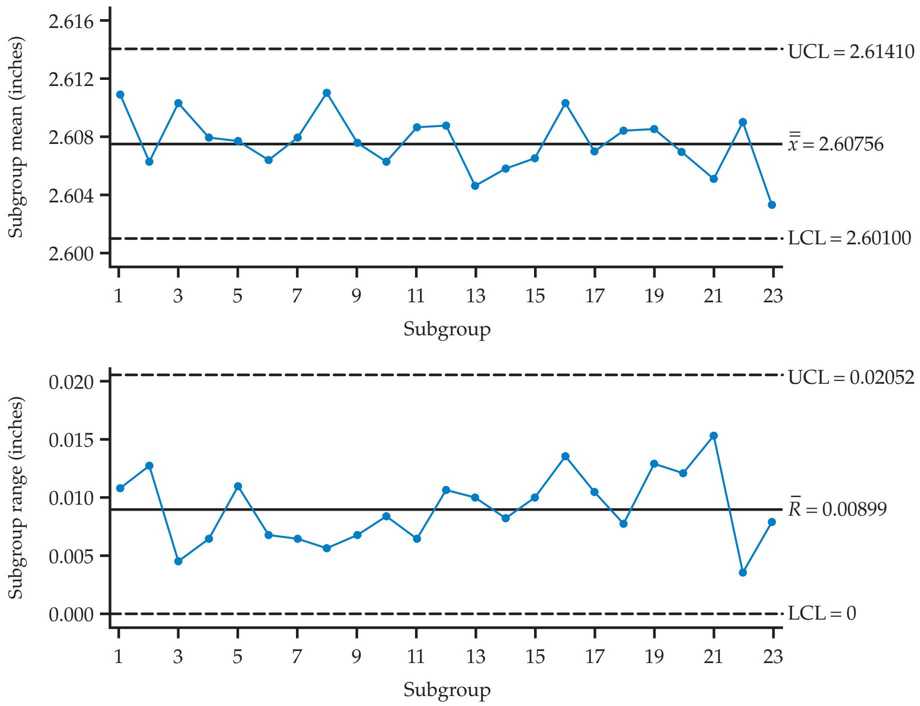


FIGURE 15.9 $\bar{x}$ and R charts for the O-ring data of Table 15.3 with Subgroups 7 and 17 removed.

APPLY YOUR KNOWLEDGE

15.11 Interpreting signals. Explain the difference in the interpretation of a point falling beyond the upper control limit of the $\bar{x}$ chart versus a point falling beyond the upper control limit of the R chart.

15.12 Auto thermostats. A maker of auto air conditioners checks a sample of four thermostatic controls from each hour's production. The thermostats are set at 75°F and then placed in a chamber where the temperature is raised gradually. The temperature at which the thermostat turns on the air conditioner is recorded. The process mean should be $\mu = 75°$. Past experience indicates that the response temperature of properly adjusted thermostats varies with $\sigma = 0.5°$. The mean response temperature $\bar{x}$ for each hour's sample is plotted on an $\bar{x}$ control chart. Calculate the center line and control limits for this chart.

CASE 15.2 **15.13 O-rings.** Show the computations that confirm the limits of the $\bar{x}$ chart and R chart shown in Figure 15.9. ORING

$\bar{x}$ and s charts

In the construction of subgroup control charts, the use of the simplistic range statistic instead of the sample standard deviation statistic is a historical artifact from when calculations were done by hand. Given the availability of computer software to do calculations, the need for computational simplicity is no longer a compelling argument. The fact that the range statistic is still in widespread use is probably due to training issues. It is much easier for corporate trainers to explain and for employees to comprehend the range statistic (largest minus smallest observation) than a statistic based on summing squared deviations, dividing the sum by $n - 1$, and then taking a square root!

The primary advantage of the sample standard deviation is that it uses all the data as opposed to the range statistic, which utilizes only two observation values (largest and smallest). For small subgroup sizes ($n \leq 10$), the range statistic competes well with the sample standard deviation statistic. However, for larger subgroup sizes, it is generally advisable to utilize the more efficient sample standard deviation.[5]

When using sample standard deviations, the R chart is replaced by an s chart. The s chart is a plot of the subgroup standard deviations with appropriate control limits superimposed. In addition, the construction of the $\bar{x}$ chart is based on the subgroup standard deviation values, not the range values. There is no difference in the calculation of the grand mean ($\bar{\bar{x}}$), but we need to calculate the average sample standard deviation from the k preliminary samples:

$$\bar{s} = \frac{1}{k}(s_1 + s_2 + \cdots + s_k)$$

As with the range method, we should always start with the s chart. Here is a summary of how to construct $\bar{x}$ and s charts.

CONSTRUCTION OF $\bar{x}$ AND s CHARTS

Take regular samples of size n from a process. The center line and control limits for an **$\bar{x}$ chart** are

$$\text{UCL} = \bar{\bar{x}} + A_3\bar{s}$$
$$\text{CL} = \bar{\bar{x}}$$
$$\text{LCL} = \bar{\bar{x}} - A_3\bar{s}$$

where $\bar{\bar{x}}$ is the average of the subgroup means and $\bar{s}$ is the average of the subgroup standard deviations. The center line and control limits for an ***s* chart** are

$$\text{UCL} = B_4\bar{s}$$
$$\text{CL} = \bar{s}$$
$$\text{LCL} = B_3\bar{s}$$

The **control chart constants** A_3, B_3, and B_4 depend on the sample size n and are provided in Table 15.1 (page 15-13).

EXAMPLE 15.4

CASE 15.1

$\bar{x}$ and s Charts We leave the specific computations of the $\bar{x}$ and s charts for the lab-testing data to Exercise 15.15. Figure 15.10 shows the $\bar{x}$ and s charts produced by software. Comparing these control charts with the $\bar{x}$ and R charts of Figures 15.3 and 15.4 (pages 15-16 and 15-17), we are left with the same conclusion: the lab-testing process is in control in terms of both level and variability. ■

LAB

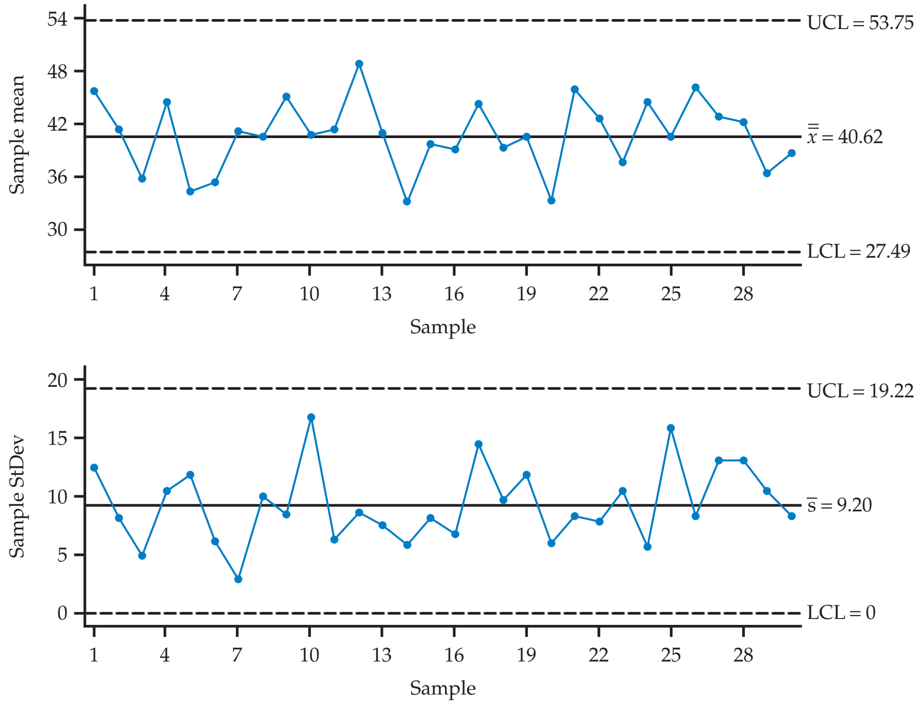


FIGURE 15.10 $\bar{x}$ and s charts for the lab-testing data of Table 15.2.

APPLY YOUR KNOWLEDGE

15.14 Hospital losses. Both nonprofit and for-profit hospitals are financially pressed by restrictions on reimbursement by insurers and the government. One hospital looked at its losses broken down by diagnosis. The leading source was joint replacement surgery. Table 15.4 gives data on the losses (in dollars) incurred by this hospital in treating major joint replacement patients.[6] The hospital has taken from its records a random sample of eight such patients each month for 15 months. HLOSS

TABLE 15.4 Hospital losses for 15 samples of joint replacement patients

Subgroup	Losses (dollars)								Sample mean	Standard deviation
1	6835	5843	6019	6731	6362	5696	7193	6206	6360.63	521.72
2	6452	6764	7083	7352	5239	6911	7479	5549	6603.63	817.12
3	7205	6374	6198	6170	6482	4763	7125	6241	6319.75	749.12
4	6021	6347	7210	6384	6807	5711	7952	6023	6556.88	736.51
5	7000	6495	6893	6127	7417	7044	6159	6091	6653.25	503.72
6	7783	6224	5051	7288	6584	7521	6146	5129	6465.75	1034.26
7	8794	6279	6877	5807	6076	6392	7429	5220	6609.25	1103.96
8	4727	8117	6586	6225	6150	7386	5674	6740	6450.63	1032.96
9	5408	7452	6686	6428	6425	7380	5789	6264	6479.00	704.70
10	5598	7489	6186	5837	6769	5471	5658	6393	6175.13	690.46
11	6559	5855	4928	5897	7532	5663	4746	7879	6132.38	1128.64
12	6824	7320	5331	6204	6027	5987	6033	6177	6237.88	596.56
13	6503	8213	5417	6360	6711	6907	6625	7888	6828.00	879.82
14	5622	6321	6325	6634	5075	6209	4832	6386	5925.50	667.79
15	6269	6756	7653	6065	5835	7337	6615	8181	6838.88	819.46

(a) Calculate $\bar{x}$ and s for the first two subgroups to verify the table entries.

(b) Make an s control chart using center lines and limits calculated from these past data. There are no points out of control.

(c) Because the s chart is in control, base the $\bar{x}$ chart on all 15 samples. Make this chart. Is it also in control?

CASE 15.1 **15.15 Lab testing.** Show the computations that confirm the limits of the $\bar{x}$ chart and s chart shown in Figure 15.10. LAB

Assumptions underlying subgroup charts

There are two fundamental assumptions underlying the development of subgroup charts and their associated control chart constants. First, it is assumed that the individual observations within the subgroups are independent. Second, it is assumed that the individual observations within the subgroups are Normally distributed.

Under these ideal assumptions, the subgroup means of an in-control process will behave independently and exactly follow the Normal model. The premise is that the central limit theorem mechanism will take hold to give approximate Normality to the subgroup means when the individual measurements are not Normally distributed. However, a separate question is, what is the impact of non-Normality on the applicability of the control chart constants? There have been extensive studies on the impact of non-Normality of individual measurements on the control chart constants used in the construction of subgroup charts. The general conclusion is that control chart constants for the construction of $\bar{x}$ and R charts are quite robust to moderate departures from Normality. Unless the departure from Normality is severe, the application of these standard control chart constants is generally satisfactory. *However, in contrast to the range method, even moderate departures from Normality can adversely affect the appropriateness of the s chart.*

The fact that the s chart is less robust to non-Normality than the R chart does not disqualify its consideration. Remember that the sample standard deviation statistic has the advantage of using all the data as opposed to the range statistic. To retain that advantage, one strategy is to transform (e.g., logarithm) the individual measurements so that approximate Normality is achieved. Thereafter, standard subgroup charts can be applied to the transformed individual observations. As a convenience, Minitab actually provides an option within its control chart platform of transforming the individual observations prior to the construction of the subgroup charts.

Let's consider now the other fundamental assumption underlying the development of subgroup charts: namely, it is assumed that the individual measurements within the subgroups are independent, that is, behave randomly.

EXAMPLE 15.5

ISH

$\bar{x}$ Limits Too Wide Consider a data set taken from a classic and widely used quality control manual, *Guide to Quality Control*, written by Kaoru Ishikawa. The data consist of five measurements of moisture content of a textile product taken at successive times within the day (6:00, 10:00, 14:00, 18:00, and 22:00) for 25 consecutive days (subgroups).

We will soon see that the systematic sampling within the days is critical information to better understand the process workings. However, we ignore this fact and follow Ishikawa's construction of an $\bar{x}$ and R chart for the 25 subgroups. Figure 15.11 shows Minitab-generated $\bar{x}$ and R charts. No points exceed either set of control limits. Indeed, Ishikawa provides no indications of any unusual process behavior and, implicitly, leaves his readers with the impression that the process is in control. However, upon close inspection, the $\bar{x}$ chart limits seem a bit too wide, with the subgroup means tightly dispersed around the center line. Remember that $\bar{x}$ chart limits are based only on the within-subgroup variation as estimated with the subgroup ranges. If the within-subgroup variation is substantially different from the between-subgroup variation, this is typically due to the presence of systematic nonrandom variability within the subgroups.

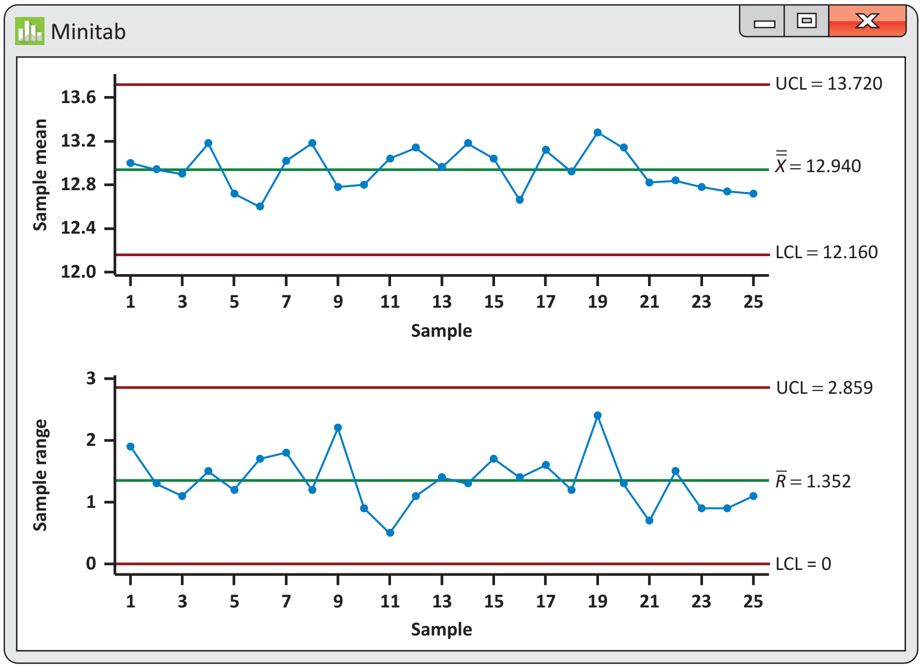
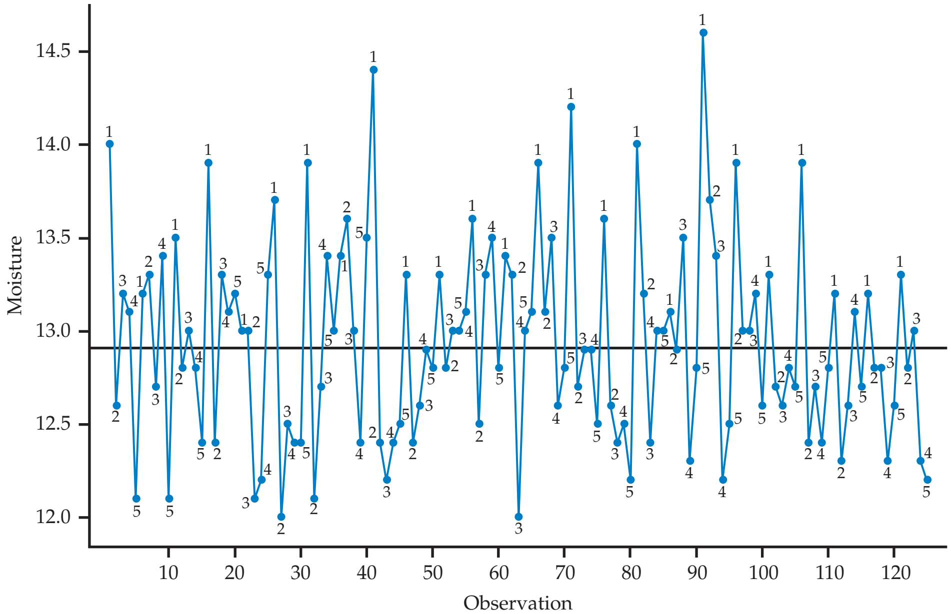


FIGURE 15.11 $\bar{x}$ and R charts for Ishikawa's moisture data, Example 15.5.

