CHAPTER 11 Appendix

Density Curves and the Normal Distributions

Normal distributions are important in statistics and probability in part because many random variables have (at least approximately) the properties of a Normal ("bell-shaped") distribution. They are also used for inference on some population parameters.

Normal Distributions

Excel

Normal Distribution Calculations

Excel does not provide a means to visualize areas under the Normal curve, but it can compute areas under the Normal curve or work backward (find quantiles).

1. Click an empty cell in the spreadsheet.
2. **Formulas ➔ More Functions ➔ Statistical**. Scroll down to find **NORM.DIST**.
3. To find the area to the left of the point of interest for any Normal distribution, enter the value, the mean (μ), the standard deviation (σ), and "1" for Cumulative.
4. **OK**

For a Standard Normal (Z) distribution, use **NORM.S. DIST**.

1. Enter the z-score, and "1" for Cumulative.
2. **OK**

To find quantiles, use **NORM.S.INV** or **NORM.INV**.

1. Enter the probability to the left of the point, the mean (μ), and the standard deviation (σ).
2. **OK**

Normal Quantile Plots

Neither Normal quantile plots nor Normal probability plots are available as a basic function in Excel. To create these plots, this author recommends watching the Excel videos. See the Excel Video Technology Manuals *Normal Calculations* and *Normal Quantile Plots*.

Normal Distribution Calculations

1. **Help ➔ Sample Data ➔ Teaching Scripts** (in Teaching Resources) **➔ Interactive Teaching Modules ➔ Distribution Calculator**
2. Enter a mean and standard deviation if not Standard Normal ($\mu = 0$, $\sigma = 1$).
3. Use the radio button to select the **Type of Calculation** desired: either input values to obtain a probability or enter a probability to obtain a value (the inverse calculation).
4. In the Calculations section, use the radio button to select the probability or percentile option of interest, and enter the value(s) of interest. The graph should update and the numerical probability will be displayed below the options. If this calculation does not update automatically, press the Enter or Tab key on your keyboard to see shading on the distribution.

Note: For JMP 11 and earlier versions, the Distribution Calculator (and many other interactive simulators) can be downloaded for free from jmp.com/tools.

Normal Quantile Plot

1. **Analyze ➔ Distribution**
2. Select the variable(s) of interest, and click **Y, Columns** to cast the variables into that role.
3. **OK**
4. Click the Red Triangle next to the variable name, then select **Normal Quantile Plot.**

Note: The option to produce a Normal quantile plot is available for only continuous variables, since such a plot makes sense for only continuous variables.

For videos to help with these topics, see the JMP Video Technology Manuals *Normal Calculations* and *Normal Quantile Plots*.

Minitab

Normal Distribution Calculations

1. **Graph ➔ Probability Distribution Plot**
2. **View Probability — OK**
3. Enter values for the mean and/or standard deviation.
4. **Shaded Area ➔** Click the appropriate picture.
5. Enter the specified value(s) in the *X* value box.
6. **OK**

For more precise values:

1. **Calc ➔ Probability Distributions ➔ Normal**
2. To find the *area to the left* of a specified value, select the **Cumulative probability** option. To find the *variable value associated with a specified area to the left* of that value, select the **Inverse cumulative probability** option.
3. Enter values for the mean and/or standard deviation.
4. Move the radio button to **Input constant**.
5. Enter the specified value of x or z or enter the specified area.
6. **OK**

Normal Quantile Plots

1. **Graph ➔ Probability Plot**
2. **Single** (for only one variable) **➔ OK**
3. Highlight and **Select** the data column of interest into the **Variable** box.
4. **OK**

For videos to help with these topics, see the Minitab Video Technology Manuals *Normal Calculations* and *Normal Quantile Plots*.

SPSS

Normal Distribution Calculations

SPSS does not have a probability lookup graphical interface. Cumulative probabilities can be found and inverse calculations performed using the Compute Variable function.

To find the probability below (to the left) of the given variable value, $P(X < k)$ or $P(X \leq k)$:

1. **Transform ➔ Compute Variable**
2. In "Target Variable," enter a name for the new variable (a new column with the name will be added to the spreadsheet).

3. In the Function Group area, select **CDF and Noncentral ➔ CDF.normal** (this will enter the formula shell in the editor). Enter the desired parameters into the formula (quantity to look up, mean, and standard deviation).
4. Click **OK.** You may have to go to the **Variable View** tab and change the number of decimal places displayed for this new variable.

