UNIT 6

Inference for Categorical Data: Proportions

Chapter 8

Estimating Proportions with Confidence

Bill Gozansky/Alamy

INTRODUCTION

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How long does a battery last on the newest iPhone, on average? What proportion of college undergraduates attended all of their classes last week? How much does the weight of a quarter-pound hamburger at a fast-food restaurant vary after cooking? These are the types of questions we would like to answer.

It wouldn't be practical to determine the lifetime of *every* iPhone battery, to ask *all* undergraduates about their attendance, or to weigh *every* burger after cooking. Instead, we choose a random sample of individuals (batteries, undergraduates, burgers) to represent the population and collect data from those individuals. From what we learned in Chapter 4, if we randomly select the sample, we should be able to generalize our results to the population of interest. However, we cannot be certain that our conclusions are correct—a different sample would likely yield a different estimate. Probability helps us account for the chance variation due to random selection or random assignment.

Chapter 8 begins the formal study of statistical inference—using information from a sample to draw conclusions about a population parameter such as p or μ. This is an important transition from Chapter 7, where you were given information about a population and asked questions about the distribution of a sample statistic, such as the sample proportion $\hat{p}$ or the sample mean $\bar{x}$.

The following activity gives you an idea of what lies ahead.

ACTIVITY The beads

Before class, your teacher prepared a large population of different-colored beads and put them into a container. In this activity, you and your team will create an interval of plausible values for $p =$ the true proportion of beads in the container that are a particular color (e.g., red).

1. As a class, discuss how to use the cup provided to select a simple random sample of beads from the container.
2. Have one student select an SRS of beads. Separate the beads into two groups: those that are red and those that are not red. Count the number of beads in each group.
3. Calculate $\hat{p} =$ the sample proportion of beads in the container that are red. Do you think this value is equal to the true proportion of red beads in the container? Explain your answer.
4. In teams of 3 or 4 students, determine an interval of plausible (believable) values for the true proportion p using the value of $\hat{p}$ from Step 3 and what you learned in Section 7.2 about the sampling distribution of a sample proportion.
5. Compare your results with those of the other teams in the class. Discuss any problems you encountered and how you dealt with them.

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In this chapter and the next, we will introduce the two most common types of formal statistical inference. Chapter 8 concerns *confidence intervals* for estimating the value of a parameter. Chapter 9 presents *significance tests*, which assess the evidence for a claim about a parameter. Both types of inference are based on the sampling distributions you studied in Chapter 7.

In this chapter, we start by presenting the idea of a confidence interval in a general way that applies to estimating any unknown parameter. In Section 8.2, we show how to estimate a population proportion using a confidence interval. Section 8.3 focuses on confidence intervals for a difference in proportions.

SECTION 8.1 Confidence Intervals: The Basics

LEARNING TARGETS *By the end of the section, you should be able to:*

- Identify an appropriate point estimator and calculate the value of a point estimate.
- Interpret a confidence interval in context.
- Determine the point estimate and margin of error from a confidence interval.
- Use a confidence interval to make a decision about the value of a parameter.
- Interpret a confidence level in context.
- Describe how the sample size and confidence level affect the margin of error.
- Explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval.

Mr. Buckley's class did "The beads" activity from the Introduction. In their sample of 251 beads, they selected 107 red beads and 144 other beads. If we had to give a single number to estimate p = the true proportion of beads in the container that are red, what would it be? Because the sample proportion $\hat{p}$ is an unbiased estimator of the population proportion p, we use the statistic $\hat{p}$ as a **point estimator** of the parameter p. The best guess for the value of p is $\hat{p} = 107/251 = 0.426$. This value is known as a **point estimate**.

A statistic is called a *point* estimate because it represents a single point on a number line.

DEFINITION Point estimator, Point estimate

A **point estimator** is a statistic that provides an estimate of a population parameter.

The value of that statistic from a sample is called a **point estimate**.

As we saw in Chapter 7, the ideal point estimator will have no bias and little variability. Here's an example involving some of the more common point estimators.

EXAMPLE

From batteries to smoking
Point estimators

TPopova/Getty Images

PROBLEM: Identify the point estimator you would use to estimate the parameter in each of the following settings and calculate the value of the point estimate.

(a) Quality control inspectors want to estimate the mean lifetime μ of the AA batteries produced each hour at a factory. They select a random sample of 50 batteries during each hour of production and then drain them under conditions that mimic normal use. Here are the lifetimes (in hours) of the batteries from one such sample:

16.73	15.60	16.31	17.57	16.14	17.28	16.67	17.28	17.27	17.50
15.59	17.54	16.46	15.63	16.82	17.16	16.62	16.71	16.69	17.98
15.99	15.64	17.20	17.24	16.68	16.55	17.48	15.58	17.61	15.98
15.46	16.50	16.19	16.36	17.80	16.61	16.99	16.93	16.01	16.46
17.54	17.41	16.91	16.60	16.78	15.75	17.31	16.50	16.72	17.55

(b) What proportion p of U.S. adults would classify themselves as vegan or vegetarian? A Pew Research Center report surveyed 1473 randomly selected U.S. adults. Of these, 124 said they were vegan or vegetarian.[1]

(c) The quality control inspectors in part (a) want to investigate the variability in battery lifetimes by estimating the population standard deviation σ.

SOLUTION:

(a) Use the sample mean $\bar{x}$ as a point estimator for the population mean μ. The point estimate is

$$\bar{x} = \frac{16.73 + 15.60 + \cdots + 17.55}{50} = 16.718 \text{ hours.}$$

(b) Use the sample proportion $\hat{p}$ as a point estimator for the population proportion p. The point estimate is

$$\hat{p} = \frac{124}{1473} = 0.084.$$

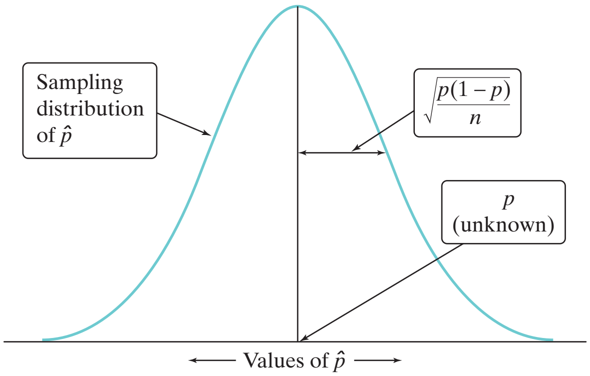
(c) Use the sample standard deviation s_x as a point estimator for the population standard deviation σ. The point estimate is $s_x = 0.664$ hour.

FOR PRACTICE, TRY EXERCISE 1

The Idea of a Confidence Interval

When Mr. Buckley's class did the beads activity, they obtained a sample proportion of $\hat{p} = 107/251 = 0.426$. To account for sampling variability, one team created an interval of plausible values by adding 0.062 to and subtracting 0.062 from $\hat{p} = 0.426$ to get an interval from 0.364 to 0.488. Where did the 0.062 come from? Their reasoning was based on the sampling distribution of the sample proportion from Section 7.2:

- Because the number of successes (107) and the number of failures (144) were both at least 10, the Large Counts condition is met. Therefore, the sampling distribution of $\hat{p}$ is approximately Normal.



- In about 95% of samples, the value of $\hat{p}$ will be within 2 standard deviations $\left(2\sigma_{\hat{p}} \approx 2\sqrt{\frac{0.426(1-0.426)}{251}} = 0.062\right)$ of the true proportion p.
- Therefore, in about 95% of samples, the value of the true proportion p will be within 2 standard deviations $\left(2\sigma_{\hat{p}} \approx 2\sqrt{\frac{0.426(1-0.426)}{251}} = 0.062\right)$ of $\hat{p}$.

When the estimate of a parameter is reported as an interval of values, it is called a **confidence interval**.

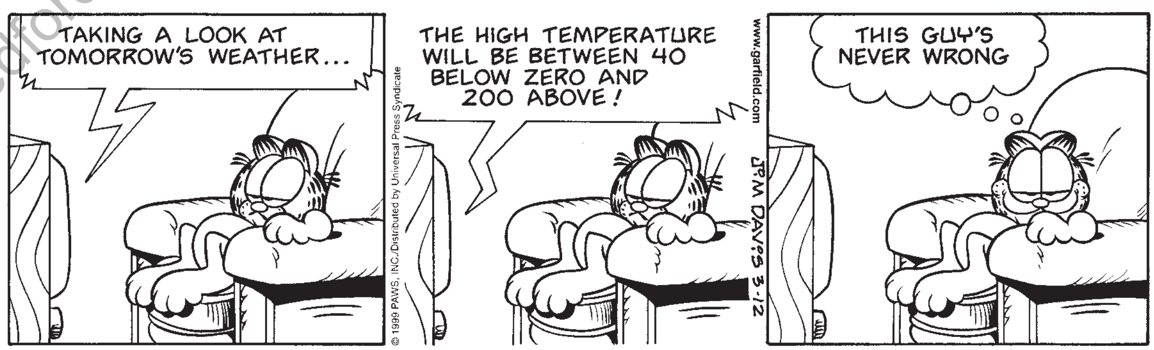
A confidence interval is called an *interval estimate* because it represents an interval of values on a number line, rather than a single point.

DEFINITION Confidence interval

A **confidence interval** gives an interval of plausible values for a parameter based on sample data.

Plausible does not mean the same thing as possible. You could argue that just about any value of a parameter is *possible*. *Plausible* means that we shouldn't be surprised if any one of the values in the interval is equal to the value of the parameter. Based on their calculations, the class shouldn't be surprised if Mr. Buckley revealed that the true proportion of red beads in the container is any value from 0.364 to 0.488. However, it would be surprising if the true proportion was less than 0.364 or greater than 0.488.

We use an interval of plausible values rather than a single point estimate to account for sampling variability and increase our confidence that we have a correct value for the parameter. Of course, as the cartoon illustrates, there is a trade-off between the amount of confidence we have that our estimate is correct and how much information the interval provides.



Confidence intervals are constructed so that we know *how much* confidence we should have in the interval. The most common **confidence level** is 95%. You will learn how to interpret confidence levels shortly.

DEFINITION Confidence level

The **confidence level** ***C*** gives the overall success rate of the method used to calculate the confidence interval. That is, the interval computed from the sample data will capture the true parameter value in *C*% of all possible samples when the conditions for inference are met.

The Associated Press and the NORC Center for Public Affairs Research recently asked a random sample of U.S. adults how much financial difficulty they would experience if they had to pay an unexpected bill of $1000 right away. Overall, 65% of respondents admitted they would have "a little" or "a lot" of difficulty. A summary of the study reported that the 95% confidence interval for the proportion of U.S. adults who would admit to experiencing some financial difficulty is 0.613 to 0.687. That is, they are 95% confident that the interval from 0.613 to 0.687 captures the true proportion of all U.S. adults who would admit to experiencing some financial difficulty paying an unexpected bill of $1000 right away.

Some people include the phrase "based on the sample" when interpreting a confidence interval: "Based on the sample, we are *C*% confident that the interval from ____________ to ____________ captures the [parameter in context]."