It would seem wise to look at the individual observations of the data set. Figure 15.12 shows a time plot of the 125 consecutive observations with labels corresponding to the five sampling periods each day. An immediate pattern emerges! Namely, the first sampling period of each day is consistently high

FIGURE 15.12 Time plot of the individual moisture measurements labeled by sampling period.



relative to other sampling periods. This process behavior *escaped* Ishikawa in his study of only the subgroup statistics. The systematic component represents a seasonality in the data that can be modeled using a seasonal regression model; refer to Section 14.4 (page 740) for examples of fitting regression models to seasonal data. A regression analysis would show that the systematic first-period effect accounts for about 42% of the variation in the data. Efforts should be concentrated on finding the causes for the systematic variation, perhaps due to some sort of "start-up" effect or higher ambient moisture in the morning. If the systematic variation is removed, then the process can be substantially improved in terms of variation reduction and, thus, a more consistent process. ■

The previous example illustrated that key information for process understanding and improvement can be lost by studying only subgroup statistics. The problem with the Ishikawa application of subgroup charts clearly stems from the design of the data collection scheme. In particular, it is clear that the principle of rational subgroups discussed earlier (page 15–10) was violated. If individual observations were taken closer together to form rational subgroups, we can expect that the seasonality buried within the subgroups will come to the surface and be reflected in the behavior of the subgroups themselves.

Finally, Example 15.5 illustrated how one type of systematic variation within the subgroups can result in the $\bar{x}$ chart limits being inappropriately too wide. As it turns out, other types of systematic behavior within the subgroups can have the opposite effect of having the $\bar{x}$ chart limits being too narrow.

EXAMPLE 15.6

SP500

$\bar{x}$ Limits Too Narrow To illustrate the phenomenon of too narrow limits, consider the daily closing prices of the Standard & Poor (S&P) 500 composite index from January 2, 2018 to December 28, 2018.[7] To bring in subgrouping, let us define a subgroup as a business week starting on Monday and ending on Friday. Figure 15.13 shows a JMP-produced $\bar{x}$ chart for the 52 formed S&P subgroups. There are a few places where the control limits vary slightly. This is due to the fact that some subgroups are based on four observations rather five observations because of holidays. There are a couple of facts that

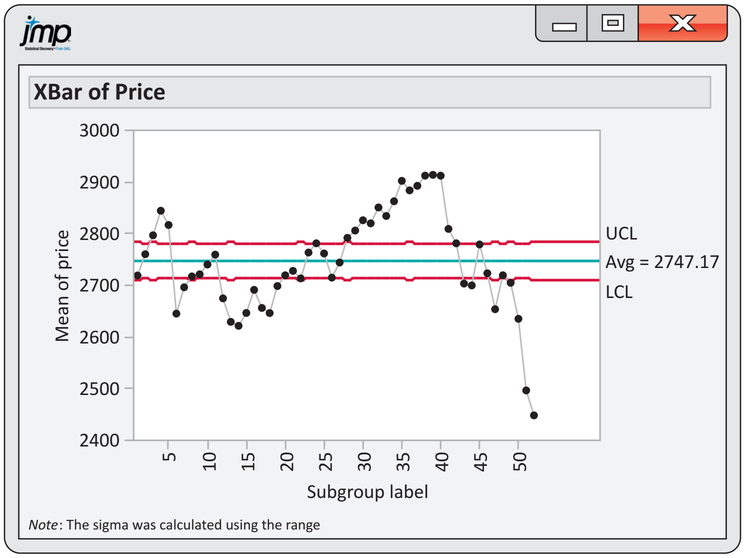


FIGURE 15.13 $\bar{x}$ chart for the S&P 500 subgroups (January 2018 to December 2018).

nonrandom processes, p. 679

are plainly seen from the control chart. First, the subgroup means show nonrandom behavior. Second, the limits are so narrow that more than half of the subgroups fall outside the limits!

positive autocorrelation, p. 748

The problem stems from the fact that consecutive individual observations within the subgroups are not randomly varying but rather are close together. Such a behavior is known as positive autocorrelation, as discussed in Chapter 14. The closeness of consecutive observations deflates the range estimate and, in turn, the width of the control limits. Forming subgroups based on bringing individual observations even closer together as called for by the principle of rational subgrouping cannot rescue this issue. ■

With Examples 15.5 and 15.6, we have explored the impact of systematic variation within the subgroups on the effectiveness of standard subgroup charts. In cases where the systematic variation is undesirable, the goal should be to remove the systematic variation from the process to bring the process in control in the sense of exhibiting only simple random variation. If done, the process is amenable to standard SPC methods for purposes of process monitoring. The Ishikawa example seems conducive to such a strategy. Presumably, a team of process improvement members can find and eliminate the root cause of the persisting first-period effect.

A more difficult scenario is the issue of positive autocorrelation illustrated in Example 15.6. Even though we showed a nontraditional application of the $\bar{x}$ chart with Example 15.6, positive autocorrelation is frequently encountered in traditional SPC settings (manufacturing and service). However, the sources of such behavior are difficult to identify and often impossible to remove. For such processes, standard SPC methods are not well suited for process monitoring. More generally than positive autocorrelation, not all systematic variation is undesirable. For example, we might wish to monitor a yield process that is systematically trending up. Our desire is not to eliminate this systematic variation. Instead, we wish to monitor the process for any unusual changes relative to the trending behavior. For situations where we cannot remove or do not want to remove systematic variation, a broader class of data analysis tools, such as the time series methods of Chapter 14, is required for effective monitoring.

Charts for individual observations

Up to this point we have concentrated on the application of control charts to statistics based on samples of two or more observations. We did, however, illustrate the importance of looking at the individual observations with Example 15.5 (page 15-25).

There are many applications where it is not practical to gather a sample of observations at a given time. For example, in low-volume manufacturing environments, the production rate is often slow, and therefore, the time between measurements is too long to allow rational subgroups to be formed. Some processes are just naturally viewed as a series of individual measurements. Data arising once a day, once a week, or once every two weeks do not lend themselves to being grouped into subgroups. Weekly sales or inventory levels are common company performance measurements monitored as individual observations.

A series of individual observations can be viewed as a special case of the $\bar{x}$ chart with $n = 1$. For an in-control process with mean μ and standard deviation σ, the sample mean control chart with known parameters and $n = 1$ is

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{1}} = \mu + 3\sigma$$

$$\text{CL} = \mu$$

$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{1}} = \mu - 3\sigma$$

If the goal is to maintain the process at a target level mean of μ, then μ is used to establish the control limits. In other cases, we need to estimate μ. For a set of k consecutive observations $x_1, x_2, \ldots, x_k$, the estimate of μ is simply given by the sample mean $\bar{x}$.

In the area of quality control, several estimators have been developed in the estimation of σ. Interestingly, the estimator used by many statistical software packages is not based on the sample standard deviation. Instead, a more common alternative estimate of process variability is based on the variability observed between successive observations. For a series of k observations, we define **moving ranges** as

moving ranges

$$MR_t = |x_t - x_{t-1}|$$

for $t = 2, 3, \ldots, k$. The moving range statistic MR is simply a special case of the general range statistic R, which is the largest observation minus the smallest observation in a given sample. In this case, successive observations are paired to form samples of size 2, allowing for the range to simply be computed as the absolute value of the difference between the two observations. For a series of k observations, there will be a series of $k - 1$ moving ranges. As a result, the mean moving range is given by:

$$\overline{MR} = \frac{\sum MR_t}{k - 1}$$

By using the mean moving range, it can be shown that an unbiased estimate for the process standard deviation is given by $\overline{MR}/d_2$, where the value of d_2 depends on the number of observations used in determining the individual ranges. From Table 15.1 (page 15-13), we find that $d_2 = 1.128$ when $n = 2$. The control limits based on $\overline{MR}$ are given by

$$\text{UCL} = \bar{x} + 3\left(\frac{\overline{MR}}{1.128}\right) = \bar{x} + 2.66\overline{MR}$$

$$\text{CL} = \bar{x}$$

$$\text{LCL} = \bar{x} - 3\left(\frac{\overline{MR}}{1.128}\right) = \bar{x} - 2.66\overline{MR}$$

individuals (*I*) chart

A control chart based on these limits is known as an **individuals (*I*) chart.**

In conjunction with the individuals chart, some practitioners will plot the moving ranges with limits for the detection of changes in process variability. The moving range limits are simply the earlier R chart limits with $\overline{MR}$ used in place of $\bar{R}$:

$$\text{UCL} = D_4\overline{MR}$$
$$\text{CL} = \overline{MR}$$
$$\text{LCL} = D_3\overline{MR}$$

Referring to Table 15.1 (page 15-13), we find that for ranges determined from two observations $D_4 = 3.267$ and $D_3 = 0$. The result limits are then

$$\text{UCL} = 3.267\overline{MR}$$
$$\text{CL} = \overline{MR}$$
$$\text{LCL} = 0$$

moving-range (*MR*) chart

A plot of the moving ranges with the preceding limits is known as a **moving-range (*MR*) chart.** Here is a summary of how to construct I and MR charts.

CONSTRUCTION OF *I* AND *MR* CHARTS

For a series of individual observations, the center line and control limits for an ***I* chart** are

$$\text{UCL} = \bar{x} + 2.66\overline{MR}$$
$$\text{CL} = \bar{x}$$
$$\text{LCL} = \bar{x} - 2.66\overline{MR}$$

where $\bar{x}$ is the average of individual observations and $\overline{MR}$ is the average of the moving ranges $|x_t - x_{t-1}|$. The center line and control limits for an ***MR* chart** are

$$\text{UCL} = 3.267\overline{MR}$$
$$\text{CL} = \overline{MR}$$
$$\text{LCL} = 0$$

EXAMPLE 15.7

LEBRON1

Is LeBron in Control? Sports enthusiasts use a variety of statistics to prognosticate team and player performance for general entertainment, betting purposes, and fantasy league play. Indeed, there are literally hundreds of thousands of websites providing sports statistics on amateur and professional athletes and teams. Let us consider the performance of the professional basketball superstar LeBron James during the 2017–2018 season, his last with the Cleveland Cavaliers before joining the LA Lakers. Basketball players can be tracked on a variety of offensive and defensive measures. The number of minutes played from game to game varies, so it makes sense to consider measures in terms of a rate, such as points per minute or rebounds per minute.

Table 15.5 provides the points per minute (ppm) that LeBron had for each of the 82 consecutive regular-season games he played in.[8] The sample mean for the 82 observations is

$$\bar{x} = \frac{0.703883 + 0.641425 + \cdots + 0.672994 + 0.947867}{82} = 0.745425$$

TABLE 15.5 Points per minute scored by LeBron James each game played during the 2017–2018 regular season (read left to right)

0.703883	0.641425	0.705128	0.912752	0.701048	0.585049	0.405405	0.880783
1.335416	0.640657	0.817439	0.824656	0.454727	0.634191	0.846224	0.845376
0.660147	0.987039	0.682393	0.966184	0.738570	0.638864	0.868455	0.671206
0.779537	0.735729	0.774527	0.713946	0.645717	0.788043	0.483676	1.139241
0.869565	0.499376	0.424403	0.774366	0.709709	0.581040	0.885114	0.376176
0.806618	0.667491	0.882759	0.459990	0.469157	0.737813	0.651357	0.661376
0.556291	0.626905	0.350691	0.755668	0.765781	0.540319	0.860215	0.931208
0.874317	0.490909	0.825688	0.799656	0.768574	0.589855	1.078886	0.996593
0.642949	0.763116	0.842950	0.851927	0.827413	0.999584	0.888325	0.922551
0.972405	0.473892	1.111613	0.646707	0.415225	0.724508	0.850515	1.094981
0.672994	0.947867						

The mean moving range is

$$\overline{MR} = \frac{|0.641425 - 0.703883| + \cdots + |0.947867 - 0.672994|}{81} = 0.216931$$

The center line and control limits for the I chart are

$$\begin{aligned} \text{UCL} &= \bar{x} + 2.66\overline{MR} = 0.745425 + 2.66(0.216931) = 1.322461 \\ \text{CL} &= \bar{x} = 0.745425 \\ \text{LCL} &= \bar{x} - 2.66\overline{MR} = 0.745425 - 2.66(0.216931) = 0.168389 \end{aligned}$$

The center line and control limits for the MR chart are

$$\begin{aligned} \text{UCL} &= 3.267\overline{MR} = 3.267(0.216931) = 0.708714 \\ \text{CL} &= \overline{MR} = 0.216931 \\ \text{LCL} &= 0 \end{aligned}$$

Figure 15.14 displays both the I and the MR chart produced by Minitab. For the most part, the I chart shows a process that is stable around the center line. There is, however, a three-sigma signal with observation 9, labeled by Minitab with the symbol "1" and a red dot. In that game on November 3, 2017, LeBron scored 1.335 points per minute, which is much greater than expected by the upper control limit. Investigation of observation 9 reveals that LeBron scored 57 points, the second-highest total of his career, and tied the Cleveland Cavaliers franchise scoring record for a single game. After the game, LeBron was quoted saying, "This is the best I've felt in my career right now."

Aside from LeBron's record performance, the control charts show that LeBron's offensive performance is an in-control process. This does not mean that we can predict his future individual game outcomes precisely, but rather it means that the level and average variability of his performance can be predicted with a high degree of confidence. ■

The individuals chart, like other control charts, is a three-sigma control chart with interpretation grounded in the Normal distribution. Unlike the mean chart based on sample sizes greater than one, the individuals chart does not have the advantage of the central limit theorem effect. Checking for Normality of the individual observations then becomes a particularly important step in the implementation of the individuals chart. Figure 15.15 shows the Normal quantile plot for the points per minute data. Aside from the one noted outlier, we see that the data are compatible with the Normal distribution.

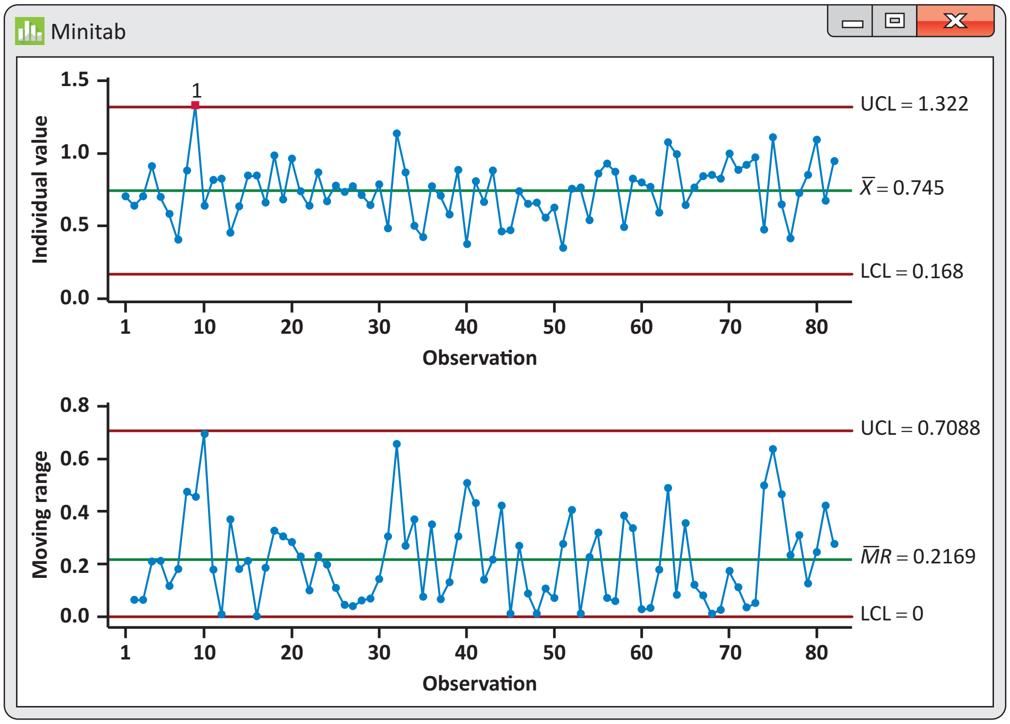


FIGURE 15.14 Minitab-generated individuals (*I*) chart and moving-range (*MR*) charts for LeBron James's points per minute (ppm). Observation 9 is highlighted as a three-sigma signal on the *I* chart.