To find a variable value (quantile) given a cumulative probability:

1. **Transform ➔ Compute Variable**
2. In "Target Variable," enter a name for the new variable (a new column with the name will be added to the spreadsheet).
3. In the Function Group area, select **Inverse DF ➔ IDF.Normal**.
4. Enter the parameters (area to the left, mean, and standard deviation).
5. **OK**

Normal Quantile Plots

1. **Analyze ➔ Descriptive Statistics ➔ Q-Q Plots**
2. Select the variable of interest on the left, then click the right arrow to move the variable to the Variables section.
3. **OK**

For videos to help with these topics, see the SPSS Video Technology Manuals *Normal Calculations* and *Normal Quantile Plots*.

CRUNCHIT!

Normal Distribution Calculations

1. **Distribution Calculator ➔ Normal**
2. Enter the mean and standard deviation (or accept the defaults for standard Normal).
3. Use **Probability** to find the probability given a variable value, or **Quantile** (percentile) to find a value of the variable given a cumulative probability.
4. Enter the appropriate value.
5. **Calculate**

Normal Quantile Plots

1. **Graphics ➔ QQ Plot**
2. For "Sample," select the column of interest.
3. **Calculate**

For videos to help with these topics, see the CrunchIt! Help Video on *Distribution Calculators*.

TI-83/-84

Normal Distribution Calculations

There are two ways to find Normal distribution probabilities: using normalcdf from the [2nd] [VARS] = [DISTR] menu, or using ShadeNorm from the DRAW menu in the same location.

For probabilities such as $P(X > k)$ or $P(X < k)$, the upper (lower) end of the area is + (−) infinity. For the calculator, $+\infty$ is [1] [2nd] [,] [9] [9] = 1E99, but practically speaking, any "very large" value will work.

To look up probabilities for a Normal distribution, use the following. If you are using the standard Normal distribution ($\mu = 0$, $\sigma = 1$), there is no need to enter the mean and standard deviation.

1. Press [2nd] [VARS] = [DISTR] ➔ **2:normalcdf(** and press [ENTER] (or just press [2]).
2. The syntax for 2:normalcdf(is **normalcdf(lowervalue,uppervalue,mu,sigma)**. To look up proportions from a Normal distribution with mean = μ and standard deviation = σ:
   A. To find the proportion of values below k, $P[X < k]$ or $P[X \leq k]$: **normalcdf(-1E99,k,μ, σ)**
   B. To find the proportion of values above k, $P[X > k]$ or $P[X \geq k]$: **normalcdf(k, 1E99, μ,σ)**
   C. To find the proportion of values between a and b, $P[a < X < b]$ or $P[a \leq X \leq b]$**: normalcdf(a,b, μ,σ)**

3. If using ShadeNorm:
   A. Use [2nd] [PRGM] = [DRAW]**, 1:ClrDraw** between calculations. Otherwise, the Normal curve will become increasingly filled in, and you will not be able to discern the area of interest.
   B. Size the window. An appropriate size is $\mu \pm 3\sigma$ for the x axis. The y sizing will depend on the distribution. For standard Normal distributions, Xmin = −3, Xmax = 3, Ymin = −0.1, and Ymax = 0.4 work well.
   C. [2nd] [DISTR] [▶]**, 2:ShadeNorm(**. The parameters are the same.

To find value in a standard Normal distribution with a particular proportion to the left (a quantile or percentile):

1. Press [2nd] [VARS] = [DISTR] ➔ **3:invNorm(** and press [ENTER] (or just press [3] on the DISTR menu).
2. Enter the proportion desired, a comma, the mean, and standard deviation and press [ENTER]. If you are using a standard Normal distribution, there is no need to enter the mean and standard deviation. For example, invNorm(.50) will return 0 (at $z = 0$, half of the distribution is to the left)

Normal Quantile Plots

1. Enter the data in a list.
2. Press [2nd] [Y=] = **STAT PLOT**, and select one of the three plots. Turn that plot **On** if needed; select the **Quantile** plot, ∠.
3. Enter L1 (as [2nd] [1]) or the list with your data as **Xlist**.
4. Select a data axis (your text uses the y axis) and a mark type for each data point.
5. Press [ZOOM] [9] [.]

For videos to help with these topics, see the TI Video Technology Manuals *Normal Calculations* and *Normal Quantile Plots*.

Normal Distribution Calculations

To find the cumulative probability, $P(X \leq x)$, use the command

```
> Pnorm(x,mu,sd)
```

To find value in a standard Normal distribution with a particular proportion to the left (a quantile):

```
> qnorm(Prob,mu,sd)
```

Normal Quantile Plots

The command is

```
> Qqnorm(mydat$xvar)
```

For videos to help with these topics, see the R Video Technology Manuals *Normal Calculations* and *Normal Quantile Plots*.