INTERPRETING A CONFIDENCE INTERVAL

To interpret a *C*% confidence interval for an unknown parameter, say, "We are *C*% confident that the interval from _____ to _____ captures the [parameter in context]."

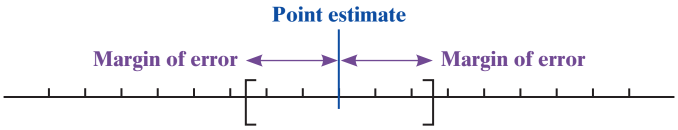
AP® EXAM TIP

When interpreting a confidence interval, make sure that you are describing the parameter and not the statistic. It's wrong to say that we are 95% confident the interval from 0.613 to 0.687 captures the proportion of U.S. adults who *admitted* they would experience financial difficulty. The "proportion who *admitted* they would experience financial difficulty" is the sample proportion, which is known to be 0.65. The interval gives plausible values for the proportion who *would admit* to experiencing some financial difficulty if asked.

To create an interval of plausible values for a parameter based on data from a sample, we need two components: a point estimate to use as the midpoint of the interval and a **margin of error** to account for sampling variability. The structure of a confidence interval is

$$\text{point estimate} \pm \text{margin of error}$$

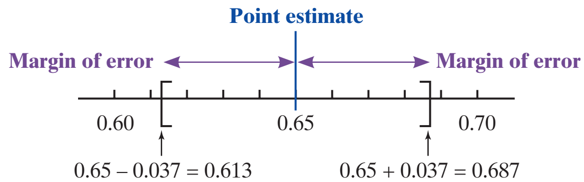
We can visualize a *C*% confidence interval like this:



***C*% confidence interval for parameter**

Earlier, we learned that the 95% confidence interval for the proportion of all U.S. adults who would admit to experiencing some financial difficulty paying an unexpected bill of $1000 right away is 0.613 to 0.687. This interval could also be expressed as

$$0.65 \pm 0.037$$



95% confidence interval for p = proportion of all U.S. adults who would admit to experiencing some financial difficulty

Confidence intervals reported in the media are often presented as a point estimate and a margin of error.

DEFINITION Margin of error

The **margin of error** of an estimate describes how far, at most, we expect the estimate to vary from the true population value. That is, in a C% confidence interval, the distance between the point estimate and the true parameter value will be less than the margin of error in C% of all samples.

In addition to estimating a parameter, we can also use confidence intervals to assess claims about a parameter, as in the following example.

EXAMPLE Who will win the election?
Interpreting a confidence interval

PROBLEM: Two weeks before a presidential election, a polling organization asked a random sample of registered voters the following question: "If the presidential election were held today, would you vote for Candidate A or Candidate B?" Based on this poll, the 95% confidence interval for the population proportion who favor Candidate A is (0.48, 0.54).

Burlingham/Shutterstock.com

(a) Interpret the confidence interval.

(b) What is the point estimate that was used to create the interval? What is the margin of error?

(c) Based on this poll, a political reporter claims that the majority of registered voters favor Candidate A. Use the confidence interval to evaluate this claim.

SOLUTION:

(a) *We are 95% confident that the interval from 0.48 to 0.54 captures the true proportion of all registered voters who favor Candidate A in the election.*

(b) point estimate $= \dfrac{0.48+0.54}{2} = 0.51$

margin of error $= 0.54 - 0.51 = 0.03$

The point estimate is the midpoint of the interval. The margin of error is the distance from the point estimate to the endpoints of the interval.

Another way to calculate the margin of error is to divide the width of the confidence interval by 2: $(0.54 - 0.48)/2 = 0.03$.

(c) *Because there are plausible values of p less than or equal to 0.50 in the confidence interval, the interval does not give convincing evidence that a majority (more than 50%) of registered voters favor Candidate A.*

Any value from 0.48 to 0.54 is a plausible value for the true proportion who favor Candidate A.

FOR PRACTICE, TRY EXERCISE 5

Interpreting Confidence Level

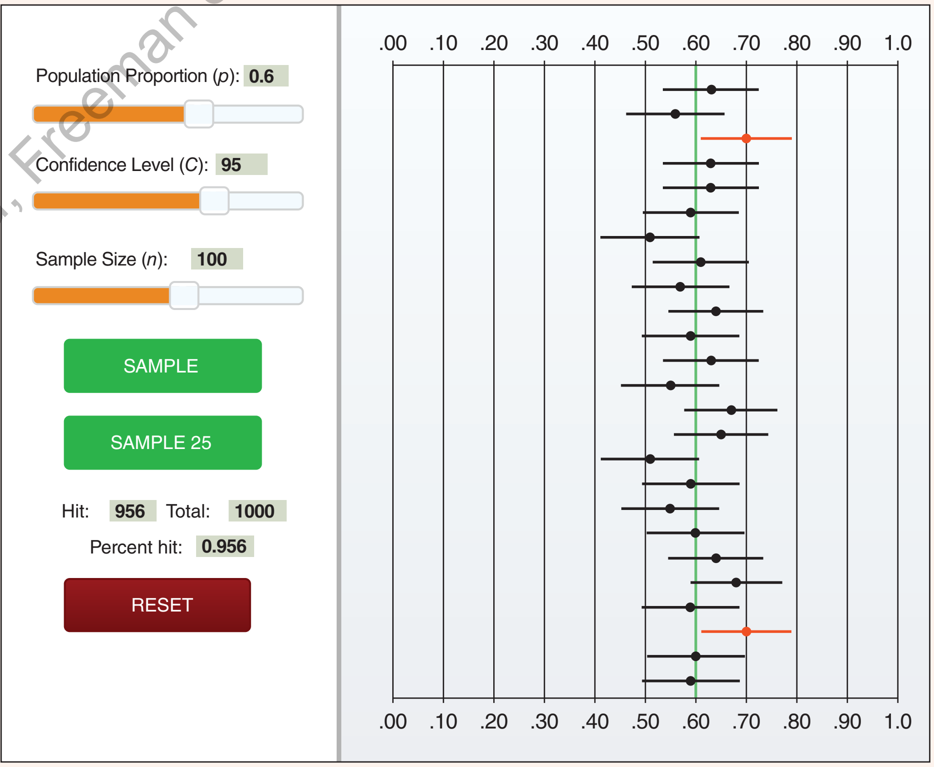
What does it mean to be 95% confident? The following activity gives you a chance to explore the meaning of the confidence level.

ACTIVITY The *Confidence Intervals for Proportions* applet

APPLET

In this activity, you will use the *Confidence Intervals for Proportions* applet to learn what it means to say that we are "95% confident" that our confidence interval captures the parameter value.

1. Go to highschool.bfwpub.com/updatedtps6e and launch the applet. Change the settings to: Population Proportion (p): 0.6, confidence level: 95, and sample size (n): 100. The display shows the values from 0.00 to 1.00, with a green line at $p = 0.60$ indicating the value of the true proportion.
2. Click "Sample" to choose an SRS of size $n = 100$ and display the resulting confidence interval. The confidence interval is shown as a horizontal line segment with a dot representing the sample proportion $\hat{p}$ in the middle of the interval.
3. Did the interval capture the population proportion p (what the applet calls a "hit")? Click "Sample" a total of 10 times. How many of the intervals captured the population proportion p? *Note*: So far, you have used the applet to take 10 SRSs, each of size $n = 100$. Be sure you understand the difference between sample size and the number of samples taken.
4. Reset the applet. Click "Sample 25" 40 times to choose 1000 SRSs and display the confidence intervals based on those samples. What percent of the intervals captured the true proportion p?



5. Change the confidence level to 99%. The applet will automatically recalculate all 1000 confidence intervals using a 99% confidence level. What percent of the intervals capture the true proportion p?
6. Repeat Step 5 using a 90% confidence level.
7. Summarize what you have learned about the relationship between confidence level and capture rate (percent hit) after taking many samples.

We will investigate the effect of changing the sample size later.

As the activity confirms, *when the conditions are met and the method is used many times, the capture rate will be very close to the stated confidence level.*

INTERPRETING A CONFIDENCE LEVEL

To interpret a confidence level C, say, "If we were to select many random samples of the same size from the same population and construct a C% confidence interval using each sample, about C% of the intervals would capture the [parameter in context]."

Let's revisit the presidential election poll to practice interpreting a confidence level.

EXAMPLE

Another look at the election poll
Interpreting a confidence level

PROBLEM: Two weeks before a presidential election, a polling organization asked a random sample of registered voters the following question: "If the presidential election were held today, would you vote for Candidate A or Candidate B?" Based on this poll, the 95% confidence interval for the population proportion who favor Candidate A is (0.48, 0.54). Interpret the confidence level.

SOLUTION:

If we were to select many random samples of the same size from the population of registered voters and construct a 95% confidence interval using each sample, about 95% of the intervals would capture the true proportion of all registered voters who favor Candidate A in the election.

Remember that interpretations of confidence level are about the method used to construct the interval—not one particular interval. In fact, we can interpret confidence levels before data are collected!

FOR PRACTICE, TRY EXERCISE 11

AP® EXAM TIP

On a given problem, you may be asked to interpret the confidence interval, the confidence level, or both. Be sure you understand the difference: the confidence interval gives a set of plausible values for the parameter and the confidence level describes the overall capture rate of the method.

In the preceding example, there are only two possibilities:

1. The interval from 0.48 to 0.54 captures the population proportion p. Our random sample was one of the many samples for which the difference between p and $\hat{p}$ is less than the margin of error. When using a 95% confidence level, about 95% of samples result in a confidence interval that captures p.
2. The interval from 0.48 to 0.54 does *not* capture the population proportion p. Our random sample was one of the few samples for which the difference between p and $\hat{p}$ is greater than the margin of error. When using a 95% confidence level, only about 5% of all samples result in a confidence interval that fails to capture p.

Without conducting a census, we cannot know whether our sample is one of the 95% for which the interval captures p or whether it is one of the unlucky 5% that does not. The statement that we are "95% confident" is shorthand for saying, "We got these numbers using a method that gives correct results for 95% of samples."

The confidence level does not tell us the probability that a particular confidence interval captures the population parameter. Once a particular confidence interval is calculated, its endpoints are fixed. And because the value of a parameter is also a constant, a particular confidence interval either includes the parameter (probability = 1) or doesn't include the parameter (probability = 0). As Figure 8.1 illustrates, no individual 95% confidence interval has a 95% probability of capturing the true parameter value.