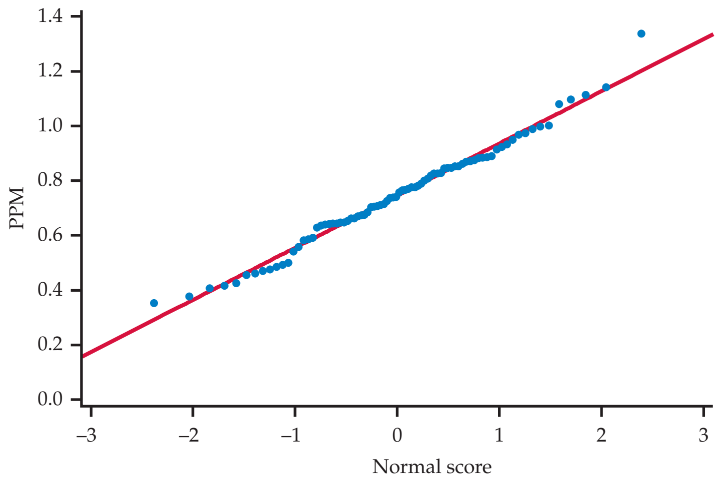


FIGURE 15.15 Normal quantile plot of LeBron James's points per minute data.

APPLY YOUR KNOWLEDGE

15.16 LeBron. In Example 15.7, we noted an unusual observation associated with a franchise record performance. Remove this point and reestimate the center lines and control limits for both the *I* chart and *MR* chart. Comment on the process relative to the revised limits. LEBRON1

15.17 Personal processes. From your personal life, provide two examples of processes for which you would collect data in the form of individual measurements that ultimately might be monitored by an *I* chart.

15.18 LeBron in the playoffs. The control charts of Example 15.7 and Exercise 15.16 are based on regular-season performance. After the conclusion of the regular season, LeBron played in 22 playoff games. Here are his points per minute in the playoffs (read left to right): LEBRON2

0.546282	1.153364	0.671731	0.693391	1.052212	0.705505	1.036468	0.553977
1.061728	0.920840	0.755208	0.415321	1.083405	0.720000	1.050537	0.662139
0.997831	0.729167	1.072931	0.661345	0.704125	0.566968		

Using the control limits from Exercise 15.16, plot LeBron's playoff numbers. What do you conclude about LeBron's playoff performance?

EXAMPLE 15.8

DELAY

Monitoring a Non-Normal Process In the management of supply chains, many organizations face significant challenges with their suppliers' delivery performance. Consider a manufacturer that records the delay (hrs) for shipments from a specific supplier. Figure 15.16(a) shows an I chart for the supplier's shipping process. Based on the control chart, there is an initial temptation to seek explanations for observations 32 and 47. However, a close examination of the observations show them to be a bit "bunched" below and around the center line, with some observations floating upward. This suggests that the data are right-skewed. The Normal quantile plot of Figure 15.16(b) confirms this suspicion.

In previous chapters, we have utilized the logarithmic transformation to achieve approximate Normality. The logarithmic transformation does a fairly reasonable job for these data. However, we will pursue an alternative transformation that will do a relatively better job for these data. In particular, a very common approach to Normalize data is to raise each observation to some power λ, that is, x^{λ}. Such a transformation is known as a power transformation. Common power transformations include the square root ($x^{1/2}$), cube root ($x^{1/3}$), and inverse (x^{-1}). When $\lambda = 0$, the transformation is not x^0 but turns

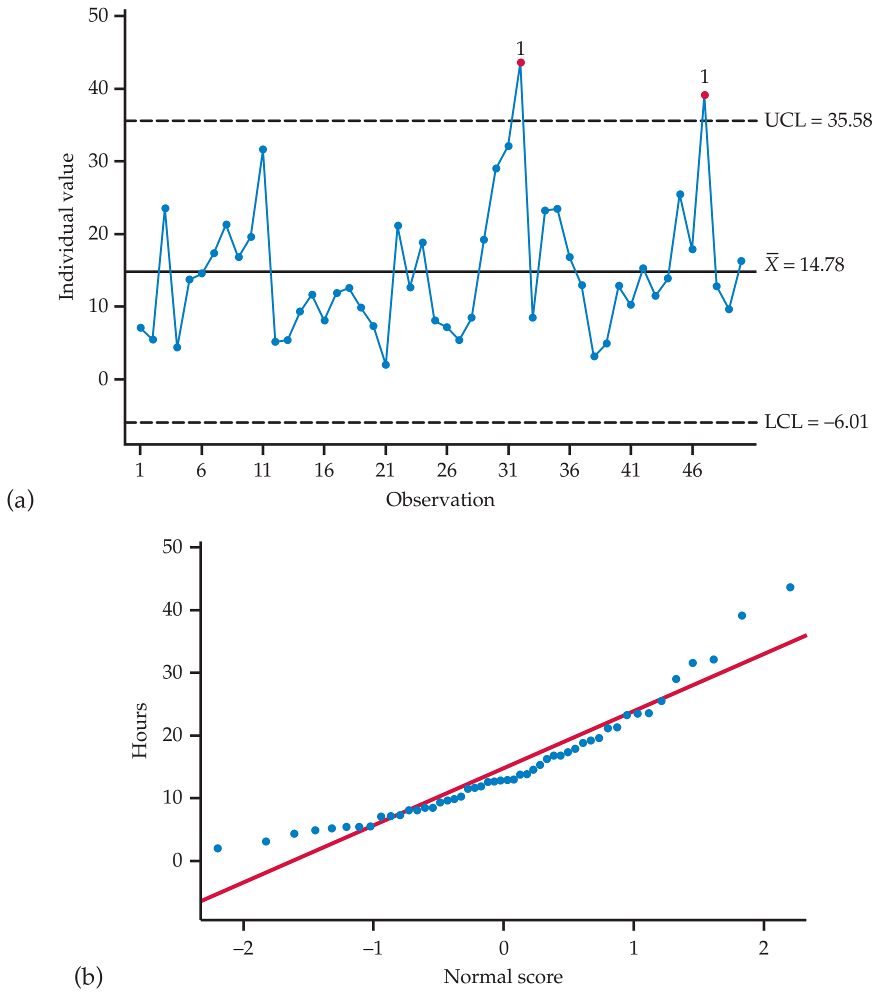


FIGURE 15.16 (a) Individuals (I) chart for shipping delay data. (b) Normal quantile plot of shipping delay data.

out to correspond to the logarithmic transformation! To find a suitable choice of λ, many software packages provide a procedure known as the Box-Cox method. In Minitab, the Box-Cox procedure is actually found within the "Control Chart" menu. In JMP (not JMP Student Edition), the Box-Cox procedure is found within the "Fit Model" platform.

Figure 15.17 shows both Minitab's and JMP's Box-Cox output for the delay data. Minitab estimates λ to be 0.26, while JMP estimates it to be 0.242. These differences are due to slightly different estimation procedures. Since these values are close to 0.25, we will take the fourth root of the data. Figure 15.18 shows the Normal quantile plot for the delay data raised to the power of 0.25. The transformed data are clearly compatible with the Normal distribution. We now construct an *I* chart for the transformed data as displayed in Figure 15.19. Unlike the *I* chart on the original skewed data, the *I* for the transformed data show no unusual observations warranting special explanation. ■

Given the two "seemingly" unusual observations on the original scale, there might be a feeling that the Normalizing transformation camouflaged these values. This is not a correct perspective. Normalizing transformations reexpress the data to a scale that allows the Normal-based control limits to serve as appropriate benchmarks. Think about it this way: given that the two transformed values are not unusual relative to Normal distribution, they are not unusual in original scale relative to the underlying skewed distribution.

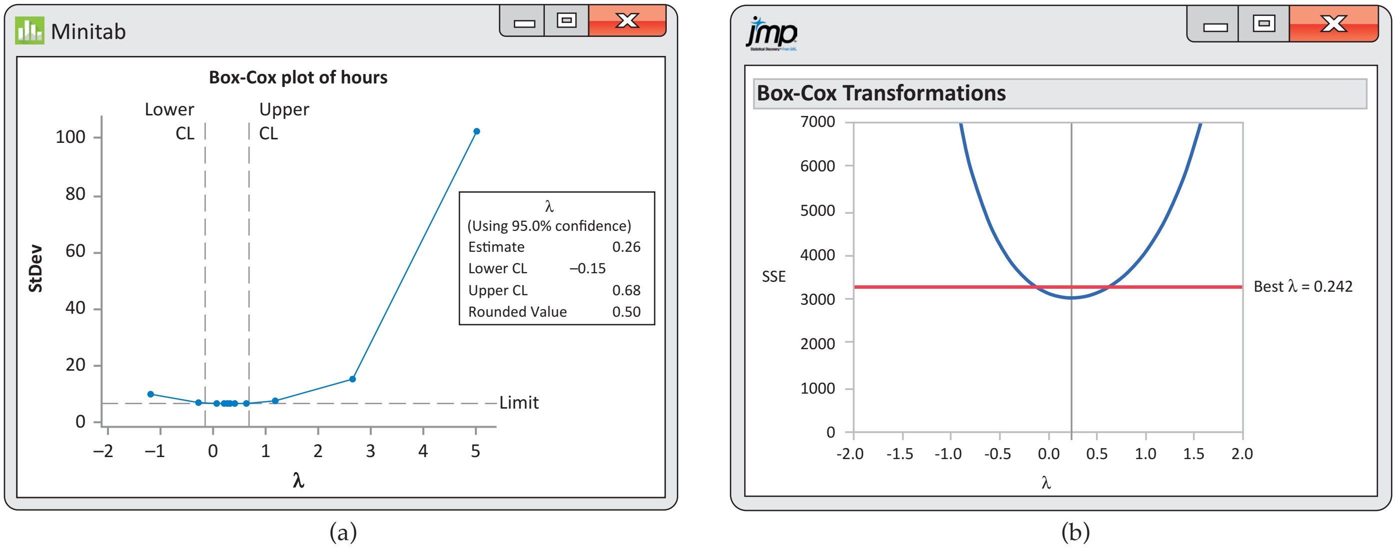


FIGURE 15.17 Box-Cox output: (a) Minitab reported optimal λ. (b) JMP reported optimal λ.

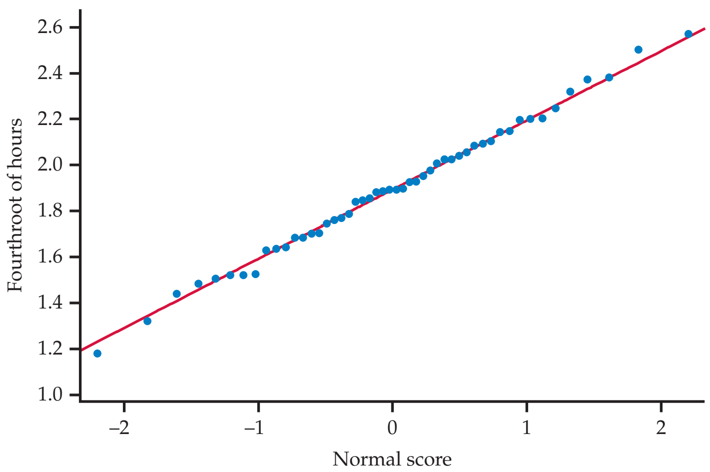


FIGURE 15.18 Normal quantile plot of the shipping delay data transformed by raising observation values to the 0.25 power.

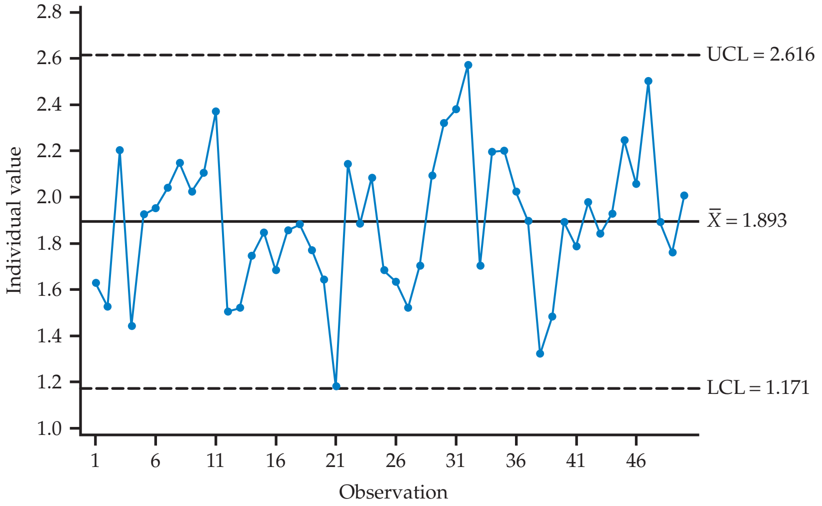


FIGURE 15.19 Individuals (*I*) chart applied to shipping delay data transformed by raising observation values to the 0.25 power.

The conclusion that the supplier shipment process is not out of control does not imply that the manufacturer customer should be satisfied with the performance of its supplier. A process is in control with an average delay of about 15 hours. From the manufacturer's perspective, this average delay may be considered unacceptable. As will be discussed in Section 15.3, there is a fundamental distinction between statistical control and and a process meeting the manufacturer's requirements.

SECTION 15.2 SUMMARY

- Standard **three-sigma control charts** plot the values of some statistic for regular samples from the process against the time order of the samples. The **center line** is set at the mean of the plotted statistic. The **control limits** lie at three standard deviations of the plotted statistic above and below the center line. A point outside the control limits is an **out-of-control signal.**

- When we measure some quantitative characteristic of the process and gather samples of two or more observations, we use **$\bar{x}$ and *R* charts** for process control. The *R* chart monitors variation within individual samples. If the *R* chart is in control, the $\bar{x}$ chart monitors variation from sample to sample. To interpret the charts, always look first at the *R* chart. For larger subgroups, the *R* chart can be replaced by an ***s* chart.**

- Control chart constants for the construction of $\bar{x}$ and *R* charts are quite robust to moderate departures from Normality. However, moderate departures from Normality can adversely affect the appropriateness of the *s* chart. In cases of severe non-Normality for $\bar{x}/R$ chart implementation or moderate non-Normality for $\bar{x}/s$ chart implementation, it is recommended that the individual measurements be transformed prior to the implementation of the subgroup charts.

- Individual measurements within subgroups that violate the independence assumption can render the $\bar{x}$ either too wide or too narrow.

- The ***I* chart** and ***MR* chart** are used for monitoring a process of individual observations. The *I* chart does not benefit from the central limit theorem effect. As a result, it is important to check if the individual observations follow the Normal distribution before constructing the *I* chart. Transformations of the observations should be considered if non-Normality is evident.