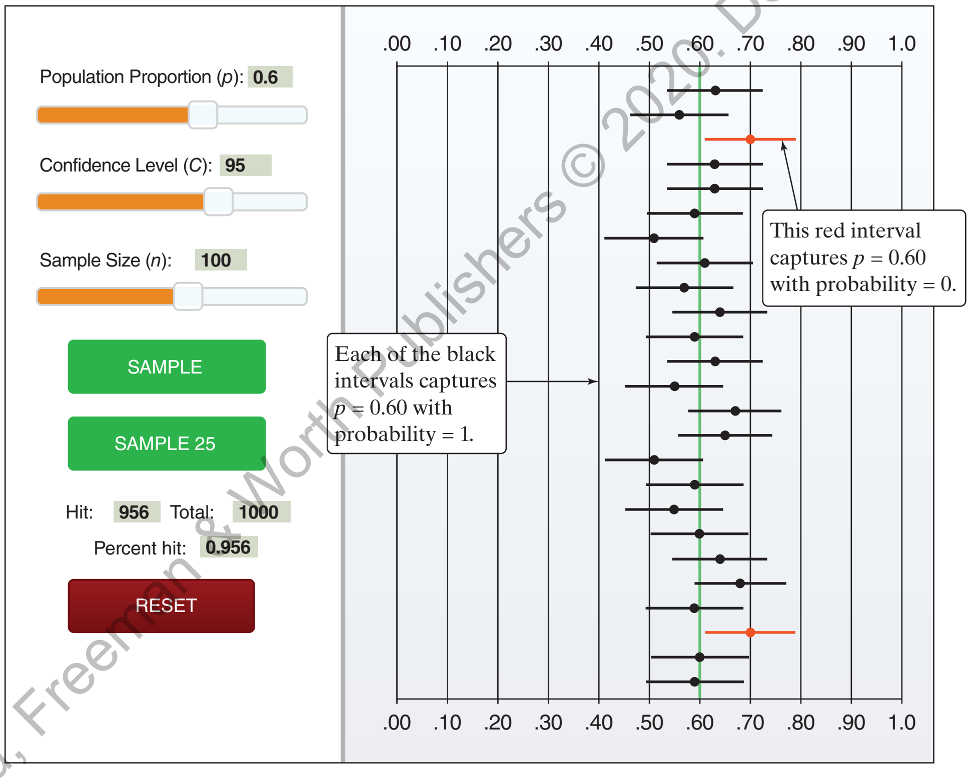


FIGURE 8.1 Image from the *Confidence Intervals for Proportions* applet showing that the probability a particular 95% confidence interval captures the true parameter value is either 0 or 1 (and not 0.95).

CHECK YOUR UNDERSTANDING

The Pew Research Center and *Smithsonian* magazine recently quizzed a random sample of 1006 U.S. adults on their knowledge of science.[2] One of the questions asked, "Which gas makes up most of the Earth's atmosphere: hydrogen, nitrogen, carbon dioxide, or oxygen?" A 95% confidence interval for the proportion who would correctly answer nitrogen is 0.175 to 0.225.

1. Interpret the confidence interval.
2. Interpret the confidence level.
3. Calculate the point estimate and the margin of error.
4. If people guess one of the four choices at random, about 25% should get the answer correct. Does this interval provide convincing evidence that less than 25% of all U.S. adults would answer this question correctly? Explain your reasoning.

What Affects the Margin of Error?

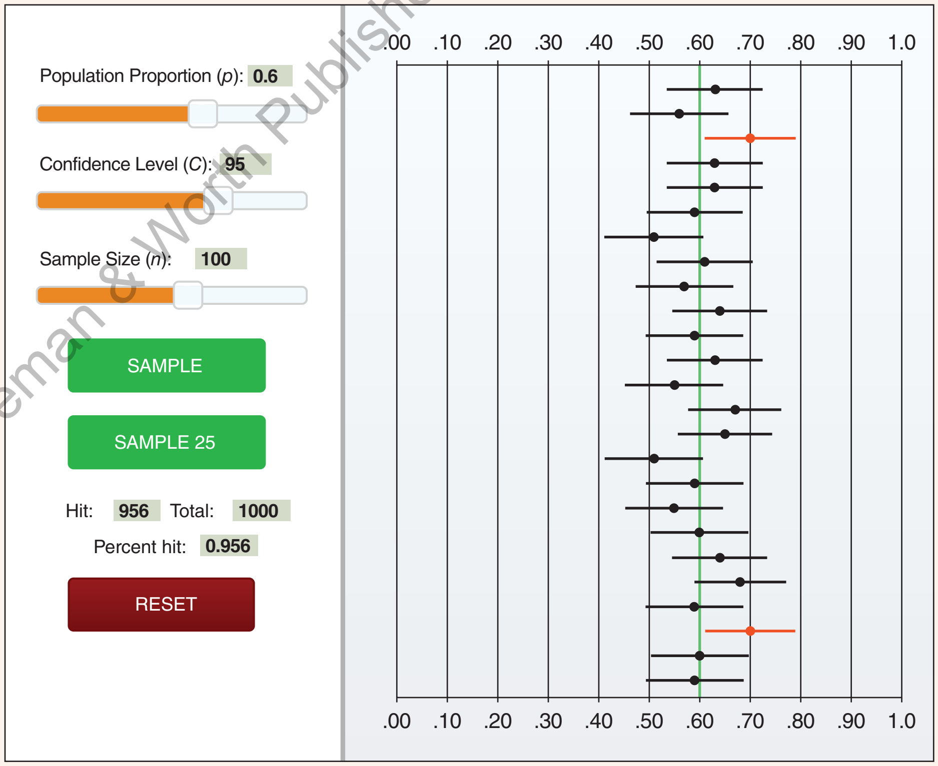
Why settle for 95% confidence when estimating an unknown parameter? Do larger random samples yield "better" intervals? The *Confidence Intervals for Proportions* applet will shed some light on these questions.

ACTIVITY Exploring margin of error with the *Confidence Intervals for Proportions* applet

In this activity, you will use the applet to explore the relationship between the confidence level, the sample size, and the margin of error.

Part 1: Adjusting the Confidence Level

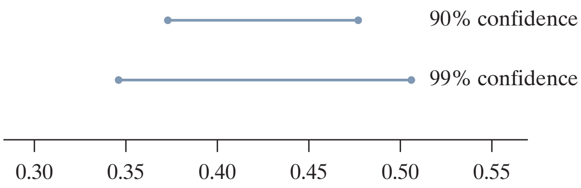
1. Go to highschool.bfwpub.com/updatedtps6e and launch the applet. Change the settings to: Population Proportion (p): 0.6, confidence level: 95, and sample size (n): 100. Click "Sample 25" 40 times to select 1000 SRSs and make 1000 confidence intervals.



2. Change the confidence level to 99%. What happens to the length of the confidence intervals? What happens to the capture rate (percent hit)? Drag the slider back and forth between 95% and 99% confidence to make sure you see what is happening.
3. Now change the confidence level to 90% and repeat Step 2.
4. Summarize what you learned about the relationship between the confidence level and the margin of error for a fixed sample size.

Part 2: Adjusting the Sample Size

5. Reset the applet settings to: Population Proportion (p): 0.6, confidence level: 95, and sample size (n): 100. Press "Sample 25" to select 25 SRSs of size $n = 100$ and make 25 confidence intervals.
6. Using the slider, increase the sample size to $n = 500$. Press "Sample 25" to select 25 SRSs of size $n = 500$ and make 25 confidence intervals. What do you notice about the length of the confidence intervals?
7. Using the slider, increase the sample size to $n = 1000$. Press "Sample 25" to select 25 SRSs of size $n = 1000$ and make 25 confidence intervals. What do you notice about the length of the confidence intervals?
8. Summarize what you learned about the relationship between the sample size and the margin of error for a fixed confidence level.
9. Does increasing the sample size increase the capture rate (percent hit)? Use the applet to investigate.



As the activity illustrates, the price we pay for greater confidence is a wider interval. If we're satisfied with 90% confidence, then our interval of plausible values for the parameter will be narrower than if we insist on 95% or 99% confidence. For example, here is a 90% confidence interval and a 99% confidence interval for the proportion of red beads in Mr. Buckley's container based on the class's sample data. Unfortunately, intervals constructed at a 90% confidence level will capture the true value of the parameter less often than intervals that use a 99% confidence level.

The activity also shows that we can get a more precise estimate of a parameter by increasing the sample size. Larger samples generally yield narrower confidence intervals at any confidence level. *In fact, the width of a confidence interval for a proportion or a mean is proportional to* $1/\sqrt{n}$*, so that quadrupling the sample size cuts the margin of error in half.* However, larger samples don't affect the capture rate and cost more time and money to obtain.

DECREASING THE MARGIN OF ERROR

In general, we prefer an estimate with a small margin of error. The margin of error gets smaller when:

- *The confidence level decreases.* To obtain a smaller margin of error from the same data, you must be willing to accept less confidence.
- *The sample size n increases.* In general, increasing the sample size n reduces the margin of error for any fixed confidence level.

To see why these facts are true, let's look a bit more closely at the method Mr. Buckley's class used to calculate a confidence interval for the true proportion of beads in the container that are red. They started with a point estimate of $\hat{p} = 107/251 = 0.426$. Then they added and subtracted 2 standard deviations to get the interval of plausible values from 0.364 to 0.488.

We could rewrite this interval as

$$\text{point estimate} \pm \text{margin of error}$$
$$\hat{p} \pm 2\sigma_{\hat{p}}$$
$$0.426 \pm 2\sqrt{\frac{0.426(1-0.426)}{251}}$$

This leads to the more general formula for a confidence interval:

$$\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})$$

The **critical value** depends on both the confidence level C and the sampling distribution of the statistic. Mr. Buckley's class used a critical value of 2 to be 95% confident. If they wanted to be 99.7% confident, they could have gone 3 standard deviations in each direction. Greater confidence requires a larger critical value.

DEFINITION Critical value

The **critical value** is a multiplier that makes the interval wide enough to have the stated capture rate.

The margin of error also depends on the standard deviation of the statistic. As you learned in Chapter 7, the sampling distribution of a statistic will have a smaller standard deviation when the sample size is larger. This is why the margin of error decreases as you increase the sample size.

WHAT THE MARGIN OF ERROR DOESN'T ACCOUNT FOR When we calculate a confidence interval, we include the margin of error because we expect the value of the point estimate to vary somewhat from the parameter. However, the margin of error accounts for *only* the variability we expect from random sampling. It does not account for practical difficulties, such as undercoverage and nonresponse in a sample survey. These problems can produce estimates that are much farther from the parameter than the margin of error would suggest. Remember this unpleasant fact when reading the results of an opinion poll or other sample survey. **The margin of error does *not* account for any sources of bias in the data collection process.**

caution !

EXAMPLE What's your GPA?

Factors that affect the margin of error

PROBLEM: As part of a project about response bias, Ellery surveyed a random sample of 25 students from her school. One of the questions in the survey required students to state their GPA aloud. Based on the responses, Ellery said she was 90% confident that the interval from 0.40 to 0.72 captures the proportion of all students at her school with GPAs greater than 3.0.[3]

OJO Images Ltd/Alamy

(a) Explain what would happen to the width of the interval if the confidence level were increased to 99%.

(b) How would the width of a 90% confidence interval based on a sample of size 100 compare to the original 90% interval, assuming the sample proportion remained the same?

(c) Describe one potential source of bias in Ellery's study that is not accounted for by the margin of error.

SOLUTION:

(a) *The confidence interval would be wider because increasing the confidence level increases the margin of error.*

To increase the confidence level (capture rate), we need to use a larger critical value, which increases the margin of error.

(b) *The confidence interval would be half as wide because the sample size is 4 times as big.*

Increasing the sample size decreases the standard deviation of the sampling distribution of the sample proportion (assuming the sample proportion doesn't change).

(c) *The margin of error doesn't account for the fact that many students might lie about their GPAs when having to respond without anonymity. The proportion of students with GPAs greater than 3.0 might be even less than 0.40!*

FOR PRACTICE, TRY EXERCISE 19

Section 8.1 Summary

- To estimate an unknown population parameter, start with a statistic that will provide a reasonable guess. The chosen statistic is a **point estimator** for the parameter. The specific value of the point estimator that we use gives a **point estimate** for the parameter.
- A **confidence interval** gives an interval of plausible values for an unknown population parameter based on sample data. The interval estimate has the form

$$\text{point estimate} \pm \text{margin of error}$$

  When calculating a confidence interval, it is common to use the form

$$\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})$$

- To interpret a $C\%$ confidence interval, say, "We are $C\%$ confident that the interval from ____ to ____ captures the [parameter in context]." Be sure that your interpretation describes a parameter and not a statistic.
- The **confidence level C** is the success rate (capture rate) of the method that produces the interval. If you use 95% confidence intervals often, about 95% of your intervals will capture the true parameter value when certain conditions are met. You don't know whether a particular 95% confidence interval calculated from a set of data actually captures the true parameter value.
- Other things being equal, the **margin of error** of a confidence interval gets smaller as:
  - the confidence level C decreases;
  - the sample size n increases.
- Remember that the margin of error for a confidence interval only accounts for chance variation, not other sources of error like nonresponse and undercoverage.

Section 8.1 Exercises

In Exercises 1–4, identify the point estimator you would use to estimate the parameter and calculate the value of the point estimate.

1. **Got shoes?** How many pairs of shoes, on average, do female teens have? To find out, an AP® Statistics class selected an SRS of 20 female students from their school. Then they recorded the number of pairs of shoes that each student reported having. Here are the data:

pg 538

50	26	26	31	57	19	24	22	23	38
13	50	13	34	23	30	49	13	15	51

2. **Got shoes?** The class in Exercise 1 wants to estimate the variability in the number of pairs of shoes that female students have by estimating the population standard deviation σ.

3. **Going to the prom** Tonya wants to estimate the proportion of seniors in her school who plan to attend the prom. She interviews an SRS of 50 of the 750 seniors in her school and finds that 36 plan to go to the prom.

4. **Reporting cheating** What proportion of students are willing to report cheating by other students? A student project put this question to an SRS of 172 undergraduates at a large university: "You witness two students cheating on a quiz. Do you go to the professor?" Only 19 answered "Yes."[4]

5. **Prayer in school** A *New York Times*/CBS News Poll asked a random sample of U.S. adults the question "Do you favor an amendment to the Constitution that would permit organized prayer in public schools?" Based on this poll, the 95% confidence interval for the population proportion who favor such an amendment is (0.63, 0.69).

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(a) Interpret the confidence interval.

(b) What is the point estimate that was used to create the interval? What is the margin of error?

(c) Based on this poll, a reporter claims that more than two-thirds of U.S. adults favor such an amendment. Use the confidence interval to evaluate this claim.

6. **Losing weight** A Gallup poll asked a random sample of U.S. adults, "Would you like to lose weight?" Based on this poll, the 95% confidence interval for the population proportion who want to lose weight is (0.56, 0.62).[5]

(a) Interpret the confidence interval.

(b) What is the point estimate that was used to create the interval? What is the margin of error?

(c) Based on this poll, Gallup claims that more than half of U.S. adults want to lose weight. Use the confidence interval to evaluate this claim.

7. **Bottling cola** A particular type of diet cola advertises that each can contains 12 ounces of the beverage. Each hour, a supervisor selects 10 cans at random, measures their contents, and computes a 95% confidence interval for the true mean volume. For one particular hour, the 95% confidence interval is 11.97 ounces to 12.05 ounces.

(a) Does the confidence interval provide convincing evidence that the true mean volume is different than 12 ounces? Explain your answer.

(b) Does the confidence interval provide convincing evidence that the true mean volume is 12 ounces? Explain your answer.

8. **Fun size candy** A candy bar manufacturer sells a "fun size" version that is advertised to weigh 17 grams. A hungry teacher selected a random sample of 44 fun size bars and found a 95% confidence interval for the true mean weight to be 16.945 grams to 17.395 grams.

(a) Does the confidence interval provide convincing evidence that the true mean weight is different than 17 grams? Explain your answer.

(b) Does the confidence interval provide convincing evidence that the true mean weight is 17 grams? Explain your answer.

9. **Shoes** The AP® Statistics class in Exercise 1 also asked an SRS of 20 boys at their school how many pairs of shoes they have. A 95% confidence interval for $\mu_G - \mu_B =$ the true difference in the mean number of pairs of shoes for girls and boys is 10.9 to 26.5.

(a) Interpret the confidence interval.

(b) Does the confidence interval give convincing evidence of a difference in the true mean number of pairs of shoes for boys and girls at the school? Explain your answer.

10. **Lying online** Many teens have posted profiles on sites such as Facebook. A sample survey asked random samples of teens with online profiles if they included false information in their profiles. Of 170 younger teens (ages 12 to 14) polled, 117 said "Yes." Of 317 older teens (ages 15 to 17) polled, 152 said "Yes."[6] A 95% confidence interval for $p_Y - p_O =$ the true difference in the proportions of younger teens and older teens who

include false information in their profile is 0.120 to 0.297.

(a) Interpret the confidence interval.

(b) Does the confidence interval give convincing evidence of a difference in the true proportions of younger and older teens who include false information in their profiles? Explain your answer.

11. **More prayer in school** Refer to Exercise 5. Interpret the confidence level.
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12. **More weight loss** Refer to Exercise 6. Interpret the confidence level.

13. **Household income** The 2015 American Community Survey estimated the median household income for each state. According to ACS, the 90% confidence interval for the 2015 median household income in Arizona is \$51,492 ± \$431.

(a) Interpret the confidence interval.

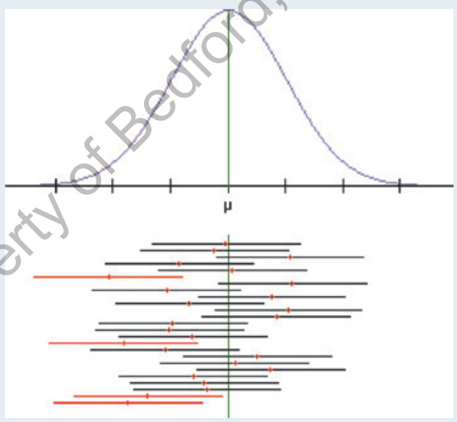
(b) Interpret the confidence level.

14. **More income** The 2015 American Community Survey estimated the median household income for each state. According to ACS, the 90% confidence interval for the 2015 median household income in New Jersey is \$72,222 ± \$610.

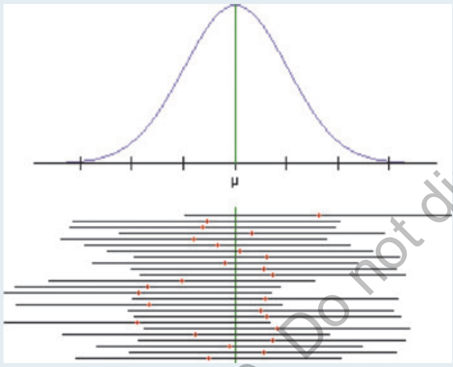
(a) Interpret the confidence interval.

(b) Interpret the confidence level.

15. **How confident?** The figure shows the result of taking 25 SRSs from a Normal population and constructing a confidence interval for the population mean using each sample. Which confidence level—80%, 90%, 95%, or 99%—do you think was used? Explain your reasoning.



16. **How confident?** The figure shows the result of taking 25 SRSs from a Normal population and constructing a confidence interval for the population mean using each sample. Which confidence level—80%, 90%, 95%, or 99%—do you think was used? Explain your reasoning.



17. **Explaining confidence** A 95% confidence interval for the mean body mass index (BMI) of young American women is 26.8 ± 0.6. Discuss whether each of the following explanations is correct, based on that information.

(a) We are confident that 95% of all young women have BMI between 26.2 and 27.4.

(b) We are 95% confident that future samples of young women will have mean BMI between 26.2 and 27.4.

(c) Any value from 26.2 to 27.4 is believable as the true mean BMI of young American women.

(d) If we take many samples, the population mean BMI will be between 26.2 and 27.4 in about 95% of those samples.

(e) The mean BMI of young American women cannot be 28.

18. **Explaining confidence** The admissions director for a university found that (107.8, 116.2) is a 95% confidence interval for the mean IQ score of all freshmen. Discuss whether each of the following explanations is correct, based on that information.

(a) There is a 95% probability that the interval from 107.8 to 116.2 contains μ.

(b) There is a 95% chance that the interval (107.8, 116.2) contains $\bar{x}$.

(c) This interval was constructed using a method that produces intervals that capture the true mean in 95% of all possible samples.

(d) If we take many samples, about 95% of them will contain the interval (107.8, 116.2).

(e) The probability that the interval (107.8, 116.2) captures μ is either 0 or 1, but we don't know which.

19. **Prayer in school** Refer to Exercise 5.

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(a) Explain what would happen to the length of the interval if the confidence level were increased to 99%.

(b) How would the width of a 95% confidence interval based on a sample size 4 times as large compare to the original 95% interval, assuming the sample proportion remained the same?

(c) The news article goes on to say: "The theoretical errors do not take into account additional errors resulting from the various practical difficulties in taking any survey of public opinion." List some of the "practical difficulties" that may cause errors which are not included in the ±3 percentage point margin of error.

20. **Losing weight** Refer to Exercise 6.

(a) Explain what would happen to the length of the interval if the confidence level was decreased to 90%.

(b) How would the width of a 95% confidence interval based on a sample size 4 times as large compare to the original 95% interval, assuming the sample proportion remained the same?

(c) As Gallup indicates, the 3 percentage point margin of error for this poll includes only sampling variability (what they call "sampling error"). What other potential sources of error (Gallup calls these "nonsampling errors") could affect the accuracy of the 95% confidence interval?

21. **California's traffic** People love living in California for many reasons, but traffic isn't one of them. Based on a random sample of 572 employed California adults, a 90% confidence interval for the average travel time to work for all employed California adults is 23 minutes to 26 minutes.[7]

(a) Interpret the confidence level.

(b) Name two things you could do to reduce the margin of error. What drawbacks do these actions have?

(c) Describe how nonresponse might lead to bias in this survey. Does the stated margin of error account for this possible bias?

22. **Employment in California** Each month the government releases unemployment statistics. The stated unemployment rate doesn't include people who choose not to be employed, such as retirees. Based on a random sample of 1000 California adults, a 99% confidence interval for the proportion of all California adults employed in the workforce is 0.532 to 0.612.[8]

(a) Interpret the confidence level.

(b) Name two things you could do to reduce the margin of error. What drawbacks do these actions have?

(c) Describe how untruthful answers might lead to bias in this survey. Does the stated margin of error account for this possible bias?

Multiple Choice: *Select the best answer for Exercises 23–26.*

Exercises 23 and 24 refer to the following setting. A researcher plans to use a random sample of houses to estimate the mean size (in square feet) of the houses in a large population.