SECTION 15.2 EXERCISES

For Exercises 15.11 to 15.13, see page 15-22; for 15.14 and 15.15, see pages 15-23–15-24; and for 15.16 to 15.18, see pages 15-31 and 15-32.

15.19 Changing sample size. For a given value of $\bar{R}$ or $\bar{s}$, what is the effect of the subgroup size on the location of R and s chart limits?

15.20 Dyeing yarn. The unique colors of the cashmere sweaters your firm makes result from heating undyed yarn in a kettle with a dye liquor. The pH (acidity) of the liquor is critical for regulating dye uptake and hence the final color. There are five kettles, all of which receive dye liquor from a common source. Twice each day, the pH of the liquor in each kettle is measured, giving a sample of size 5. The process has been operating in control with $\mu = 4.22$ and $\sigma = 0.127$. Give the center line and control limits for the $\bar{x}$ chart.

15.21 Probability out? An $\bar{x}$ chart plots the means of samples of size 4 against center line CL = 700 and control limits LCL = 687 and UCL = 713. The process has been in control. Now the process is disrupted in a way that changes the mean to $\mu = 693$ and the standard deviation to $\sigma = 12$. What is the probability that the first sample after the disruption gives a point beyond the control limits of the $\bar{x}$ chart?

15.22 Alternative control limits. American and Japanese practice uses three-sigma control charts. That is, the control limits are three standard deviations on either side of the mean. When the statistic being plotted has a Normal distribution, the probability of a point outside the limits is 0.0027 (or 0.00135 in each direction). In Europe, it is conventional to place control limits so that the probability of a point out is 0.001 in each direction. For a Normally distributed statistic, how many standard deviations on either side of the mean do these alternative control limits lie?

15.23 Monitoring packaged products. To control the fill amount of its cereal products, a cereal manufacturer monitors the net weight of the product with $\bar{x}$ and R charts using a subgroup size of $n = 5$. One of its brands, Organic Bran Squares, has a target of 10.6 ounces. Suppose that 20 preliminary subgroups were gathered, and the following summary statistics were found for the 20 subgroups:

$$\sum \bar{x}_i = 211.624 \qquad \sum R_i = 7.44$$

Assume that the process is stable in both variation and level. Compute the control limits for the $\bar{x}$ and R charts.

15.24 Measuring bone density. Loss of bone density is a serious health problem for many people, especially older women. Conventional X-rays often fail to detect loss of bone density until the loss reaches 25% or more. New equipment such as the Lunar bone densitometer is much more sensitive. A health clinic installs one of these machines. The manufacturer supplies a "phantom," an aluminum piece of known density that can be used to keep the machine calibrated. Each morning, the clinic makes two measurements on the phantom before measuring the first patient. Control charts based on these measurements alert the operators if the machine has lost calibration. Table 15.6 contains data for the first 30 days of operation.[9] The units are grams per square centimeter (for technical reasons, area rather than volume is measured). BONE

TABLE 15.6 Daily calibration subgroups for a Lunar bone densitometer

Subgroup	Measurements		Sample mean	Range
1	1.261	1.260	1.2605	0.001
2	1.261	1.268	1.2645	0.007
3	1.258	1.261	1.2595	0.003
4	1.261	1.262	1.2615	0.001
5	1.259	1.262	1.2605	0.003
6	1.269	1.260	1.2645	0.009
7	1.262	1.263	1.2625	0.001
8	1.264	1.268	1.2660	0.004
9	1.258	1.260	1.2590	0.002
10	1.264	1.265	1.2645	0.001
11	1.264	1.259	1.2615	0.005
12	1.260	1.266	1.2630	0.006
13	1.267	1.266	1.2665	0.001
14	1.264	1.260	1.2620	0.004
15	1.266	1.259	1.2625	0.007
16	1.257	1.266	1.2615	0.009
17	1.257	1.266	1.2615	0.009
18	1.260	1.265	1.2625	0.005
19	1.262	1.266	1.2640	0.004
20	1.265	1.266	1.2655	0.001
21	1.264	1.257	1.2605	0.007
22	1.260	1.257	1.2585	0.003
23	1.255	1.260	1.2575	0.005
24	1.257	1.259	1.2580	0.002
25	1.265	1.260	1.2625	0.005
26	1.261	1.264	1.2625	0.003
27	1.261	1.264	1.2625	0.003
28	1.260	1.262	1.2610	0.002
29	1.260	1.256	1.2580	0.004
30	1.260	1.262	1.2610	0.002

(a) Calculate $\bar{x}$ and R for the first two days to verify the table entries.

(b) Make an R chart and comment on control. If any points are out of control, remove them and recompute the chart limits until all remaining points are in control. (That is, assume that special causes are found and removed.)

(c) Make an $\bar{x}$ chart using the samples that remain after your work in part (b). What kind of variation will be visible on this chart? Comment on the stability of the machine over these 30 days based on both charts.

15.25 Additional out-of-control signals. A single extreme point outside of three-sigma limits represents one possible statistical signal of unusual process behavior. As we saw with Figure 15.5(a) (page 15-18), process change can also give rise to unusual variation *within* control limits. As discussed (page 15-17), a variety of statistical rules, known as runs rules, have been developed to supplement the three-sigma rule in an effort to more quickly detect special cause variation. A commonly used runs rule for the detection of smaller shifts of gradual process drifts is to signal if nine consecutive points all fall on one side of the center line. We have learned that for an in-control process and the assumption of Normality, the false alarm rate for the three-sigma rule is about three in 1000. Assuming Normality of the control chart statistics, what is the false alarm rate for the nine-in-a-row rule if the process is in control?

15.26 Alloy composition—retrospective control. Die casts are used to make molds for molten metal to produce a wide variety of products ranging from kitchen and bathroom fittings to toys, doorknobs, and a variety of auto and electronic components. Die casts themselves are made out of an alloy of metals including zinc, copper, and aluminum. For one particular die cast, the manufacturer must maintain the percent of aluminum between 3.8% and 4.2%. To monitor the percent of aluminum in the casts, three casts are periodically sampled, and their aluminum content is measured. The first 20 rows of Table 15.7 give the data for 20 preliminary subgroups. ALLOY

(a) Make an R chart and comment on control of the process variation.

(b) Using the range estimate, make an $\bar{x}$ chart and comment on the control of the process level.

15.27 Alloy composition—prospective control. Project the $\bar{x}$ and R chart limits found in the previous exercise for prospective control of aluminum content. The last 15 rows of Table 15.7 give data on the next 15 future subgroups. Refer to Exercise 15.25 and apply the nine-in-a-row rule along with the standard three-sigma rule to the new subgroups. Is the process maintaining control? If not, describe the nature of the process change and indicate the subgroups affected. ALLOY

15.28 Deming speaks. The quality guru W. Edwards Deming (1900–1993) taught (among much else) that

(a) "People work in the system. Management creates the system."

(b) "Putting out fires is not improvement. Finding a point out of control, finding the special cause and removing it, is only putting the process back to where it was in the first place. It is not improvement of the process."

TABLE 15.7 Aluminum percent measurements

Subgroup	Measurements		
1	3.99	3.90	3.98
2	4.02	3.95	3.95
3	3.99	3.90	3.90
4	3.99	3.94	3.88
5	3.99	3.93	3.91
6	3.94	3.97	3.83
7	3.89	3.95	3.99
8	3.86	3.97	4.02
9	3.98	3.98	3.95
10	3.93	3.88	4.06
11	3.97	3.91	3.92
12	3.86	3.95	3.88
13	3.92	3.97	3.95
14	4.01	3.91	3.91
15	4.00	4.02	3.93
16	4.01	3.97	3.98
17	3.92	3.92	3.95
18	3.96	3.96	3.90
19	4.04	3.93	3.95
20	3.96	3.85	4.03
21	4.06	4.04	3.93
22	3.94	4.02	3.98
23	3.95	4.07	3.99
24	3.90	3.92	3.97
25	3.97	3.96	3.94
26	3.96	4.00	3.91
27	3.90	3.85	3.91
28	3.97	3.87	3.94
29	3.97	3.88	3.83
30	3.82	3.99	3.84
31	3.95	3.87	3.94
32	3.86	3.91	3.98
33	3.94	3.93	3.90
34	3.89	3.90	3.81
35	3.91	3.99	3.83

(c) "Eliminate slogans, exhortations and targets for the workforce asking for zero defects and new levels of productivity."

Choose one of Deming's sayings. Explain carefully what facts about improving quality the saying attempts to summarize.

15.29 Accounts receivable. In an attempt to understand the bill-paying behavior of its distributors, a manufacturer samples bills and records the number of days between the issuing of the bill and the receipt of payment. The manufacturer formed subgroups of 10 randomly chosen bills per week over the course of 30 weeks. It found an overall mean $\bar{\bar{x}}$ of 30.6833 days and an average standard deviation $\bar{s}$ of 7.50638 days.

(a) Assume that the process is stable in both variation and level. Compute the control limits for the $\bar{x}$ and s charts.

(b) Here are the means and standard deviations of future subgroups:

Week	31	32	33	34	35
$\bar{x}$	31.1	29.5	33.0	33.4	33.2
s	6.1001	10.5013	8.5114	7.5011	3.7059

Week	36	37	38	39	40
$\bar{x}$	35.8	37.3	41.5	35.9	36.7
s	4.1846	6.5328	8.1548	5.8585	6.7338

Is the accounts receivable process still in control? If not, specify the nature of the process departure.

15.30 Patient monitoring. There is increasing interest in the use of control charts in health care. Many physicians are directly involving patients in proactive monitoring of health measurements such as blood pressure, glucose, and expiratory flow rate. Patients are asked to record measurement for a certain number of days. The patient then brings the measurements to the physician who, in turn, will use software to generate control limits. The patient is then asked to plot future measurements on a chart with the limits. Consider data on a patient with hypertension. The data are 30 consecutive self-recorded home systolic measurements. BP

(a) Construct a histogram of the systolic readings. How compatible is the histogram with the Normal distribution?

(b) Determine the mean and standard deviation estimates ($\hat{\mu}$ and $\hat{\sigma}$) that will be used in the construction of an I chart.

(c) Compute the UCL and LCL of the I chart.

(d) Construct the I chart for the systolic series. Discuss the stability of the process.

(e) Moving forward, based on the plotted measurements, when would you suggest the patient call in to the physician's office? In general, list some benefits from patient-based control chart monitoring from both the patient's and physician's perspective.

(f) Why do you think physicians generally recommend only the use of the I chart for their patients and not the MR chart?

15.31 Control charting your reaction times. Consider the following personal data-generating experiment. Obtain a stopwatch, a capability that many electronic watches and smartphones offer. Alternatively, you can use one of many web-based stopwatches easily found with a Google search (make sure to use a site that reports to at least 0.01 second). Attempt to start and stop your stopwatch as close as possible to 5 seconds. Record the result to as many decimal places as your stopwatch shows. Repeat the experiment 50 times. Input your results into a statistical software package.

(a) Construct a histogram of your measurements. How compatible is the histogram with the Normal distribution?

(b) Determine the mean and standard deviation estimates ($\hat{\mu}$ and $\hat{\sigma}$) that will be used in the construction of an I chart.

(c) Compute the UCL and LCL of the I chart.

(d) Construct the I chart for your data series. Discuss the stability of your process. Are you in control? Were there any out-of-control signals? If so, provide an explanation for the unusual observation(s).

15.32 Control charts for lifetime testing data. Based on an accelerated lifetime testing process, consider subgroup data on the lifetime (hrs) of an electronic component. The data are shown in Table 15.8. LIFET

TABLE 15.8 Accelerated lifetime measurements (hrs) for an electronic component

Subgroup	x1	x2	x3	x4
1	0.326	0.449	1.330	0.488
2	0.255	5.940	3.663	0.432
3	0.181	0.563	4.118	1.009
4	0.749	0.666	2.658	0.432
5	1.901	0.644	0.794	0.359
6	1.379	0.377	0.492	0.161
7	14.065	0.656	1.676	2.029
8	2.025	1.940	0.862	3.972
9	0.650	0.458	0.846	0.895
10	1.021	0.209	0.215	0.393
11	0.658	1.626	0.352	3.631
12	1.847	1.214	2.448	0.855
13	17.495	1.717	1.221	1.989
14	0.414	0.780	0.329	0.963
15	1.620	2.016	0.718	1.706
16	12.500	0.598	0.198	0.609
17	1.251	1.234	0.426	2.619
18	1.775	0.075	0.182	0.722
19	2.191	2.197	0.791	2.340
20	0.538	2.177	0.670	0.678
21	0.215	1.110	2.040	0.894
22	2.743	1.175	0.499	0.669
23	0.898	1.249	2.246	1.512
24	2.597	4.680	4.340	0.188
25	2.934	4.026	1.708	0.088

(a) Without removing any subgroups, construct $\bar{x}$ and R control charts for these data. What do the control charts suggest about the process?

(b) Stack all the individual measurements into a single column of your software. Obtain a histogram and Normal quantile plot for the individual measurements. What do you conclude?

(c) Transform the individual measurements using the logarithmic transformation and then construct $\bar{x}$ and R control charts for the transformed data. What do you conclude now?

(d) If your software has the Box-Cox procedure, find the optimal value for λ. Does the optimal value basically suggest a logarithmic transformation? Explain.

15.3 Process Capability Indices

When you complete this section, you will be able to:

- Estimate the percent of product that meets specifications using the Normal distribution.
- Explain why the percent of product meeting specifications is not a good measure of capability.
- Compute and interpret the C_p and C_{pk} capability indices.
- Identify issues that affect the interpretation of capability indices.

A process in control is stable over time. With an in-control process, we can predict the mean and the amount of variation the process will show. Control charts are, so to speak, the voice of the process telling us what state it is in. *There is no guarantee that a process in control produces products of satisfactory quality.* "Satisfactory quality" is measured by comparing the product or service to some standard outside the process, set by technical specifications, customer expectations, or the goals of the organization. These external standards are unrelated to the internal state of the process, which is all that statistical control pays attention to.

CAPABILITY

Capability refers to the ability of a process to meet or exceed the requirements placed on it.

Capability has nothing to do with control—except for the very important point that, if a process is not in control, it is hard to tell if it is capable.

EXAMPLE 15.9

CASE 15.1 **Capability** Both the $\bar{x}$ and R charts and the $\bar{x}$ and s charts showed that the lab-testing process is stable and in control. Suppose that the ER stipulates that getting lab results must take no longer than 50 minutes. Figure 15.20 compares the distribution of individual lab test times with an *upper specification limit* (USL) of 50 minutes. We can clearly see that the process is not capable of meeting the specification. ■

LAB

Managers must understand that *if a process that is in control does not have adequate capability, fundamental changes in the process are needed.* The process is doing as well as it can and displays only the chance variation that is natural to its present state. *Slogans to encourage workers or disciplining workers for poor performance will not change the state of the process.* Better training for workers is a change in the process that may improve capability. New equipment or more uniform material may also help, depending on the findings of a careful investigation.

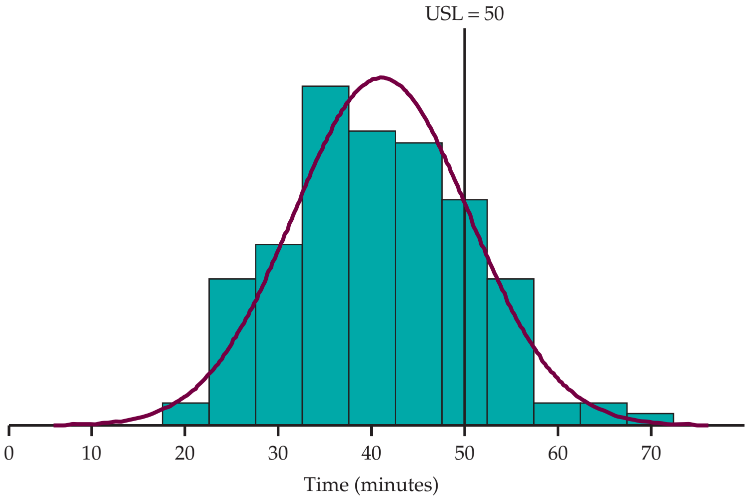


FIGURE 15.20 Comparing the distribution of individual lab test turnaround times with an upper specification limit, for Example 15.9.