23. The researcher is deciding between a 95% confidence level and a 99% confidence level. Compared with a 95% confidence interval, a 99% confidence interval will be

(a) narrower and would involve a larger risk of being incorrect.

(b) wider and would involve a smaller risk of being incorrect.

(c) narrower and would involve a smaller risk of being incorrect.

(d) wider and would involve a larger risk of being incorrect.

(e) wider and would have the same risk of being incorrect.

24. After deciding on a 95% confidence level, the researcher is deciding between a sample of size $n = 500$ and a sample of size $n = 1000$. Compared with using a sample size of $n = 500$, a confidence interval based on a sample size of $n = 1000$ will be

(a) narrower and would involve a larger risk of being incorrect.

(b) wider and would involve a smaller risk of being incorrect.

(c) narrower and would involve a smaller risk of being incorrect.

(d) wider and would involve a larger risk of being incorrect.

(e) narrower and would have the same risk of being incorrect.

25. In a poll conducted by phone,

I. Some people refused to answer questions.

II. People without telephones could not be in the sample.

III. Some people never answered the phone in several calls.

Which of these possible sources of bias is included in the ±2% margin of error announced for the poll?

(a) I only
(b) II only
(c) III only
(d) I, II, and III
(e) None of these

26. You have measured the systolic blood pressure of an SRS of 25 company employees. Based on the sample, a 95% confidence interval for the mean systolic blood pressure for the employees of this company is (122, 138). Which of the following statements is true?

(a) 95% of the sample of employees have a systolic blood pressure between 122 and 138.

(b) 95% of the population of employees have a systolic blood pressure between 122 and 138.

(c) If the procedure were repeated many times, 95% of the resulting confidence intervals would contain the population mean systolic blood pressure.

(d) If the procedure were repeated many times, 95% of the time the population mean systolic blood pressure would be between 122 and 138.

(e) If the procedure were repeated many times, 95% of the time the sample mean systolic blood pressure would be between 122 and 138.

Recycle and Review

27. **Power lines and cancer** (4.2, 4.3) Does living near power lines cause leukemia in children? The National Cancer Institute spent 5 years and $5 million gathering data on this question. The researchers compared 638 children who had leukemia with 620 who did not. They went into the homes and measured the magnetic fields in children's bedrooms, in other rooms, and at the front door. They recorded facts about power lines near the family home and also near the mother's residence when she was pregnant. *Result:* No association between leukemia and exposure to magnetic fields of the kind produced by power lines was found.[9]

(a) Was this an observational study or an experiment? Justify your answer.

(b) Does this study prove that living near power lines doesn't cause cancer? Explain your answer.

28. **Sisters and brothers** (3.1, 3.2) How strongly do physical characteristics of sisters and brothers correlate? Here are data on the heights (in inches) of 11 adult pairs:[10]

Brother	71	68	66	67	70	71	70	73	72	65	66
Sister	69	64	65	63	65	62	65	64	66	59	62

(a) Construct a scatterplot using brother's height as the explanatory variable. Describe what you see.

(b) Use technology to compute the least-squares regression line for predicting sister's height from brother's height.

(c) Interpret the slope in context.

(d) Calculate and interpret the residual for the first pair listed in the table.

SECTION 8.2 Estimating a Population Proportion

LEARNING TARGETS *By the end of the section, you should be able to:*

- State and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.
- Determine the critical value for calculating a C% confidence interval for a population proportion using a table or technology.
- Construct and interpret a confidence interval for a population proportion.
- Determine the sample size required to obtain a C% confidence interval for a population proportion with a specified margin of error.

In Section 8.1, we saw that a confidence interval can be used to estimate an unknown population parameter. We are often interested in estimating the proportion p of some outcome in a population. Here are some examples:

- What proportion of U.S. adults are unemployed right now?
- What proportion of high school students have cheated on a test?

- What proportion of pine trees in a national park are infested with beetles?
- What proportion of college students pray daily?
- What proportion of a company's laptop batteries last as long as the company claims?

This section shows you how to construct and interpret a confidence interval for a population proportion.

Constructing a Confidence Interval for p

When Mr. Buckley's class did "The beads" activity in Section 8.1, the random sample of 251 beads they selected included 107 red beads and 144 other beads. Starting with the general formula for a confidence interval from Section 8.1:

$$\begin{aligned} &\text{point estimate} \pm \text{margin of error} \\ = &\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic}) \end{aligned}$$

they determined the values to substitute into the formula using what they learned about the sampling distribution of the sample proportion $\hat{p}$ in Section 7.2.

- *Statistic:* The class decided to use $\hat{p} = 107/251 = 0.426$ because $\hat{p}$ is an unbiased estimator of p.
- *Critical value:* The class decided to use critical value $= 2$ based on the empirical rule for Normal distributions.
- *Standard deviation of statistic:* Remembering that the standard deviation of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$, the class decided to use $\hat{p} = 0.426$ in the formula to get $\sqrt{\dfrac{0.426(1-0.426)}{251}} = 0.031$.

The class's 95% confidence interval is

$$\begin{aligned} &0.426 \pm 2(0.031) \\ = &0.426 \pm 0.062 = (0.364, 0.488) \end{aligned}$$

The class is 95% confident that the interval from 0.364 to 0.488 captures the true proportion of red beads in Mr. Buckley's container.

The interval constructed by Mr. Buckley's class is nearly correct. Here is the exact formula for a *one-sample z interval for a population proportion.*

ONE-SAMPLE *z* INTERVAL FOR A POPULATION PROPORTION

When the conditions are met, a $C\%$ confidence interval for the unknown proportion p is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where z^* is the critical value for the standard Normal curve with $C\%$ of its area between $-z^*$ and z^*.

CONDITIONS FOR ESTIMATING p To make sure the formula for a one-sample z interval for a population proportion is valid, we need to verify that the observations in the sample can be viewed as independent and that the sampling distribution of $\hat{p}$ is approximately Normal. We do this by checking three conditions. Let's discuss them one at a time.

1. **The Random Condition** When our data come from a random sample, we can make an inference about the population from which the sample was selected. If the data come from a convenience sample or voluntary response sample, we should have no confidence that the resulting value of $\hat{p}$ is a good estimate of p. To be sure that $\hat{p}$ is a valid point estimate, we check the *Random condition*: The data come from a random sample from the population of interest.

Random sampling also helps ensure that individual observations in the sample can be viewed as independent. Finally, random sampling introduces chance into the data-production process. We can model random behavior with a probability distribution, like the sampling distributions of Chapter 7. Our method of calculation assumes that the data come from an SRS of size n from the population of interest. Other types of random samples (e.g., stratified or cluster) might be preferable to an SRS in a given setting, but they require more complex calculations than the ones we'll use. When an example, exercise, or AP® Statistics exam item refers to a "random sample" without saying "stratified," "cluster," or "systematic," you can assume the sample is an SRS.

2. **The 10% Condition** As you learned in Chapter 7, the formula for the standard deviation of the sampling distribution of $\hat{p}$ assumes that individual observations are independent. However, when we're sampling without replacement from a (finite) population, the observations are not independent because knowing the outcome of one trial helps us predict the outcome of future trials. Whenever we are sampling without replacement—which is nearly always—we need to check the *10% condition:* $n < 0.10N$, where n is the sample size and N is the population size.

When the 10% condition is met, the standard deviation of the sampling distribution of $\hat{p}$ is approximately

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

In practice, of course, we don't know the value of p. If we did, we wouldn't need to construct a confidence interval for it! In large random samples, $\hat{p}$ will be close to p. So we replace p in the formula for the standard deviation of the sample proportion with $\hat{p}$ to get the **standard error** (SE) of the sample proportion $\hat{p}$:

$$\text{SE}_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The formula sheet provided on the AP® Statistics exam uses the notation $s_{\hat{p}}$ rather than $\text{SE}_{\hat{p}}$ for the standard error of the sample proportion $\hat{p}$.

Like the standard deviation, the standard error describes how much the sample proportion $\hat{p}$ typically varies from the population proportion p in repeated SRSs of size n.

DEFINITION **Standard error**

When the standard deviation of a statistic is estimated from data, the result is called the **standard error** of the statistic.

3. **The Large Counts Condition** When Mr. Buckley's class used the empirical rule to determine the critical value for their confidence interval, they were assuming that the sampling distribution of $\hat{p}$ was approximately Normal. If the distribution of $\hat{p}$ is approximately Normal, then $\hat{p}$ will be within 2 standard deviations of p in about 95% of samples. This means the value of p will be within 2 standard deviations of $\hat{p}$ in about 95% of samples. Thus, using a critical value of 2 will result in approximately 95% confidence.

From what we learned in Chapter 7, we can use the Normal approximation to the sampling distribution of $\hat{p}$ as long as $np \geq 10$ and $n(1-p) \geq 10$. Like the standard error, we replace p with $\hat{p}$ when checking the *Large Counts condition*: $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$.

When the Large Counts condition is met, we can use a Normal distribution to calculate the critical value z^* for any confidence level. You'll learn how to do this soon.

Here is a summary of the three conditions for constructing a confidence interval for p.

CONDITIONS FOR CONSTRUCTING A CONFIDENCE INTERVAL ABOUT A PROPORTION

- **Random:** The data come from a random sample from the population of interest.
  - **10%:** When sampling without replacement, $n < 0.10N$.
- **Large Counts:** Both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

Let's verify that the conditions were met for the interval calculated by Mr. Buckley's class.

EXAMPLE The beads
Checking conditions


Studioshots/Alamy

PROBLEM: Mr. Buckley's class wants to construct a confidence interval for p = the true proportion of red beads in the container, which includes 3000 beads. Recall that the class's sample of 251 beads had 107 red beads and 144 other beads. Check if the conditions for constructing a confidence interval for p are met.

SOLUTION:

- Random: The class took a random sample of 251 beads from the container. ✓
  - 10%: 251 beads is less than 10% of 3000. ✓
- Large Counts:

$$n\hat{p} = 251\left(\frac{107}{251}\right) = 107 \geq 10 \quad \text{and}$$

$$n(1-\hat{p}) = 251\left(1-\frac{107}{251}\right) = 251\left(\frac{144}{251}\right) = 144 \geq 10 \checkmark$$

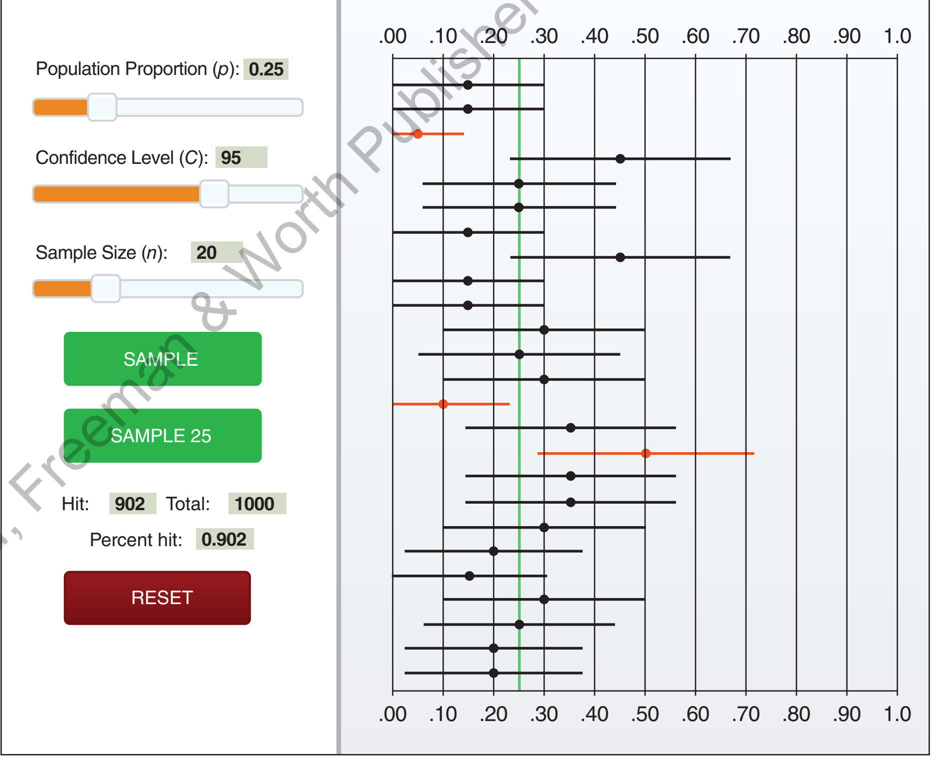
FOR PRACTICE, TRY EXERCISE 29

Notice that $n\hat{p}$ and $n(1-\hat{p})$ are the number of successes and failures in the sample. In the preceding example, we could address the Large Counts condition simply by saying, "The numbers of successes (107) and failures (144) in the sample are both at least 10."

Simulation studies have shown that a variation of our method for calculating a 95% confidence interval for p can result in closer to a 95% capture rate in the long run, especially for small sample sizes. This simple adjustment, first suggested by Edwin Bidwell Wilson in 1927, is sometimes called the *plus four* estimate. Just pretend we have four additional observations, two of which are successes and two of which are failures. Then calculate the "plus four interval" using the plus four estimate in place of $\hat{p}$ and sample size $n+4$ in our usual formula.

WHAT HAPPENS IF ONE OF THE CONDITIONS IS VIOLATED? If the data come from a voluntary response or convenience sample, there's no point in constructing a confidence interval for p. Violation of the Random condition severely limits our ability to make any inference beyond the data at hand.

The figure shows a screen shot from the *Confidence Intervals for Proportions* applet at the book's website, highschool.bfwpub.com/updatedtps6e. We set $n = 20$ and $p = 0.25$. The Large Counts condition is not met because $np = 20(0.25) = 5$ is not at least 10. We used the applet to generate 1000 random samples and construct 1000 95% confidence intervals for p. Only 902 of those 1000 intervals contained $p = 0.25$, a capture rate of 90.2%. When the Large Counts condition is violated, the capture rate will almost always be *less* than the one advertised by the confidence level when the method is used many times.



Violating the 10% condition means that we are sampling a large fraction of the population, which should be a good thing! But, as you learned in Section 7.2, the formula we use for the standard deviation of $\hat{p}$ gives a value that is too large when the 10% condition is violated. Confidence intervals based on this formula are wider than they need to be. If many 95% confidence intervals for a population proportion are constructed in this way, more than 95% of them will capture p.

The actual capture rate is almost always *greater* than the reported confidence level when the 10% condition is violated.

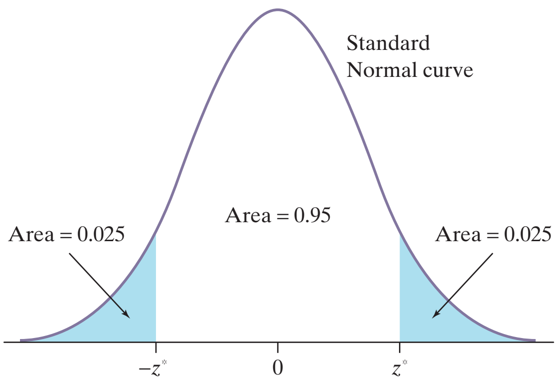


FIGURE 8.2 Finding the critical value z^* for a 95% confidence interval starts by labeling the middle 95% under a standard Normal curve and calculating the area in each tail.

CALCULATING CRITICAL VALUES How do we get the critical value z^* for our confidence interval? If the Large Counts condition is met, we can use the standard Normal curve. For their 95% confidence interval in the beads activity, Mr. Buckley's class used a critical value of 2. Based on the empirical rule for Normal distributions, they figured that $\hat{p}$ will be within 2 standard deviations of p in about 95% of all samples. Thus, p should be within 2 standard deviations of $\hat{p}$ in about 95% of all samples

We can get a more precise critical value from Table A or a calculator. As Figure 8.2 shows, the central 95% of the standard Normal distribution is marked off by 2 points, z^* and $-z^*$. We use the * to remind you that this is a critical value, not a standardized score that has been calculated from data.

Because of the symmetry of the Normal curve, the area in each tail is $0.05/2 = 0.025$. Once you know the tail areas, there are two ways to calculate the value of z^*:

- *Using Table A:* Search the body of Table A to find the point $-z^*$ with area 0.025 to its left. The entry $z = -1.96$ is what we are looking for, so $z^* = 1.96$.

z	.05	.06	.07
−2.0	.0202	.0197	.0192
−1.9	.0256	.0250	.0244
−1.8	.0322	.0314	.0307

- *Using technology:* The command invNorm(area:0.025, mean:0, SD:1) gives $z = -1.960$, so $z^* = 1.960$.

Now we can officially calculate a 95% confidence interval using the data from Mr. Buckley's class:

$$\hat{p} \pm z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.426 \pm 1.96\sqrt{\frac{0.426(1-0.426)}{251}}$$
$$= 0.426 \pm 0.061$$
$$= (0.365, 0.487)$$

Notice that the margin of error is slightly smaller for this interval than when the class used 2 for the critical value. Mr. Buckley's class is 95% confident that the interval from 0.365 to 0.487 captures the true proportion of red beads in his container. They can also be 95% confident that the interval from $3000(0.365) = 1095$ to $3000(0.487) = 1461$ captures the true *number* of red beads in his container of 3000 beads.

To find a level C confidence interval, we need to catch the central C% under the standard Normal curve. Here's an example that shows how to get the critical value z^* for a different confidence level and use it to calculate a confidence interval.

EXAMPLE

Read any good books lately?

Calculating a critical value and confidence interval

PROBLEM: According to a 2016 Pew Research Center report, 73% of American adults have read a book in the previous 12 months. This estimate was based on a random sample of 1520 American adults.[11] Assume the conditions for inference are met.

(a) Determine the critical value z^* for a 90% confidence interval for a proportion.

(b) Construct a 90% confidence interval for the proportion of all American adults who have read a book in the previous 12 months.

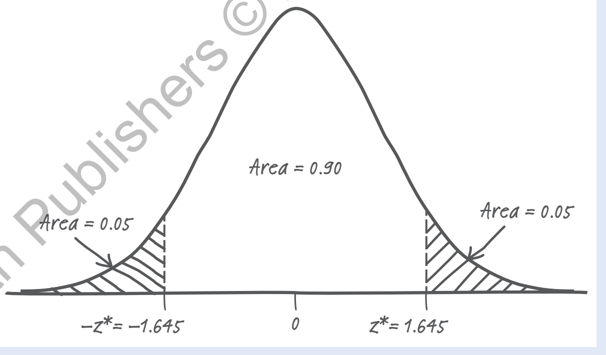
(c) Interpret the interval from part (b).

Lisa Solonynko/Alamy

SOLUTION:

(a) *Using Table A:* $z^* = 1.64$ or $z^* = 1.65$

Using technology: The command invNorm (area:0.05, mean:0, SD:1) gives $z = -1.645$, so $z^* = 1.645$.



(b) $0.73 \pm 1.645\sqrt{\dfrac{0.73(1-0.73)}{1520}}$

$= 0.73 \pm 0.019$

$= (0.711, 0.749)$

(c) We are 90% confident that the interval from 0.711 to 0.749 captures p = the true proportion of American adults who have read a book in the previous 12 months.

FOR PRACTICE, TRY EXERCISE 35

There are about 250 million U.S. adults. How *many* of them have read a book in the previous year? Using the confidence interval from the preceding example, we can be 90% confident that the interval from 250(0.711) = 177.75 million to 250(0.749) = 187.25 million captures the true number of U.S. adults who have read a book in the previous year.

CHECK YOUR UNDERSTANDING

Sleep Awareness Week begins in the spring with the release of the National Sleep Foundation's annual poll of U.S. sleep habits and ends with the beginning of daylight savings time, when most people lose an hour of sleep.[12] In the foundation's random sample of 1029 U.S. adults, 48% reported that they "often or always" got enough sleep during the past 7 nights.

1. Identify the parameter of interest.
2. Check if the conditions for constructing a confidence interval for p are met.
3. Find the critical value for a 99% confidence interval. Then calculate the interval.
4. Interpret the interval in context.

Putting It All Together: The Four-Step Process

Taken together, the examples about Mr. Buckley's class and "The Beads" activity show you how to get a confidence interval for an unknown population proportion p. Because there are many details to remember when constructing and interpreting a confidence interval, it is helpful to group them into four steps.

CONFIDENCE INTERVALS: A FOUR-STEP PROCESS

State: State the parameter you want to estimate and the confidence level.

Plan: Identify the appropriate inference method and check conditions.

Do: If the conditions are met, perform calculations.

Conclude: Interpret your interval in the context of the problem.

The next example illustrates the four-step process in action.

EXAMPLE

Distracted walking

Constructing and interpreting a confidence interval for p

mimagephotography/Shutterstock.com

PROBLEM: A recent poll of 738 randomly selected cell-phone users found that 170 of the respondents admitted to walking into something or someone while talking on their cell phone.[13] Construct and interpret a 95% confidence interval for the proportion of all cell-phone users who would admit to walking into something or someone while talking on their cell phone.

SOLUTION:

STATE: 95% CI for p = the true proportion of all cell-phone users who would admit to walking into something or someone while talking on their cell phone.

> **STATE:** State the parameter you want to estimate and the confidence level.

PLAN: One-sample z interval for p.

> **PLAN:** Identify the appropriate inference method and check conditions.

- Random: Random sample of 738 cell-phone users. ✓
  - 10%: It is reasonable to assume that 738 is less than 10% of all cell-phone users. ✓
- Large Counts: The number of successes (170) and the number of failures (738 − 170 = 568) are both at least 10. ✓

> Remember that $n\hat{p}$ is the number of successes and $n(1-\hat{p})$ is the number of failures in the sample:
>
> $$n\hat{p} = 738\left(\frac{170}{738}\right) = 170$$
>
> $$n(1-\hat{p}) = 738\left(\frac{568}{738}\right) = 568$$

DO: $\hat{p} = 170/738 = 0.230$

$$0.230 \pm 1.96\sqrt{\frac{0.230(1-0.230)}{738}}$$

$= 0.230 \pm 0.030$

$= (0.200, 0.260)$

CONCLUDE: We are 95% confident that the interval from 0.200 to 0.260 captures p = the true proportion of all cell-phone users who would admit to walking into something or someone while talking on their cell phone.