Figure 15.20 gives us a visual summary of the capability of the process, but managers often like a numerical summary of capability. One measure of capability is simply the *percent of process outcomes that meet the specifications*. When the variable we measure has a Normal distribution, we can use the estimated mean and standard deviation along with the Normal distribution to estimate this percent. When the variable is not Normal, we can use the actual percents of the measurements in the samples that meet the specifications.

Normal distribution calculations, p. 48

There is a subtle point when it comes to estimating the standard deviation of the process. When performing process capability analysis in conjunction with subgroup charts, we could derive an estimate for σ based on the subgroup variability estimates $\bar{R}$ or $\bar{s}$. However, these estimates are based solely on the within-subgroup variation. An alternative method of estimating the process standard deviation is to ignore the subgrouping of the data and to simply estimate the standard deviation from the single set of all individual observations. If the process is out of control, then the single-set estimate will be larger than the subgroup-based estimates because the single-set estimate is capturing the extra variation associated with the mean shifts occurring *between* the subgroups. It makes little sense to summarize process capability on an out-of-control process. *The summarization of process capability should occur only after the process is brought to a stable state.* When the process is in control, the issue of whether to base capability analysis on a within-subgroup estimate or a single-set estimate becomes a moot point since either type will produce nearly identical results.

When performing capability analysis on a series of individual measurements in conjunction with an I chart, software offers many options for estimating σ. One option is to simply use the sample standard deviation s. It can be shown, however, that the sample standard deviation is a biased estimate of σ. The corrected estimate is

$$\hat{\sigma} = \frac{s}{c_4}$$

For series of more than 25 observations, c_4 is well approximated[10] by

$$c_4 \approx \frac{4k - 4}{4k - 3}$$

where k is the number of individual observations. In the end, for most reasonable lengthed series, the differences between s and s/c_4 are fairly inconsequential.

EXAMPLE 15.10

CASE 15.1

Percent Meeting Specifications Figure 15.20 shows that the distribution of individual turnaround times is approximately Normal. We found in Example 15.2 (page 15-15) that $\bar{\bar{x}} = 40.62$. We now need an estimate of the standard deviation σ for the process producing the individual measurements.

If we use $\bar{R}$ as the basis for the standard deviation estimate, then the estimate is given by

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

Table 15.1 (page 15-13) gives values for d_2 for sample sizes ranging from 2 to 25. From Example 15.2, we find $\bar{R} = 21.967$. From Table 15.1, we find $d_2 = 2.326$ for $n = 5$, which gives the estimate for σ:

$$\hat{\sigma} = \frac{21.967}{2.326} = 9.444$$

We can now calculate the percent of lab tests that meet the upper specification:

$$P(\text{lab times} \leq 50) = P\left(Z \leq \frac{50 - 40.62}{9.444}\right)$$
$$= P(Z \leq 0.993) \approx 0.8389$$

It is estimated that about 84% of lab tests meet the ER specification of 50 minutes or less turnaround time. While a relatively high percent, this does not reach the ER's stipulation of acceptable process capability. ■

Even though *percent meeting specifications* seems to be a reasonable measure of process capability, there are some situations that can call into question its appropriateness. Figure 15.21 shows why. This figure compares the distributions of the diameter of the same part manufactured by two processes. The target diameter and the specification limits are marked. All the parts produced by Process A meet the specifications, but about 1.5% of those from Process B fail to do so. Nonetheless, Process B is superior to Process A because it is less variable: much more of Process B's output is close to the target. Process A produces many parts close to the lower specification limit (LSL) and the upper specification limit (USL). These parts meet the specifications, but they will fit and perform more poorly than parts with diameters close to the center of the specifications. A distribution like that for Process A might result from inspecting all the parts and discarding those whose diameters fall outside the specifications. That's not an efficient way to achieve quality.

We need a way to measure process capability that pays attention to the variability of the process (smaller is better). The standard deviation does that, but it doesn't measure capability because it does not account for the

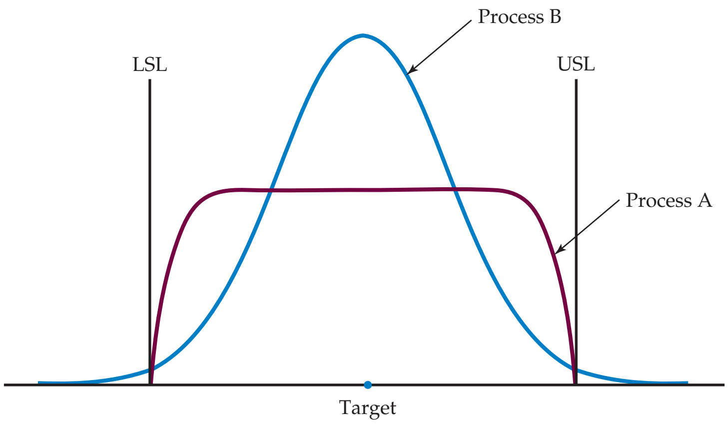


FIGURE 15.21 Two distributions for part diameters. All the parts from Process A meet the specifications, but a higher proportion of parts from Process B have diameters close to target.

capability indices

specifications that the output must meet. **Capability indices** start with the idea of comparing process variation with the specifications. Process B will beat Process A by such a measure. Capability indices also allow us to measure process improvement—we can continue to drive down variation, and so improve the process, long after 100% of the output meets specifications. *Continual improvement of processes is our goal, not just reaching "satisfactory" performance.* The real importance of capability indices is that they give us numerical measures to describe ever-better process quality. Statistical software offers many capability indices, but we consider only the most basic ones.

CAPABILITY INDICES FOR TWO-SIDED SPECIFICATIONS

Consider a process with lower and upper specification limits (LSL and USL) for some measured characteristic of its output. The process mean for this characteristic is μ and the standard deviation is σ. The **potential capability index C_p** is

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

The **performance capability index C_{pk}** is

$$C_{pk} = \min\left(\frac{\mu - \text{LSL}}{3\sigma}, \frac{\text{USL} - \mu}{3\sigma}\right)$$

Large values of C_p or C_{pk} indicate more capable processes.

68–95–99.7 rule, p. 45

Capability indices start from the fact that *Normal distributions are in practice about 6 standard deviations wide.* That's the 99.7 part of the 68–95–99.7 rule. Conceptually, C_p is the specification width as a multiple of the process width 6σ. When $C_p = 1$, the process output will just fit within the specifications if the center is midway between LSL and USL. Larger values of C_p are better—the process output can fit within the specs with room to spare. But a process with high C_p can produce poor-quality product if it is not correctly centered. As we see with the next example, C_{pk} remedies this deficiency by considering both the center μ and the variability σ of the measurements.

EXAMPLE 15.11

Interpreting Capability Indices Consider the series of pictures in Figure 15.22. We might think of a process that machines a metal part and we measure a dimension of the part that has LSL and USL as its specification limits. There is, of course, variation from part to part. The dimensions vary Normally with mean μ and standard deviation σ.

Figure 15.22(a) shows process width equal to the specification width. That is, $C_p = 1$. Almost all the parts will meet specifications *if*, as in this figure, the process mean μ is at the center of the specs. Because the mean is centered, it is 3σ from both LSL and USL, so $C_{pk} = 1$ also. In Figure 15.22(b), the mean has moved down to LSL. Only half the parts will meet the specifications. C_p is unchanged because the process width has not changed. But C_{pk} sees that the center μ is right on the edge of the specifications, $C_{pk} = 0$. The value becomes negative if μ is outside the specifications.

In Figures 15.22(c) and (d), the process σ has been reduced to half the value it had in Figures 15.22(a) and (b). The process width 6σ is now half the specification width, so $C_p = 2$. In Figure 15.22(c), the center is just 3 of the new σ's above LSL, so that $C_{pk} = 1$. Figure 15.22(d) shows the same smaller

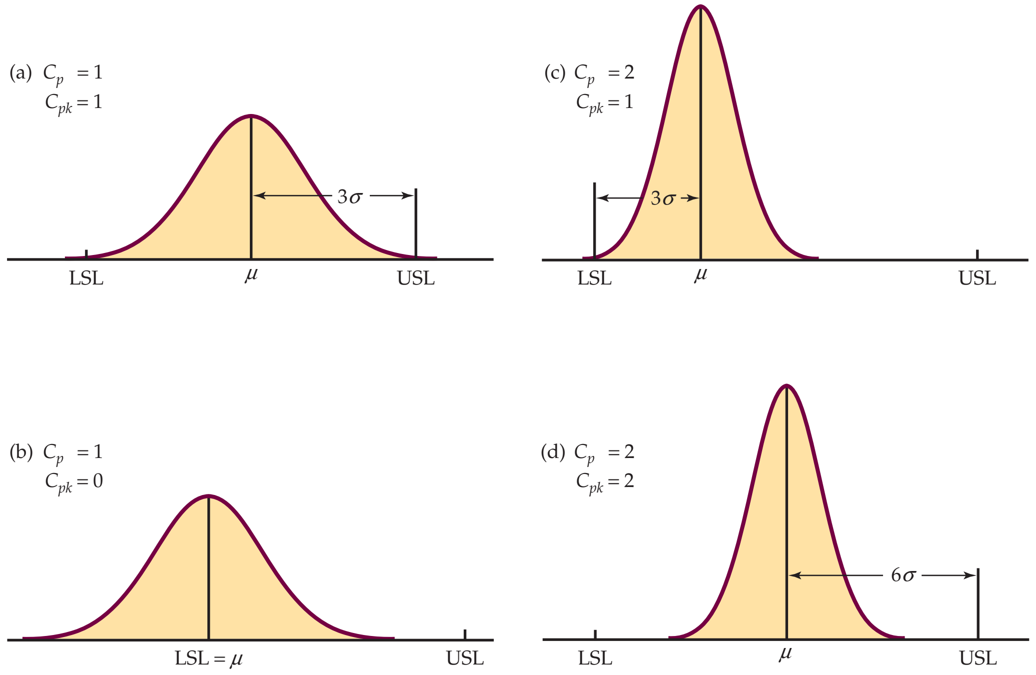


FIGURE 15.22 How capability indices work. (a) Process centered, process width equal to specification width. (b) Process off-center, process width equal to specification width. (c) Process off-center, process width equal to half the specification width. (d) Process centered, process width equal to half the specification width.

σ accompanied by mean μ correctly centered between LSL and USL. C_{pk} rewards the process for moving the center from 3σ to 6σ away from the nearer limit by increasing from 1 to 2. You see that C_p and C_{pk} are equal if the process is properly centered. If not, C_{pk} is smaller than C_p. ■

Example 15.11 shows that as a process moves off target, C_p remains constant while C_{pk} decreases in value. Off-target processes have the *potential* of having higher capability if the mean is adjusted to the center of the specs. For these reasons, C_p can be viewed as a measure of process potential while C_{pk} attempts to measure *actual* process performance.

EXAMPLE 15.12

CASE 15.2

ORING

O-Ring Process Capability At the conclusion of the process study in Example 15.3 (page 15-19), we found two special causes and eliminated from our data the subgroups on which those causes operated. From Figure 15.9 (page 15-21), after removal of the two subgroups, we find $\bar{\bar{x}} = 2.60755$ and $\bar{R} = 0.00899$. As noted in Case 15.2 (page 15-18), specification limits for the inside diameter are set at 2.612 ± 0.02 inches, which implies LSL $= 2.592$ and USL $= 2.632$. Figure 15.23 shows that the individual measurements are compatible with the Normal distribution.

From Table 15.1 (page 15-13), we find $d_2 = 2.059$ for $n = 4$, which gives the estimate for σ:

$$\hat{\sigma} = \frac{0.00899}{2.059} = 0.004366$$

In addition, the mean estimate $\hat{\mu}$ is simply the grand mean $\bar{\bar{x}}$. These estimates may be quite accurate if we have data on many past samples.

Estimates based on only a few observations may, however, be inaccurate because statistics from small samples can have large sampling variability. This

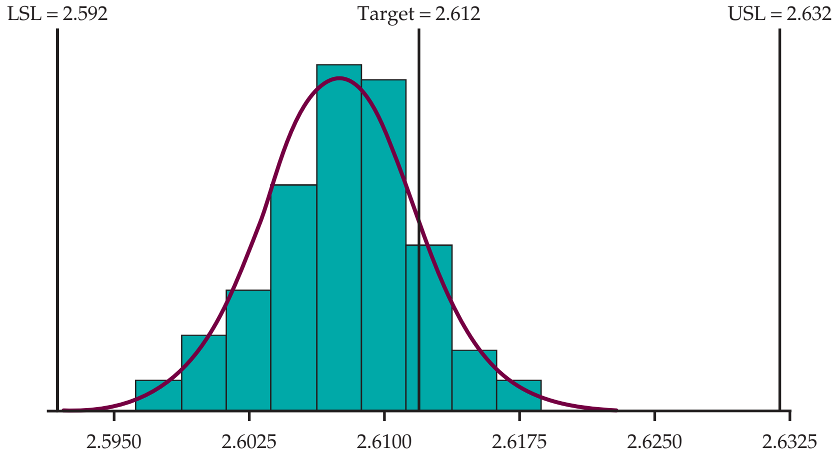


FIGURE 15.23 Comparing the distribution of O-ring measurements with lower and upper specification limits, for Example 15.12.

important point is often not appreciated when capability indices are used in practice. To emphasize that we can only estimate the indices, we write $\hat{C}_p$ and $\hat{C}_{pk}$ for values calculated from sample data. They are

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}}$$

$$= \frac{2.632 - 2.592}{(6)(0.004366)} = 1.53$$

$$\hat{C}_{pk} = \min\left(\frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}}, \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}}\right)$$

$$= \min\left(\frac{2.60755 - 2.592}{(3)(0.004366)}, \frac{2.632 - 2.60755}{(3)(0.004366)}\right)$$

$$= \min(1.19, 1.87) = 1.19$$

Both indices are well above 1, which indicates that we have a highly capable process. However, the fact that $\hat{C}_{pk}$ is markedly smaller than $\hat{C}_p$ indicates that the process is off target. Indeed, Figure 15.23 shows that the center of the distribution is to the left of the center of the specs. If we can adjust the center of the process distribution to the target of 2.612, then $\hat{C}_{pk}$ will increase and will equal $\hat{C}_p$. ■

Our discussion of capability indices has focused on capability relative to two specification limits. When there is only one specification limit involved, we can define one-sided indices as follows:

$$C_{pl} = \frac{\mu - \text{LSL}}{3\sigma}$$

$$C_{pu} = \frac{\text{USL} - \mu}{3\sigma}$$

The change in denominator from 6 to 3 standard deviations reflects the focus on one specification rather than two.

EXAMPLE 15.13

CASE 15.1 **Capability of a Lab-Testing Process** In Examples 15.2 and 15.10 (pages 15-15 and 15-40), we found the estimates for the mean and standard deviation:

$$\hat{\mu} = 40.62$$
$$\hat{\sigma} = 9.444$$

In this application, the upper specification limit was set to 50 minutes. We estimate the one-sided upper capability index to be

$$\hat{C}_{pu} = \frac{50 - 40.62}{(3)(9.444)} = 0.33$$

The estimated capability index is considerably less than 1. As can be seen from Figure 15.20 (page 15-39), this reflects the fact that the process mean is close enough to the upper specification limit to result in a high percent of unacceptable outcomes. ■

We end our discussion on process capability indices on two cautionary notes. First, their interpretation is based on the assumption that the individual process measurements are Normally distributed. It is hard to interpret indices when the measurements are strongly non-Normal. It is best to apply capability indices only when a Normal quantile plot or histogram shows that the distribution is at least roughly Normal. Second, as we saw with Examples 15.12 and 15.13, process indices need to be *estimated* from the process data. The implication is that estimated indices are statistics and thus are subject to sampling variation. A supplier under pressure from a large customer to measure C_{pk} often may base calculations on small samples from the process. The resulting estimate $\hat{C}_{pk}$ can differ greatly from the true process C_{pk} in either direction. As a rough rule of thumb, it is best to rely on indices computed from samples of at least 50 measurements.