DO: If the conditions are met, perform calculations.

CONCLUDE: Interpret your interval in the context of the problem.

Make sure your conclusion is about the population proportion (users who *would admit*) and not the sample proportion (those who *admitted*).

FOR PRACTICE, TRY EXERCISE 41

AP® EXAM TIP

If a free response question asks you to construct and interpret a confidence interval, you are expected to do the entire four-step process. That includes clearly defining the parameter, identifying the procedure, and checking conditions.

caution

Remember that the margin of error in a confidence interval only accounts for sampling variability! There are other sources of error that are not taken into account. As is the case with many surveys, we are forced to assume that respondents answer truthfully. If they don't, then we shouldn't be 95% confident that our interval captures the truth. Other problems like nonresponse and question wording can also affect the results of a survey. *Lesson:* Sampling beads is much easier than sampling people!

Your calculator will handle the "Do" part of the four-step process, as the following Technology Corner illustrates.

18. Technology Corner CONSTRUCTING A CONFIDENCE INTERVAL FOR A POPULATION PROPORTION

TI-Nspire and other technology instructions are on the book's website at highschool.bfwpub.com/updatedtps6e.

The TI-83/84 can be used to construct a confidence interval for an unknown population proportion. We'll demonstrate using the previous example. Of $n = 738$ cell-phone users surveyed, $X = 170$ admitted to walking into something or someone while talking on their cell phone. To construct a confidence interval:

- Press STAT, then choose TESTS and 1-PropZInt.

- When the 1-PropZInt screen appears, enter $X = 170$, $n = 738$, and confidence level = 0.95. *Note:* The value you enter for X is the number of successes (not the proportion of successes) and must be an integer.

- Highlight "Calculate" and press ENTER. The 95% confidence interval for p is reported, along with the sample proportion $\hat{p}$ and the sample size, as shown here.

AP® EXAM TIP

You may use your calculator to compute a confidence interval on the AP® Statistics exam. But there's a risk involved. If you just give the calculator answer with no work, you'll get either full credit for the "Do" step (if the interval is correct) or no credit (if it's wrong). If you opt for the calculator-only method, be sure to complete the other three steps, including identifying the procedure (e.g., one-sample *z* interval for *p*) and give the interval in the Do step (e.g., 0.19997 to 0.26073).

Choosing the Sample Size

In planning a study, we may want to choose a sample size that allows us to estimate a population proportion within a given margin of error. The formula for the margin of error (ME) in the confidence interval for p is

$$ME = z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

To calculate the sample size, substitute values for ME, z^*, and $\hat{p}$, and solve for n. Unfortunately, we won't know the value of $\hat{p}$ until *after* the study has been conducted. This means we have to guess the value of $\hat{p}$ when choosing n. Here are two ways to do this:

1. Use a guess for $\hat{p}$ based on a pilot (preliminary) study or past experience with similar studies.
2. Use $\hat{p} = 0.5$ as the guess. The margin of error ME is largest when $\hat{p} = 0.5$, so this guess yields an upper bound for the sample size that will result in a given margin of error. If we get any other $\hat{p}$ when we do our study, the margin of error will be smaller than planned.

Once you have a guess for $\hat{p}$, the formula for the margin of error can be solved to give the required sample size n.

SAMPLE SIZE FOR DESIRED MARGIN OF ERROR WHEN ESTIMATING *p*

To determine the sample size n that will yield a $C\%$ confidence interval for a population proportion p with a maximum margin of error ME, solve the following inequality for n:

$$z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

where $\hat{p}$ is a guessed value for the sample proportion. The margin of error will always be less than or equal to ME if you use $\hat{p} = 0.5$.

Here's an example that shows you how to determine the sample size.

EXAMPLE

Customer satisfaction
Determining sample size

PROBLEM: A company has received complaints about its customer service. The managers intend to hire a consultant to carry out a survey of customers. Before contacting the consultant, the company president wants some idea of the sample size that she will be required to pay for. One value of interest is the proportion p of customers who are satisfied with the company's customer service. She decides that she wants the estimate to be within 3 percentage points (0.03) at a 95% confidence level. How large a sample is needed?

wbritten/Getty Images

SOLUTION:

$$1.96\sqrt{\frac{0.5(1-0.5)}{n}} \le 0.03$$

We have no idea about the true proportion p of satisfied customers, so we use $\hat{p} = 0.5$ as our guess to be safe.

$$\sqrt{\frac{0.5(1-0.5)}{n}} \le \frac{0.03}{1.96}$$

Divide both sides by 1.96.

$$\frac{0.5(1-0.5)}{n} \le \left(\frac{0.03}{1.96}\right)^2$$

Square both sides.

$$0.5(1-0.5) \le n\left(\frac{0.03}{1.96}\right)^2$$

Multiply both sides by n.

$$\frac{0.5(1-0.5)}{\left(\frac{0.03}{1.96}\right)^2} \le n$$

Divide both sides by $\left(\frac{0.03}{1.96}\right)^2$.

$$n \ge 1067.111$$

Make sure to follow the inequality when rounding your answer.

The sample needs to include at least 1068 customers.

FOR PRACTICE, TRY EXERCISE 49

Why not round to the nearest whole number—in this case, 1067? Because a smaller sample size will result in a larger margin of error, possibly more than the desired 3 percentage points for the poll. In general, we round to the next highest integer when solving for sample size to make sure the margin of error is less than or equal to the desired value.

CHECK YOUR UNDERSTANDING

Refer to the preceding example about the company's customer satisfaction survey.

1. In the company's prior-year survey, 80% of customers surveyed said they were satisfied. Using this value as a guess for $\hat{p}$, find the sample size needed for a margin of error of at most 3 percentage points with 95% confidence. How does this compare with the required sample size from the example?
2. What if the company president demands 99% confidence instead of 95% confidence? Would this require a smaller or larger sample size, assuming everything else remains the same? Explain your answer.

Section 8.2 Summary

- When constructing a confidence interval for a population proportion p, we need to ensure that the observations in the sample can be viewed as independent and that the sampling distribution of $\hat{p}$ is approximately Normal. The required conditions are:
    - **Random:** The data come from a random sample from the population of interest.
        - **10%:** When sampling without replacement, $n < 0.10N$.
    - **Large Counts:** Both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10. That is, the number of successes and the number of failures in the sample are both at least 10.
- When the conditions are met, the $C\%$ **one-sample z interval for p is**

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

  where z^* is the **critical value** for the standard Normal curve with $C\%$ of its area between $-z^*$ and z^*.
- When we use the value of $\hat{p}$ to estimate the standard deviation of the sampling distribution of $\hat{p}$, the result is the **standard error** of $\hat{p}$: $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- When asked to construct and interpret a confidence interval, follow the four-step process:

4 STEP

STATE: State the parameter you want to estimate and the confidence level.

PLAN: Identify the appropriate inference method and check conditions.

DO: If the conditions are met, perform calculations.

CONCLUDE: Interpret your interval in the context of the problem.

- The sample size needed to obtain a confidence interval with a maximum margin of error ME for a population proportion involves solving

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

  for n, where $\hat{p}$ is a guessed value for the sample proportion, and z^* is the critical value for the confidence level you want. Use $\hat{p} = 0.5$ if you don't have a good idea about the value of $\hat{p}$.

8.2 Technology Corner

TI-Nspire and other technology instructions are on the book's website at highschool.bfwpub.com/updatedtps6e.

18. Constructing a confidence interval for a population proportion Page 560

Section 8.2 Exercises

For Exercises 29 to 32, check whether each of the conditions is met for calculating a confidence interval for the population proportion p.

29. **Rating school food** Latoya wants to estimate the proportion of the seniors at her boarding school who like the cafeteria food. She interviews an SRS of 50 of the 175 seniors and finds that 14 think the cafeteria food is good.
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30. **High tuition costs** Glenn wonders what proportion of the students at his college believe that tuition is too high. He interviews an SRS of 50 of the 2400 students and finds 38 of those interviewed think tuition is too high.

31. **Salty chips** A quality control inspector takes a random sample of 25 bags of potato chips from the thousands of bags filled in an hour. Of the bags selected, 3 had too much salt.

32. **Whelks and mussels** The small round holes you often see in seashells were drilled by other sea creatures, who ate the former dwellers of the shells. Whelks often drill into mussels, but this behavior appears to be more or less common in different locations. Researchers collected whelk eggs from the coast of Oregon, raised the whelks in the laboratory, then put each whelk in a container with some delicious mussels. Only 9 of 98 whelks drilled into a mussel.[14]

33. **The 10% condition** When constructing a confidence interval for a population proportion, we check that the sample size is less than 10% of the population size.

(a) Why is it necessary to check this condition?

(b) What happens to the capture rate if this condition is violated?

34. **The Large Counts condition** When constructing a confidence interval for a population proportion, we check that both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

(a) Why is it necessary to check this condition?

(b) What happens to the capture rate if this condition is violated?

35. **Selling online** According to a recent Pew Research Center report, many American adults have made money by selling something online. In a random sample of 4579 American adults, 914 reported that they earned money by selling something online in the previous year.[15] Assume the conditions for inference are met.
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(a) Determine the critical value z^* for a 98% confidence interval for a proportion.

(b) Construct a 98% confidence interval for the proportion of all American adults who would report having earned money by selling something online in the previous year.

(c) Interpret the interval from part (b).

36. **Reporting cheating** What proportion of students are willing to report cheating by other students? A student project put this question to an SRS of 172 undergraduates at a large university: "You witness two students cheating on a quiz. Do you go to the professor?" Only 19 answered "Yes."[16] Assume the conditions for inference are met.

(a) Determine the critical value z^* for a 96% confidence interval for a proportion.

(b) Construct a 96% confidence interval for the proportion of all undergraduates at this university who would go to the professor.

(c) Interpret the interval from part (b).

37. **More online sales** Refer to Exercise 35. Calculate and interpret the standard error of $\hat{p}$ for these data.

38. **More cheating** Refer to Exercise 36. Calculate and interpret the standard error of $\hat{p}$ for these data.

39. **Going to the prom** Tonya wants to estimate what proportion of her school's seniors plan to attend the prom. She interviews an SRS of 50 of the 750 seniors in her school and finds that 36 plan to go to the prom.

(a) Identify the population and parameter of interest.

(b) Check conditions for constructing a confidence interval for the parameter.

(c) Construct a 90% confidence interval for p.

(d) Interpret the interval in context.

40. **Student government** The student body president of a high school claims to know the names of at least 1000 of the 1800 students who attend the school. To test this claim, the student government advisor randomly selects 100 students and asks the president to identify each by name. The president successfully names only 46 of the students.

(a) Identify the population and parameter of interest.

(b) Check conditions for constructing a confidence interval for the parameter.

(c) Construct a 99% confidence interval for p.

(d) Interpret the interval in context.