APPLY YOUR KNOWLEDGE

15.33 Specification limits versus control limits. The manager you report to is confused by LSL and USL versus LCL and UCL. The notations look similar. Carefully explain the conceptual difference between specification limits for individual measurements and control limits for $\bar{x}$.

15.34 C_p versus C_{pk}. Sketch Normal curves that represent measurements on products from a process with

(a) $C_p = 3$ and $C_{pk} = 1$.

(b) $C_p = 3$ and $C_{pk} = 2$.

(c) $C_p = 3$ and $C_{pk} = 3$.

CASE 15.2 **15.35 O-ring capability in terms of percent defective.** Refer to Example 15.12 for the mean and standard deviation estimates for the O-ring application. Using software, estimate the percent of O-rings that do not meet specs. In quality applications, it is common to report defective rates in units of parts per million (ppm). What is the defective rate for the O-ring process in ppm? ORING

SECTION 15.3 SUMMARY

- **Capability indices** measure process variability (C_p) or process center and variability (C_{pk}) against the standard provided by external specifications for the output of the process. Larger values indicate higher capability.

- Interpretation of C_p and C_{pk} requires that measurements on the process output have a roughly Normal distribution. These indices are not meaningful unless the process is in control so that its center and variability are stable.

- Estimates of C_p and C_{pk} can be quite inaccurate when based on small numbers of observations, due to sampling variability. It is generally recommended that capability index estimates be based on at least 50 measurements.

SECTION 15.3 EXERCISES

For Exercises 15.33 to 15.35, see page 15-44.

15.36 Estimating nonconformance rate. Suppose a Normally distributed process is centered on target, with the target being halfway between specification limits. If $\hat{C}_p = 0.80$, what is the estimated rate of nonconformance of the process to the specifications?

15.37 Measuring capability. You are in charge of a process that makes metal clips. The critical dimension is the opening of a clip, which has specifications 15 ± 0.5 millimeters (mm). The process is monitored by $\bar{x}$ and R charts based on samples of five consecutive clips each hour. Control has recently been excellent. The past week's 40 samples have

$$\bar{\bar{x}} = 14.99 \text{ mm} \qquad \bar{R} = 0.5208 \text{ mm}$$

A Normal quantile plot shows no important deviations from Normality.

(a) What percent of clip openings will meet specifications if the process remains in its current state?

(b) Estimate the capability index C_{pk}.

15.38 Hospital losses again. Table 15.4 (page 15-24) gives data on a hospital's losses for 120 joint replacement patients, collected as 15 monthly samples of eight patients each. The process has been in control, and losses have a roughly Normal distribution. The sample standard deviation (s) for the individual measurements is 811.53. The hospital decides that a suitable specification limit for its loss in treating one such patient is USL = \$8000. HLOSS

(a) Estimate the percent of losses that meet the specification.

(b) Estimate C_{pu}.

15.39 Measuring your personal capability. Refer to Exercise 15.31 (page 15-37), in which you collected 50 sequential observations on your ability to measure 5 seconds. Suppose we define acceptable performance as 5 ± 0.15 seconds.

(a) Assume that the Normal distribution is sufficiently adequate to describe your distribution of times. Estimate the percent of stopwatch recordings that will meet specifications if your process remains in its current state.

(b) Estimate your personal C_p.

(c) Estimate your personal C_{pk}.

(d) Are your C_p and C_{pk} close in value? If not, what does that suggest about your stopwatch recording ability?

15.40 Alloy composition process capability. Refer to Exercise 15.26 (page 15-36) as it relates to the 20 preliminary subgroups on percents of aluminum content. The acceptable range for the percents of aluminum is 3.8% to 4.2%. ALLOY

(a) Obtain the individual observations and make a Normal quantile plot of them. What do you conclude? (If your software will not make a Normal quantile plot, use a histogram to assess Normality.)

(b) Estimate C_p.

(c) Estimate C_{pk}.

(d) Comparing your results from parts (b) and (c), what would you recommend to improve process capability?

15.41 Six Sigma quality. A process with $C_p \geq 2$ is sometimes said to have "Six Sigma quality." Sketch the specification limits and a Normal distribution of individual measurements for such a process when it is properly centered. Explain from your sketch why this is called Six Sigma quality.

15.42 More on Six Sigma quality. The originators of the Six Sigma quality standard reasoned as follows. Short-term process variation is described by σ. In the long term, the process mean μ will also vary. Studies show that in most manufacturing processes, $\pm 1.5\sigma$ is adequate to allow for changes in μ. The Six Sigma standard is intended to allow the mean μ to be as much as 1.5σ away from the center of the specifications and still meet high standards for percent of output lying outside the specifications.

(a) Sketch the specification limits and a Normal distribution for process output when $C_p = 2$ and the mean is 1.5σ away from the center of the specifications.

(b) What is C_{pk} in this case? Is Six Sigma quality as strong a requirement as $C_{pk} \geq 2$?

(c) Because most people don't understand standard deviations, Six Sigma quality is usually described as guaranteeing a certain level of parts per million of output that fail to meet specifications. Based on your sketch in part (a), what is the probability of an outcome outside the specification limits when the mean is 1.5σ away from the center? How many parts per million is this? (You will need software or a calculator for Normal probability calculations because the value you want is beyond the limits of the standard Normal table.)

15.4 Attribute Control Charts

When you complete this section, you will be able to:

- Distinguish the settings for the application of a proportions (*p*) chart versus the application of a counts (*c*) chart.
- Compute the center line and control limits for a *p* chart.
- Compute the center line and control limits for a *c* chart.

We have considered control charts for just one kind of data: measurements of a quantitative variable in some continuous scale of units. We described the distribution of measurements by its center and spread and use $\bar{x}$ and R (or $\bar{x}$ and s) charts for process control. In contrast to continuous data, discrete data typically result from counting. Examples of counting are the number (or proportion) of defective parts in a production run, the daily number of patients in a clinic, and the number of invoice errors. In the quality area, discrete data are known as attribute data. We consider the two most common control charts dedicated to attribute data: namely, the p chart for use when the data are proportions and the c chart for use when the data are counts of events that can occur in some interval of time, area, or volume.

Control charts for sample proportions

binomial distribution, p. 250

We studied the sampling distribution of a sample proportion $\hat{p}$ in Chapter 6. When the binomial distribution underlying the sample proportions is well approximated by the Normal distribution, then the standard three-sigma framework can be applied to the sample proportion data. We ought to call such charts $\hat{p}$ charts because they plot sample proportions. Unfortunately, they have always been called p charts in business practice. We will keep the traditional name but also keep our usual notation: p is a *process* proportion and $\hat{p}$ is a *sample* proportion.

CONSTRUCTION OF A *p* CHART

Take regular samples from a process that has been in control. The samples need not all be the same size. Denote the sample size for the ith sample as n_i. Estimate the process proportion p of "successes" by

$$\bar{p} = \frac{\text{total number of successes in past samples}}{\text{total number of opportunities in these samples}}$$

The center line and control limits for the ***p* chart** are

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

$$\text{CL} = \bar{p}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

where n_i is the sample size for sample i. If the lower control limit computes to a negative value, then LCL is set to 0 because negative proportions are not possible.

If we have k preliminary samples of the *same* size n, then $\bar{p}$ is just the average of the k sample proportions. In some settings, you may meet samples of unequal size—differing numbers of students enrolled in a month or differing numbers of parts inspected in a shift. The average $\bar{p}$ estimates the process proportion p even when the sample sizes vary. In cases of unequal sample sizes, the width of the control limits will vary from sample to sample, as will be shown in Example 15.15 (page 15-49).

photographee.eu/Deposit Photos

CASE 15.3 **Reducing Absenteeism** Unscheduled absences by clerical and production workers are an important cost in many companies. Reducing the rate of absenteeism is, therefore, an important goal for a company's human relations department. A rate of absenteeism above 5% is a serious concern. Many companies set 3% absent as a desirable target. You have been asked to improve absenteeism in a production facility where 12% of the workers are now absent on a typical day.

You first do some background study—in greater depth than this very brief summary. Companies try to avoid hiring workers who are likely to miss work often, such as substance abusers. They may have policies that reward good attendance or penalize frequent absences by individual workers. Changing those policies in this facility will have to wait until the union contract is renegotiated. What might you do with the current workers under current policies? Studies of absenteeism of clerical and production workers who do repetitive, routine work under close supervision point to unpleasant work environment and harsh or unfair treatment by supervisors as factors that increase absenteeism. It's now up to you to apply this general knowledge to your specific problem. ■

First, collect data. Daily absenteeism data are already available. You carry out a sample survey that asks workers about their absences and the reasons for them (responses are anonymous, of course). Workers who are absent more often complain about their supervisor and about the lighting at their workstation. Workers complain that the restrooms are dirty and unpleasant. You do more data analysis:

- A chart of average absenteeism rate for the past month broken down by supervisor (Figure 15.24) shows important differences among supervisors. Only supervisors B, E, and H meet your goal of 5% or less absenteeism. Workers supervised by I and D have particularly high rates.
- Further data analysis (not shown) shows that certain workstations have substantially higher rates of absenteeism.

Now you take action. You retrain all the supervisors in human relations skills, using B, E, and H as discussion leaders. In addition, a trainer works individually with supervisors I and D. You ask supervisors to talk with any

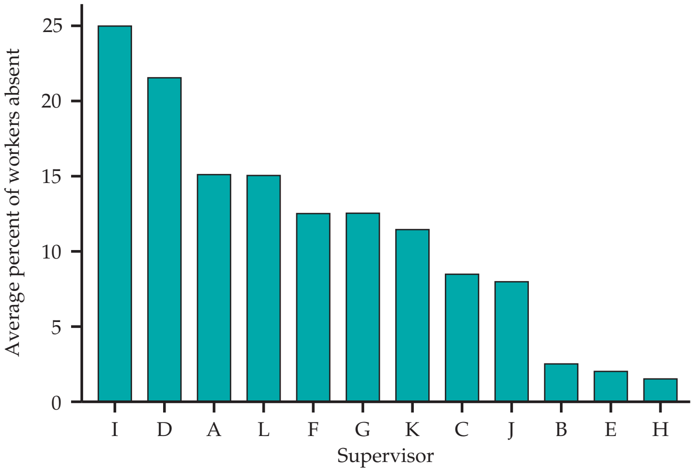


FIGURE 15.24 Chart of the average percent of days absent for workers reporting to each of 12 supervisors.

absent worker when he or she returns to work. Working with the engineering department, you study the workstations with high absenteeism rates and make changes such as better lighting. You refurbish the restrooms and schedule more frequent cleaning.

EXAMPLE 15.14

CASE 15.3

ABSENT

Absenteeism Rate *p* Chart Are your actions effective? You hope to see a reduction in absenteeism. To view progress (or lack of progress), you will keep a p chart of the proportion of absentees. The plant has 987 production workers. For simplicity, you record just the number who are absent from work each day. Only unscheduled absences count, not planned time off such as vacations.

Each day you will plot

$$\hat{p} = \frac{\text{number of workers absent}}{987}$$

You first look back at data for the past three months. There were 64 workdays in these months. The total workdays available for the workers was

$$(64)(987) = 63{,}168 \text{ person-days}$$

Absences among all workers totaled 7580 person-days. The average daily proportion absent was therefore

$$\bar{p} = \frac{\text{total days absent}}{\text{total days available for work}}$$
$$= \frac{7580}{63{,}168} = 0.120$$

The daily rate has been in control at this level.

These past data allow you to set up a p chart to monitor future proportions absent:

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.120 + 3\sqrt{\frac{(0.120)(0.880)}{987}}$$
$$= 0.120 + 0.031 = 0.151$$
$$\text{CL} = \bar{p} = 0.120$$
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.120 - 3\sqrt{\frac{(0.120)(0.880)}{987}}$$
$$= 0.120 - 0.031 = 0.089$$

Table 15.9 gives the data for the next four weeks. Figure 15.25 is the p chart. ■

TABLE 15.9 Proportions of workers absent during four weeks

Day	M	T	W	Th	F	M	T	W	Th	F
Workers absent	129	121	117	109	122	119	103	103	89	105
Proportion $\hat{p}$	0.131	0.123	0.119	0.110	0.124	0.121	0.104	0.104	0.090	0.106

Day	M	T	W	Th	F	M	T	W	Th	F
Workers absent	99	92	83	92	92	115	101	106	83	98
Proportion $\hat{p}$	0.100	0.093	0.084	0.093	0.093	0.117	0.102	0.107	0.084	0.099

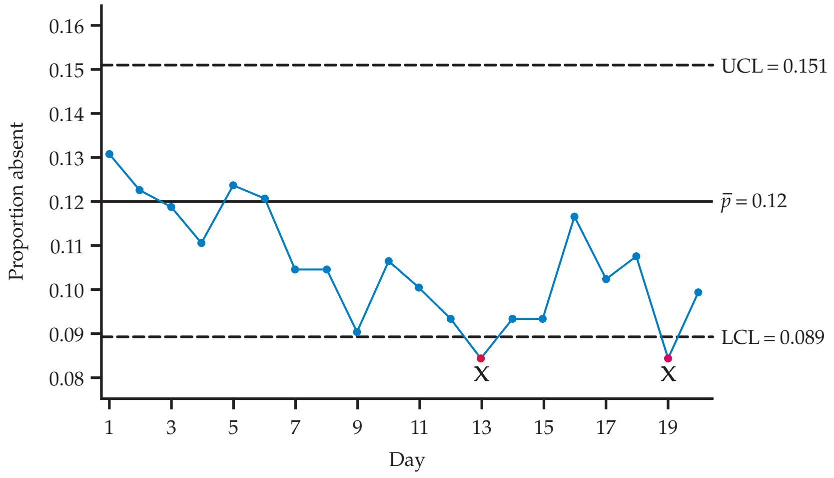


FIGURE 15.25 Prospective-monitoring *p* chart for daily proportion of workers absent over a four-week period, for Example 15.14. The lack of control shows an improvement (decrease) in absenteeism. Update the chart to continue monitoring the process.

Figure 15.25 shows a clear downward trend in the daily proportion of workers who are absent. Days 13 and 19 lie below LCL, and a run of nine days below the center line is achieved at Day 15 and continues. (See Exercise 15.25 (page 15-36) for discussion of the "nine-in-a-row" out-of-control signal.) It appears that a special cause (the various actions you took) has reduced the absenteeism rate from around 12% to around 10%. The last two weeks' data suggest that the rate has stabilized at this level. You will update the chart based on the new data. If the rate does not decline further (or even rises again as the effect of your actions wears off), you will consider further changes.

Example 15.14 is a bit oversimplified. The number of workers available did not remain fixed at 987 each day. Hirings, resignations, and planned vacations change the number a bit from day to day. The control limits for a day's $\hat{p}$ depend on n, the number of workers that day. If n varies, the control limits will move in and out from day to day. In this case, n is fairly large, which means that as long as the count of workers remains close to 987, the greater detail provided by variable limits will not likely change your conclusion. We demonstrate the construction of variable limits in the next example.

A single p chart for all workers is not the only, or even the best, choice in this setting. Because of the important role of supervisors in absenteeism, it would be wise to also keep separate p charts for the workers under each supervisor. These charts may show that you must reassign some supervisors.

EXAMPLE 15.15

BELLIN

Patient Satisfaction *p* Chart Nationwide, health care organizations are instituting process improvement methods to improve the quality of health care delivery, including patient outcomes and patient satisfaction. Bellin Health (bellin.org) is a leader in the implementation of quality methods in a health care setting. Located in Green Bay, Wisconsin, Bellin Health serves nearly half a million people in northeastern Wisconsin and in the Upper Peninsula of Michigan. As part of its quality initiative, Bellin instituted a measurement control system of more than 250 quality indicators, which they later expanded to include more than 1200 quality indicators. Most of these quality indicators are monitored by control charts.

Table 15.10 gives the numbers of Bellin ambulatory (outpatient) surgery patients sampled each quarter for 17 consecutive quarters. Also provided is the number of patients out of each sample who said they would likely recommend Bellin to others for ambulatory surgery.[11] The number of patients who would likely recommend Bellin can be divided by the sample size to give the

TABLE 15.10 Proportions of ambulatory surgery patients of Bellin Health System likely to recommend Bellin for ambulatory surgery

Quarter	Patients likely to recommend	Total number of patients	Proportion
1	164	222	0.7387
2	239	306	0.7810
3	186	245	0.7592
4	219	293	0.7474
5	219	287	0.7631
6	170	216	0.7870
7	199	256	0.7773
8	189	249	0.7590
9	177	245	0.7224
10	209	260	0.8038
11	227	275	0.8255
12	253	322	0.7857
13	278	350	0.7943
14	247	315	0.7841
15	234	285	0.8211
16	251	341	0.7361
17	319	405	0.7877

sample proportion of patients who are likely to recommend Bellin. These proportions are also provided in Table 15.10.

The average quarterly proportion of patients likely to recommend Bellin is computed as follows:

$$\bar{p} = \frac{164 + 239 + \cdots + 319}{222 + 306 + \cdots + 405} = \frac{3780}{4872} = 0.7759$$

The upper and lower control limits for each sample are given by

$$\text{UCL} = 0.7759 + 3\sqrt{\frac{(0.7759)(0.2241)}{n_i}} = 0.7759 + \frac{1.2510}{\sqrt{n_i}}$$

$$\text{LCL} = 0.7759 + 3\sqrt{\frac{(0.7759)(0.2241)}{n_i}} = 0.7759 - \frac{1.2510}{\sqrt{n_i}}$$

For the first recorded quarter, the control limits are

$$\text{UCL} = 0.7759 + \frac{1.2510}{\sqrt{222}} = 0.8599$$

$$\text{LCL} = 0.7759 - \frac{1.2510}{\sqrt{222}} = 0.6919$$

Figure 15.26 displays the p chart for all the proportions. Notice first that the control limits are of varying widths. The sample proportions are behaving as an in-control process around the center line with no out-of-control signals. Even though the stability of the process implies that Bellin is sustaining a fairly high level of satisfaction, management's goal is no doubt to find ways to increase satisfaction to even higher levels and thus cause an upward trend or upward shift in the process. ■

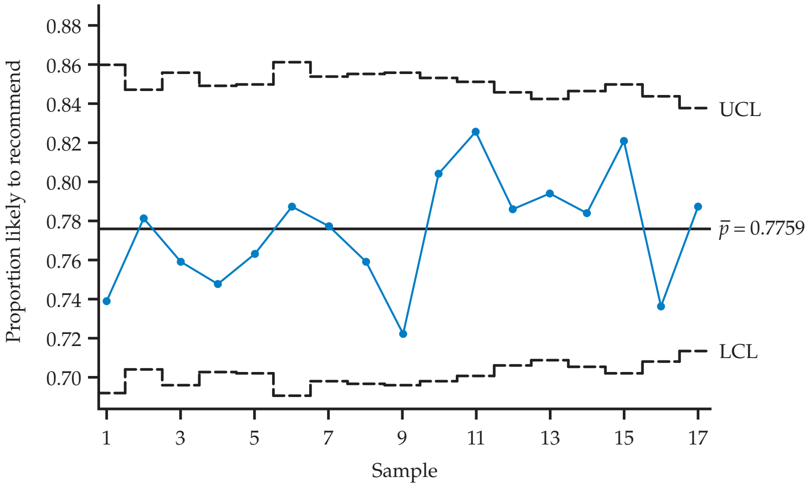


FIGURE 15.26 The p chart for proportions of ambulatory surgery patients of Bellin Health who are likely to recommend Bellin for ambulatory surgery, for Example 15.15.

APPLY YOUR KNOWLEDGE

15.43 Unpaid invoices. The controller's office of a corporation is concerned that invoices that remain unpaid after 30 days are damaging relations with vendors. To assess the magnitude of the problem, a manager searches payment records for invoices that arrived in the past 10 months. The average number of invoices is 2875 per month, with relatively little month-to-month variation. Of all these invoices, 960 remained unpaid after 30 days.

(a) What is the total number of opportunities for unpaid invoices? What is $\bar{p}$?

(b) Give the center line and control limits for a p chart on which to plot the future monthly proportions of unpaid invoices.

CASE 15.3 **15.44 Setting up a p chart.** After inspecting Figure 15.25 (15-49), you decide to monitor the future absenteeism rates using a center line and control limits calculated from the second two weeks of data recorded in Table 15.9 (page 15-48). Find $\bar{p}$ for these 10 days and give the new values of CL, LCL, and UCL. ABSENT

Control charts for counts per unit of measure

In the discussion of the p chart, there is a limit to the number of occurrences we can count. For example, if 100 parts are inspected, the most defective parts we could find would be 100. In contrast, if we were counting the number of stitch flaws in an area of carpet, then the count could be 0, 1, 2, 3, and so on indefinitely. This latter example represents the Poisson setting discussed in Section 5.4. The Poisson distribution accounts for random occurrence of events within a continuous interval of time, area, or volume. From Section 5.4, we learned that a Poisson distribution with mean μ has a standard deviation of $\sqrt{\mu}$. The Normal distribution can be used to approximate the Poisson distribution. As a rule of thumb, the Normal distribution is an adequate approximation for the Poisson distribution when $\mu \geq 5$. With these facts in mind, we can establish a three-sigma control chart for Poisson count data.

Poisson distribution, p. 262

CONSTRUCTION OF A c CHART

Suppose that k nonoverlapping units are sampled and $c_1, c_2, \ldots, c_k$ are the observed counts. Estimate the process mean count $\bar{c}$ by

$$\bar{c} = \frac{1}{k}(c_1 + c_2 + \cdots + c_k)$$

The center line and control limits for the **c chart** are

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{CL} = \bar{c}$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

If the lower control limit computes to a negative value, then LCL is set to 0 because negative counts are not possible.

EXAMPLE 15.16

SAFETY

Work Safety *c* Chart State and federal laws require employers to provide safe working conditions for their workers. Beyond the legal requirements, many companies have implemented process improvement measures, such as Six Sigma methods, to improve worker safety. Companies recognize that such efforts have beneficial effects on employee satisfaction and reduce liability and loss of workdays, all of which can improve productivity and corporate profitability. For a manufacturing facility, Table 15.11 provides 24 months of the counts of Occupational Safety and Health Administration (OSHA) reportable injuries.

TABLE 15.11 Counts of OSHA reportable injuries per month for 24 consecutive months

Month:	1	2	3	4	5	6	7	8	9	10	11	12
Injuries:	12	6	6	10	5	2	6	5	5	4	6	6
Month:	13	14	15	16	17	18	19	20	21	22	23	24
Injuries:	16	9	10	5	7	3	5	7	12	4	4	6

The average monthly number of injuries is calculated as follows:

$$\bar{c} = \frac{12 + 6 + \cdots + 6}{24} = \frac{161}{24} = 6.7083$$

The center line and control limits are given by

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 6.7083 + 3\sqrt{6.7083} = 14.48$$
$$\text{CL} = \bar{c} = 6.7083$$
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 6.7083 - 3\sqrt{6.7083} = -1.06 \rightarrow 0$$

Figure 15.27 shows the sequence plot of the counts along with the previously computed control limits. We can see that the 13th count falls above the upper control limit. This unusually high number of injuries is a signal for management investigation. If a special cause can be found for the out-of-control signal, then the associated observation should be removed from the preliminary data and the control limits should be recomputed. We leave the recomputation of this *c* chart application to Exercise 15.45. ■

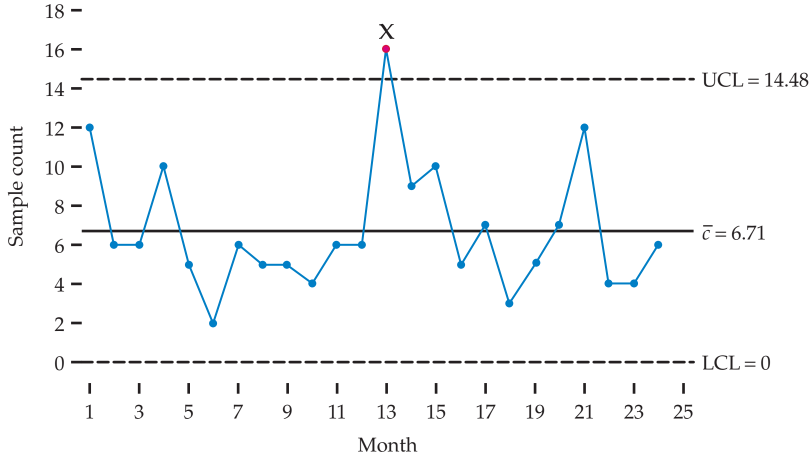


FIGURE 12.27 The *c* chart for OSHA reportable injuries, for Example 15.16.

APPLY YOUR KNOWLEDGE

15.45 Worker safety. An investigation of the out-of-control signal seen in Figure 15.27 revealed that not only were there new hires that month, but new machinery was installed. The combination of relatively inexperienced employees and unfamiliarity with the new machinery resulted in an unusually high number of injuries. Remove the out-of-control observation from the data count series and recompute the c chart control limits. Comment on control of the remaining counts. SAFETY

15.46 Positive lower control limit? What values of $\bar{c}$ are associated with a positive lower control limit for the c chart?

SECTION 15.4 SUMMARY

- **Attribute control charts** are dedicated to monitoring and control of counting type data in the form of proportions or direct counts.
- A ***p* chart** is used to monitor sample proportions $\hat{p}$. However, when monitoring counts over continuous intervals of time, area, or volume and there is no definite limit on the number of counts that can be observed, a ***c* chart** is considered.
- The interpretation of p and c charts is very similar to that of variable control charts. The out-of-control signals are also the same.

SECTION 15.4 EXERCISES

For Exercises 15.43 and 15.44, see page 15-51; and for 15.45 and 15.46, see page 15-53.

15.47 Aircraft rivets. After completion of an aircraft wing assembly, inspectors count the number of missing or deformed rivets. There are hundreds of rivets in each wing, but the total number varies depending on the aircraft type. Recent data for wings with a total of 34,700 rivets show 208 missing or deformed. The next wing contains 1070 rivets. What are the appropriate center line and control limits for plotting the $\hat{p}$ from this wing on a p chart?

15.48 Call center. A large nationwide retail chain keeps track of a variety of statistics on its service call center. One of those statistics is the length of time a customer has to wait before talking to a representative. Based on call center research and general experience, the retail chain has determined that it is unacceptable for any customer to be on hold for more than 90 seconds. To monitor the performance of the call center, a random sample of 200 calls per shift (three shifts per day) is obtained. Here are the number of unacceptable calls in each sample for 15 consecutive shifts over the course of one business week: CALLC

Shift	1	2	3	1	2	3	1	2	3
Unacceptable	6	17	6	9	16	10	8	14	5
Shift	1	2	3	1	2	3			
Unacceptable	6	16	6	9	14	7			

(a) What is $\bar{p}$ for the call center process?

(b) What are the center line and control limits for a p chart for plotting proportions of unacceptable calls?

(c) Label the data points on the p chart by the shift. What do you observe that the p chart limits failed to pick up?

15.49 School absenteeism. Here are data from an urban school district on the number of eighth-grade students with three or more unexcused absences from school during each month of a school year. Because the total number of eighth-graders changes a bit from month to month, these totals are also given for each month. SCHOOL

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
Students	911	947	939	942	918	920	931	925	902	883
Absent	291	349	364	335	301	322	344	324	303	344

(a) Find $\bar{p}$. Because the number of students varies from month to month, also find $\bar{n}$, the average per month.

(b) Make a p chart using control limits based on $\bar{n}$ students each month. Comment on control.

(c) The exact control limits are different each month because the number of students n is different each month. This situation is common when using p charts. What are the exact limits for October and June, the months with the largest and smallest n? Add these limits to your p chart, using short lines spanning a single month. Do exact limits affect your conclusions?

15.50 *p* charts and high-quality processes. A manufacturer of consumer electronic equipment makes full use not only of statistical process control but of automated testing equipment that efficiently tests all completed products. Data from the testing equipment show that finished products have only 3.5 defects per million opportunities.

(a) What is $\bar{p}$ for the manufacturing process? If the process turns out 5000 pieces per day, how many defects do you expect to see at this rate? In a typical month of 24 working days, how many defects do you expect to see?

(b) What are the center line and control limits for a p chart for plotting daily defect proportions?

(c) Explain why a p chart is of no use at such high levels of quality.

15.51 Monitoring lead time demand. Refer to the lead time demand process discussed in Exercise 5.135 (page 290). Assuming the Poisson distribution given in the exercise, what would be the appropriate control chart limits for monitoring lead time demand?

15.52 Purchase order errors. Purchase orders are checked for two primary mistakes: incorrect charge account number and missing required information. Each day, 10 purchase orders are randomly selected, and the number of mistakes in the sample is recorded. Here are the numbers of mistakes observed for 20 consecutive days: POERR

6	4	11	6	3	7	3	10	14	6
3	5	6	7	5	7	7	4	3	7

(a) What is $\bar{c}$ for the purchase order process? How many mistakes would you expect to see in 50 randomly selected purchase orders?

(b) What are the center line and control limits for a c chart for plotting counts of purchase order mistakes per 10 orders? Are there any indications of out-of-control behavior?

(c) Remove any out-of-control observation(s) from the data count series and recompute the c chart control limits. Comment on control of the remaining counts.

15.53 Implications of out-of-control signal. For attribute control charts, explain the difference in implications for a process and in actions to be taken when the plotted statistic falls beyond the upper control limit versus beyond the lower control limit.

CHAPTER 15 REVIEW EXERCISES

15.54 Enlighten management. A manager who knows no statistics asks you, "What does it mean to say that a process is in control? Is being in control a guarantee that the quality of the product is good?" Answer these questions in plain language that the manager can understand.

15.55 Pareto charts. You manage the customer service operation for a maker of electronic equipment sold to business customers. Traditionally, the most common complaint is that equipment does not operate properly when installed, but attention to manufacturing and installation quality will reduce these complaints. You hire an outside firm to conduct a sample survey of your customers. Here are the percent of customers with each of several kinds of complaints:

Category	Percent
Accuracy of invoices	25
Clarity of operating manual	8
Complete invoice	24
Complete shipment	16
Correct equipment shipped	15
Ease of obtaining invoice adjustments/credits	33
Equipment operates when installed	6
Meeting promised delivery date	11
Sales rep returns calls	4
Technical competence of sales rep	12

(a) Why do the percents not add to 100%?

(b) Make a Pareto chart. What area would you choose as a target for improvement?

15.56 Purchased material. At the present time, about five out of every 1000 lots of material arriving at a plant site from outside vendors are rejected because they are incorrect. The plant receives about 300 lots per week. As part of an effort to reduce errors in the system of placing and filling orders, you will monitor the proportion of rejected lots each week. What type of control chart will you use? What are the initial center line and control limits?

You have just installed a new system that uses an interferometer to measure the thickness of polystyrene film. To control the thickness, you plan to measure three film specimens every 10 minutes and keep $\bar{x}$ and s charts. To establish control, you measure 22 samples of three films each at 10-minute intervals. Table 15.12 gives $\bar{x}$ and s for these samples. The units are millimeters $\times 10^{-4}$. Exercises 15.57 through 15.59 are based on this process improvement setting. FILM

TABLE 15.12 $\bar{x}$ and s for samples of film thickness

Sample	$\bar{x}$	s	Sample	$\bar{x}$	s
1	848	20.1	12	823	12.6
2	832	1.1	13	835	4.4
3	826	11.0	14	843	3.6
4	833	7.5	15	841	5.9
5	837	12.5	16	840	3.6
6	834	1.8	17	833	4.9
7	834	1.3	18	840	8.0
8	838	7.4	19	826	6.1
9	835	2.1	20	839	10.2
10	852	18.9	21	836	14.8
11	836	3.8	22	829	6.7

15.57 *s* chart. Calculate control limits for s, make an s chart, and comment on control of short-term process variation.

15.58 $\bar{x}$ chart. Interviews with the operators reveal that in Samples 1 and 10, mistakes in operating the interferometer resulted in one high-outlier thickness reading that was clearly incorrect. Recalculate $\bar{s}$ after removing Samples 1 and 10. Recalculate UCL for the s chart and add the new UCL to your s chart from the previous exercise. Control for the remaining samples is excellent. Now find the appropriate center line and control limits for an $\bar{x}$ chart, make the $\bar{x}$ chart, and comment on control.

15.59 Categorizing the output. Previously, control of the process was based on categorizing the thickness of each film inspected as satisfactory or not. Steady improvement in process quality has occurred, so that just 15 of the last 5000 films inspected were unsatisfactory.

(a) What type of control chart discussed in this chapter might be considered for this setting, and what would be the control limits for a sample of 100 films?

(b) Explain why the chart in part (a) would have limited practical value at current quality levels.

15.60 Hospital losses revisited. Refer to Exercise 15.14 (page 15-23), in which you were asked to construct $\bar{x}$ and s charts for the hospital losses data shown in Table 15.4. HLOSS

(a) Make an R chart and comment on control of the process variation.

(b) Using the range estimate, make an $\bar{x}$ chart and comment on control of process level.

15.61 Bone density revisited. Refer to Exercise 15.24 (page 15-35), in which you were asked to construct $\bar{x}$ and s charts for the calibration data from a Lunar bone densitometer shown in Table 15.6. BONE

(a) Make an s chart and comment on control of the process variation.

(b) Based on the standard deviations, make an $\bar{x}$ chart and comment on control of process level.

15.62 Even more signals. There are other out-of-control signals that are sometimes used with $\bar{x}$ charts. One is "15 points in a row within the 1σ level." That is, 15 consecutive points fall between $\mu - \sigma/\sqrt{n}$ and $\mu + \sigma/\sqrt{n}$. This signal suggests that either the value of σ used for the chart is too large or that careless measurement is producing results that are suspiciously close to the target. Find the probability that the next 15 points will give this signal when the process remains in control with the given μ and σ.

15.63 It's all in the wrist. Consider the saga of a professional basketball player plagued with poor free-throw shooting performance. Here are the number of free throws he made out of 50 attempts on 20 consecutive practice days: FTHROW

25	27	31	28	22	21	27	20	25	27
23	22	29	34	30	27	26	25	28	25

(a) Construct a p chart for the data. Does the process appear to be in control?

(b) Recognizing that the player needed insight into his free-throw shooting problems, the coach hired an outside consultant to work with the player. The consultant noticed a subtle flaw in the player's technique. Namely, the player was bending back his wrist only 85 degrees when, ideally, the wrist needs to be bent back 90 degrees for proper flick motion. Part of the problem was due to the player's stiff wrist. Over the course of the next week or so, the player was given techniques to loosen his wrist. After implementing a modification to wrist movement, he got the following results on 10 new samples (again out of 50 attempts):

34	38	35	43	31	35	32	36	28	39

Plot the new sample proportions along with the control limits determined in part (a). What are your conclusions? What should be the values of the control limits for future samples?

15.64 Monitoring rare events. In certain SPC applications, we are concerned with monitoring the occurrence of events that can occur at any point within a continuous interval of time, such as the number of computer operator errors per day or plant injuries per month. However, for highly capable processes, the occurrence of events is rare. As a result, the data will plot as many strings of zeros with an occasional nonzero observation. Under such circumstances, a control chart will be fairly useless. In light of this issue, SPC practitioners monitor the time between successive events; for example, the time between accidental contaminated needle sticks in a health care setting. For this exercise, consider data on the time between fatal commercial airline accidents worldwide between January 2010 and December 2018.[12] AFATAL

(a) Construct an individuals chart for the time-between-fatalities data. If the lower control limit computes to a negative number, set it to 0 because negative data values are not possible. Report the lower and upper control limits. Identify any observations flagged as unusual.

(b) Time-between-events data tend to be non-Normal and most often are positively skewed. Construct a histogram and Normal quantile plot for the fatalities data. Is that the case for these data?

(c) For time-between-events data, transforming the data by raising them to the 0.2777 power ($y_i = x_i^{0.2777}$) often Normalizes the data. Apply this transformation to the time-between-fatalities data, and construct a histogram and Normal quantile plot for the transformed data. Are the plots generally consistent with the Normal distribution?

(d) Construct an individuals chart for the transformed data. If the lower control limit computes to a negative number, set it to 0. Report the lower and upper control limits. Identify any points that are unusual. What is the implication of the out-of-control point(s)?

(e) Remove the unusual observation(s) found in part (d), and reestimate and report the control limits. What impressions do you have about the time-between-fatalities process when plotted with the revised limits? Is there evidence of improvement or worsening of the process over the almost 10-year time span?

15.65 Monitoring budgets. Control charts are used for a wide variety of applications in business. In the accounting area, control charts can be used to monitor budget variances. A budget variance is the difference between planned spending and actual spending for a given time period. Often, budget variances are measured in percents. For improved budget planning, it is important to identify unusual variances on both the low and high sides. The data file for this exercise includes variance percents for 40 consecutive weeks for a manufacturing work center. BUDGET

(a) Construct an I chart for the variance percents. Report the lower and upper control limits. Identify any observations that are outside the control limits.

(b) Apply the runs rule based on nine consecutive observations being on one side of the center line. Is there an out-of-control signal based on this rule? If so, what are the associated observations?

(c) Remove all observations associated with out-of-control signals found in parts (a) and (b). Reestimate the I chart control limits, and apply them to the remaining observations. Are there any more out-of-control signals? If so, identify them, remove them, and reestimate limits. Continue this process until no out-of-control signals are present. Report the final control limits to be used for future monitoring.

(d) Construct an MR chart based on the final data of part (c). What do you conclude?

15.66 Is it really Poisson? Certain manufacturing environments, such as semiconductor manufacturing and biotechnology, require a low level of environmental pollutants (e.g., dust, airborne microbes, and aerosol particles). For such industries, manufacturing occurs in ultraclean environments known as *cleanrooms*. There are federal and international classifications of cleanrooms that specify the maximum number of pollutants of a particular size allowed per volume of air. Consider a manufacturer of integrated circuits. One cubic meter of air is sampled at constant intervals of time, and the number of pollutants of size 0.3 microns or larger is recorded. Here are the count data for 25 consecutive samples: CLEAN

7	3	13	1	17	3	6	9	12	5	5	0	6
2	9	1	12	2	3	3	7	5	0	3	13	

(a) Construct a c chart for the data. Does the process appear to be in control?

(b) Remove any out-of-control signals found in part (a), and reestimate the c chart limits. Does the process now appear to be in control?

(c) Remove all observations associated with out-of-control signals found in parts (a) and (b). Reestimate the control limits, and apply them to the remaining observations. Are there any more out-of-control signals? If so, identify them, remove them, and reestimate limits. Continue this process until no out-of-control signals are present. Report the final control limits.

(d) A quality control manager took a look at the data and was suspicious of the numerous rounds of data point removal. Even the final control limits were bothersome to the manager because the variation within the limits seemed too large. The manager made the following statement: "I am not so sure the c chart is applicable here. I have a hunch that the process is not influenced by only Poisson variation. I suggest we look at the estimated mean and variance of the data values." Calculate the sample variance s^2 of the original 25 values, and compare this variance estimate with the mean estimate. Explain how such a comparison can suggest the possibility that a Poisson distribution may not fully describe the process.

Answers to Odd-Numbered Exercises

15.1 Answers will vary.

15.3 Answers will vary. Examples might be size of the video or connection speed.

15.5 An example of common cause variation would include long lines. An example of a special cause might be a delayed flight.

15.7 Answers will vary.

15.9 Answers will vary.

15.11 A point falling beyond the upper control limit of the $\bar{x}$ chart indicates a change in the process level; a point falling beyond the upper control limit of the R chart indicates a change in variation.

15.13 For $n = 4$, $A_2 = 0.729$; CL $= \bar{\bar{x}} = 2.60755$; UCL $= 2.60755 + (0.729)(0.00899) = 2.6141$; LCL $= 2.60755 - (0.729)(0.00899) = 2.601$.

15.15 For $n = 5$, $A_3 = 1.427$; CL $= \bar{\bar{x}} = 40.62$; UCL $= 40.62 + (1.427)(9.20) = 53.75$; LCL $= 40.62 - (1.427)(9.20) = 27.49$. Using $B_3 = 0$ and $B_4 = 2.089$: CL $= \bar{S} = 9.20$; UCL $= (2.089)(9.20) = 19.22$; LCL $= (0)(9.20) = 0$.

15.17 Answers will vary.

15.19 The chart limits get smaller as the subgroup size increases.

15.21 $P(Out\ of\ control) = 1 - P(687 < \bar{X} < 713) = 1 - P\left(\frac{687-693}{12/\sqrt{4}} < Z < \frac{713-693}{12/\sqrt{4}}\right) = 1 - P(-1 < Z < 3.33) = 0.1591$.

15.23 For $n = 5$, $A_2 = 0.577$; CL $= \bar{\bar{x}} = 10.5812$; UCL $= 10.5812 + (0.577)(0.372) = 10.7958$; LCL $= 10.5812 - (0.577)(0.372) = 10.3666$. Using $D_3 = 0$ and $D_4 = 2.114$, CL $= \bar{R} = 0.372$; UCL $= (2.114)(0.372) = 0.7864$; LCL $= (0)(0.372) = 0$.

15.25 Assuming each point falls on either side of the center line with probability 0.5, a run of nine in a row would occur with probability $(0.5)^9 = 0.001953$, or about 2 in 1000.

15.27 Although the data points are all within the control limits, starting with subgroup 27, there are nine data points below the center line, violating the nine-in-a-row rule. It is likely the process level has shifted and/or there is a special cause acting on the process that needs to be determined.

15.29 **(a)** For $n = 10$, $A_3 = 0.975$; CL $= \bar{\bar{x}} = 30.6833$; UCL $= 30.6833 + (0.975)(7.50638) = 38.002$; LCL $= 30.6833 - (0.975)(7.50638) = 23.365$. Using $B_3 = 0.284$ and $B_4 = 1.716$: CL $= \bar{S} = 7.50638$; UCL $= (1.716)(7.50638) = 12.881$; LCL $= (0.284)(7.50638) = 2.132$. **(b)** The process variation is in control. The process level is out of control starting at week 38.

15.31 Answers will vary.

15.33 Specification limits track whether a process can meet a specification, often designated by a customer. The USL and LSL are determined by the customer, and then the process is monitored to see if it can satisfy the customer's demands. Control limits track whether a process is in control, without special cause variation acting on it. They measure inherent variation in the process and do not specify if the process can meet a customer's specifications, only whether the process is free from outside special cause variation.

15.35 $P(defective) = 1 - P(2.592 < X < 2.632) = 1 - P\left(\frac{2.592 - 2.60755}{0.004366} < Z < \frac{2.632 - 2.60755}{0.004366}\right) = 1 - P(-3.56 < Z < 5.60) = 0.000185438$, or 185 ppm.

15.37 **(a)** Using $n = 5$, $d_2 = 2.326$; $\hat{\sigma} = 0.5208/2.326 = 0.2239$; $\hat{C}_p = 1/[6(0.2239)] = 0.74438$. 74.44% of 6σ gives $Z = \pm 2.23$. So 97.42% will meet specifications. **(b)** $\hat{C}_{pk} = 0.49/[3(0.2239)] = 0.7295$.

15.39 Answers will vary.

15.41 It is called Six-Sigma Quality because it allows for 6 or more standard deviations on either side of the mean instead of the Normal 3 required.

15.43 **(a)** 28,750; $\bar{p} = 0.0334$. **(b)** CL $= 0.0334$;

$$\text{UCL} = 0.0334 + 3\sqrt{\frac{0.0334(1-0.0334)}{2875}} = 0.0435;$$

$$\text{LCL} = 0.0334 - 3\sqrt{\frac{0.0334(1-0.0334)}{2875}} = 0.0233.$$

15.45 CL $= 6.304$; UCL $= 6.304 + 3\sqrt{6.304} = 13.84$; LCL $= 6.3 - 3\sqrt{6.304} = -1.23$, so use 0. The remaining points are in control.

15.47 $\bar{p} = 0.006$; CL $= 0.006$; UCL $= 0.006 + 3\sqrt{\frac{0.006(1-0.006)}{1070}} = 0.013$; LCL $= 0.006 - 3\sqrt{\frac{0.006(1-0.006)}{1070}} = -0.001$, so use 0.

15.49 **(a)** $\bar{p} = 0.3554$; $\bar{n} = 922$. **(b)** UCL $= 0.4027$; LCL $= 0.3081$. The process is in control. **(c)** For October, UCL $= 0.402$ and LCL $= 0.309$. For June, UCL $= 0.404$ and LCL $= 0.307$. Exact limits do not affect the conclusions.

15.51 $\bar{c} = 15$; UCL $= 15 + 3\sqrt{15} = 26.62$; LCL $= 15 - 3\sqrt{15} = 3.38$.

15.53 Usually with attribute control charts, we are keeping track of count data that have a negative aspect, such as the number of mistakes. So if we get an out-of-control point above the UCL, we want to find the source of the special cause and try to correct it. However, if we get an out-of-control point below the LCL, we want to find the source of the special cause and mimic it in the future to potentially change (reduce) the level of the process.

15.55 **(a)** The customers could make zero or many complaints, not just one. **(b)** The three biggest complaints are problems with invoices, so that should be the focus.

15.57 $\bar{s} = 7.65$; using $n = 3$, $B_3 = 0$ and $B_4 = 2.568$: CL = 7.65; UCL = (2.568)(7.65) = 19.65; LCL = (0)(7.65) = 0. The first sample is out of control.

15.59 **(a)** A p chart would be appropriate. CL = 0.003;

$$\text{UCL} = 0.003 + 3\sqrt{\frac{0.003(1-0.003)}{100}} = 0.019;$$

$$\text{LCL} = 0.003 - 3\sqrt{\frac{0.003(1-0.003)}{100}} = -0.013,$$

so use 0. **(b)** The chance of an unsatisfactory film is so small that we expect only 0.3 in each sample of 100. But if a sample has 2 defects, $\hat{p} = 2/100$ is already over the UCL and would signal an out-of-control process, which isn't true.

15.61 **(a)** $\bar{s} = 0.0028$; using $n = 2$, $B_3 = 0$ and $B_4 = 3.267$: CL = 0.0028; UCL = (3.267)(0.0028) = 0.00916; LCL = (0)(0.0028) = 0. The s chart shows the process is in control. **(b)** For $n = 2$, $A_2 = 1.881$; CL = $\bar{\bar{x}} = 1.2619$; UCL = 1.2619 + (1.881)(0.0028) = 1.2693; LCL = 1.2619 − (1.881)(0.0028) = 1.2544; the process is in control.

15.63 **(a)** CL = 0.52; $\text{UCL} = 0.52 + 3\sqrt{\frac{0.52(1-0.52)}{50}} = 0.73$; $\text{LCL} = 0.52 - 3\sqrt{\frac{0.52(1-0.52)}{50}} = 0.31$; the process is in control. **(b)** The process appears out of control because the process mean has shifted. The new control limits are CL = 0.7; $\text{UCL} = 0.7 + 3\sqrt{\frac{0.7(1-0.7)}{50}} = 0.90$; $\text{LCL} = 0.7 - 3\sqrt{\frac{0.7(1-0.7)}{50}} = 0.51$.

15.65 **(a)** UCL = 11.5, LCL = −10.0. Subgroup 5 is out of control, below the LCL. **(b)** Subgroups 20 to 28 are all above the CL, violating the nine-in-a-row rule. **(c)** The process is now in control, and there are no out-of-control signals. UCL = 10.7, LCL = −9.7. **(d)** The MR chart shows the process is in control.