41. **Video games** A Pew Research Center report on gamers and gaming estimated that 49% of U.S. adults play video games on a computer, TV, game console, or portable device such as a cell phone. This estimate was based on a random sample of 2001 U.S. adults. Construct and interpret a 95% confidence interval for the proportion of all U.S. adults who play video games.[17]
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42. **September 11** A recent study asked U.S. adults to name 10 historic events that occurred in their lifetime that have had the greatest impact on the country. The most frequently chosen answer was the September 11, 2001, terrorist attacks, which was included by 76% of the 2025 randomly selected U.S. adults. Construct and interpret a 95% confidence interval for the proportion of all U.S. adults who would include the 9/11 attacks on their list of 10 historic events.

43. **Age and video games** Refer to Exercise 41. The study also estimated that 67% of adults aged 18–29 play video games, but only 25% of adults aged 65 and older play video games.

(a) Explain why you do not have enough information to give confidence intervals for these two age groups separately.

(b) Do you think a 95% confidence interval for adults aged 18–29 would have a larger or smaller margin of error than the estimate from Exercise 41? Explain your answer.

44. **Age and September 11** Refer to Exercise 42. The study also reported that 86% of millennials included 9/11 in their top-10 list and 70% of baby boomers included 9/11.

(a) Explain why you do not have enough information to give confidence intervals for millennials and baby boomers separately.

(b) Do you think a 95% confidence interval for baby boomers would have a larger or smaller margin of error than the estimate from Exercise 42? Explain your answer.

45. **Food fight** A 2016 survey of 1480 randomly selected U.S. adults found that 55% of respondents agreed with the following statement: "Organic produce is better for health than conventionally grown produce."[18]

(a) Construct and interpret a 99% confidence interval for the proportion of all U.S. adults who think that organic produce is better for health than conventionally grown produce.

(b) Does the interval from part (a) provide convincing evidence that a majority of all U.S. adults think that organic produce is better for health? Explain your answer.

46. **Three branches** According to a recent study by the Annenberg Foundation, only 36% of adults in the United States could name all three branches of government. This was based on a survey given to a random sample of 1416 U.S. adults.[19]

(a) Construct and interpret a 90% confidence interval for the proportion of all U.S. adults who could name all three branches of government.

(b) Does the interval from part (a) provide convincing evidence that less than half of all U.S. adults could name all three branches of government? Explain your answer.

47. **Prom totals** Use your interval from Exercise 39 to construct and interpret a 90% confidence interval for the total number of seniors planning to go to the prom.

48. **Student body totals** Use your interval from Exercise 40 to construct and interpret a 99% confidence interval for the total number of students at the school that the student body president can identify by name. Then use your interval to evaluate the president's claim.

49. **School vouchers** A small pilot study estimated that 44% of all American adults agree that parents should be given vouchers that are good for education at any public or private school of their choice. pg 562

(a) How large a random sample is required to obtain a margin of error of at most 0.03 with 99% confidence? Answer this question using the pilot study's result as the guessed value for $\hat{p}$.

(b) Answer the question in part (a) again, but this time use the conservative guess $\hat{p} = 0.5$. By how much do the two sample sizes differ?

50. **Can you taste PTC?** PTC is a substance that has a strong bitter taste for some people and is tasteless for others. The ability to taste PTC is inherited. About 75% of Italians can taste PTC, for example. You want to estimate the proportion of Americans who have at least one Italian grandparent and who can taste PTC.

(a) How large a sample must you test to estimate the proportion of PTC tasters within 0.04 with 90% confidence? Answer this question using the 75% estimate as the guessed value for $\hat{p}$.

(b) Answer the question in part (a) again, but this time use the conservative guess $\hat{p} = 0.5$. By how much do the two sample sizes differ?

51. **Starting a nightclub** A college student organization wants to start a nightclub for students under the age of 21. To assess support for this proposal, they will select an SRS of students and ask each respondent if he or she would patronize this type of establishment. What sample size is required to obtain a 90% confidence interval with a margin of error of at most 0.04?

52. **Election polling** Gloria Chavez and Ronald Flynn are the candidates for mayor in a large city. We want to estimate the proportion p of all registered voters in the city who plan to vote for Chavez with 95% confidence and a margin of error no greater than 0.03. How large a random sample do we need?

53. **Teens and their TV sets** According to a Gallup Poll report, 64% of teens aged 13 to 17 have TVs in their rooms. Here is part of the footnote to this report:

These results are based on telephone interviews with a randomly selected national sample of 1028 teenagers in the Gallup Poll Panel of households, aged 13 to 17.

For results based on this sample, one can say . . . that the maximum error attributable to sampling and other random effects is ±3 percentage points. In addition to sampling error, question wording and practical difficulties in conducting surveys can introduce error or bias into the findings of public opinion polls.[20]

(a) We omitted the confidence level from the footnote. Use what you have learned to estimate the confidence level, assuming that Gallup took an SRS.

(b) Give an example of a "practical difficulty" that could lead to bias in this survey.

54. Gambling and the NCAA Gambling is an issue of great concern to those involved in college athletics. Because of this concern, the National Collegiate Athletic Association (NCAA) surveyed randomly selected student athletes concerning their gambling-related behaviors.[21] Of the 5594 Division I male athletes who responded to the survey, 3547 reported participation in some gambling behavior. This includes playing cards, betting on games of skill, buying lottery tickets, betting on sports, and similar activities. A report of this study cited a 1% margin of error.

(a) The confidence level was not stated in the report. Use what you have learned to estimate the confidence level, assuming that the NCAA took an SRS.

(b) The study was designed to protect the anonymity of the student athletes who responded. As a result, it was not possible to calculate the number of students who were asked to respond but did not. How does this fact affect the way that you interpret the results?

Multiple Choice: *Select the best answer for Exercises 55–58.*

55. A Gallup poll found that only 28% of American adults expect to inherit money or valuable possessions from a relative. The poll's margin of error was ±3 percentage points at a 95% confidence level. This means that

(a) the poll used a method that gets an answer within 3% of the truth about the population 95% of the time.

(b) the percent of all adults who expect an inheritance must be between 25% and 31%.

(c) if Gallup takes another poll on this issue, the results of the second poll will lie between 25% and 31%.

(d) there's a 95% chance that the percent of all adults who expect an inheritance is between 25% and 31%.

(e) Gallup can be 95% confident that between 25% and 31% of the sample expect an inheritance.

56. Refer to Exercise 55. Suppose that Gallup wanted to cut the margin of error in half from 3 percentage points to 1.5 percentage points. How should they adjust their sample size?

(a) Multiply the sample size by 4.

(b) Multiply the sample size by 2.

(c) Multiply the sample size by 1/2.

(d) Multiply the sample size by 1/4.

(e) There is not enough information to answer this question.

57. Most people can roll their tongues, but many can't. The ability to roll the tongue is genetically determined. Suppose we are interested in determining what proportion of students can roll their tongues. We test a simple random sample of 400 students and find that 317 can roll their tongues. The margin of error for a 95% confidence interval for the true proportion of tongue rollers among students is closest to which of the following?

(a) 0.0008
(b) 0.02
(c) 0.03
(d) 0.04
(e) 0.05

58. A newspaper reporter asked an SRS of 100 residents in a large city for their opinion about the mayor's job performance. Using the results from the sample, the C% confidence interval for the proportion of all residents in the city who approve of the mayor's job performance is 0.565 to 0.695. What is the value of C?

(a) 82
(b) 86
(c) 90
(d) 95
(e) 99

Review and Recycle

59. **Oranges** (6.1, 7.3) A home gardener likes to grow various kinds of citrus fruit. One of his mandarin orange trees produces oranges whose circumferences follow a Normal distribution with mean 21.1 cm and standard deviation 1.8 cm.

(a) What is the probability that a randomly selected orange from this tree has a circumference greater than 22 cm?

(b) What is the probability that a random sample of 20 oranges from this tree has a mean circumference greater than 22 cm?

60. More oranges (1.2, 2.2) The gardener in the previous exercise randomly selects 20 mandarin oranges from the tree and counts the number of seeds in each orange. Here are the data:

3	4	6	6	9	11	11	12	13	13	14	14	16	17	22	23	23	24	28	30

(a) Graph the data using a dotplot.

(b) Based on your graph, is it plausible that the number of seeds from oranges on this tree follows a distribution that is approximately Normal? Explain your answer.

SECTION 8.3 Estimating a Difference in Proportions

LEARNING TARGETS *By the end of the section, you should be able to:*

- Determine whether the conditions are met for constructing a confidence interval about a difference between two proportions.
- Construct and interpret a confidence interval for a difference between two proportions.

In Section 8.2, you learned how to calculate and interpret a confidence interval for a population proportion p. Many interesting statistical questions involve *comparing* the proportion of successes in two populations. What is the difference between the proportion of Democrats and the proportion of Republicans who favor the death penalty? How has the proportion of teenagers with a smartphone changed from 10 years ago? In both of these cases, we want to estimate the value of $p_1 - p_2$, where p_1 and p_2 are the proportions of success in Population 1 and Population 2.

Other statistical questions involve comparing the effectiveness of two treatments in an experiment. For example, how much more effective is a new medication for relieving headaches? What is the difference in the survival rate for two cancer treatments? In these cases, we want to estimate the value of $p_1 - p_2$, where p_1 and p_2 are the true proportions of success for individuals like the ones in the experiment who receive Treatment 1 or Treatment 2.

Confidence Intervals for $p_1 - p_2$

When data come from two independent random samples or two groups in a randomized experiment (the Random condition), the statistic $\hat{p}_1 - \hat{p}_2$ is our best guess for the value of $p_1 - p_2$. The method we use to calculate a confidence interval for $p_1 - p_2$ requires that the sampling distribution of $\hat{p}_1 - \hat{p}_2$ be approximately Normal. In Section 7.2, you learned that this will be true whenever n_1p_1, $n_1(1-p_1)$, n_2p_2, and $n_2(1-p_2)$ are all at least 10. Because we don't know the value of p_1 or p_2 when we are estimating their difference, we use $\hat{p}_1$ and $\hat{p}_2$ when checking the Large Counts condition.

CONDITIONS FOR CONSTRUCTING A CONFIDENCE INTERVAL ABOUT A DIFFERENCE IN PROPORTIONS

- **Random:** The data come from two independent random samples or from two groups in a randomized experiment.
    - **10%:** When sampling without replacement, $n_1 < 0.10N_1$ and $n_2 < 0.10N_2$.
- **Large Counts:** The counts of "successes" and "failures" in each sample or group—$n_1\hat{p}_1$, $n_1(1-\hat{p}_1)$, $n_2\hat{p}_2$, $n_2(1-\hat{p}_2)$—are all at least 10.

Recall from Chapter 4 that the Random condition is important for determining the scope of inference. Random sampling allows us to generalize our results to the populations of interest; random assignment in an experiment permits us to draw cause-and-effect conclusions.

EXAMPLE

Do you prefer brand names?
Checking conditions

Ariel Skelley/Getty Images

PROBLEM: A Harris Interactive survey asked independent random samples of adults from the United States and Germany about the importance of brand names when buying clothes. Of the 2309 U.S. adults surveyed, 26% said brand names were important, compared with 22% of the 1058 German adults surveyed. Let p_U = the true proportion of all U.S. adults who think brand names are important when buying clothes and p_G = the true proportion of all German adults who think brand names are important when buying clothes. Check if the conditions for calculating a confidence interval for $p_U - p_G$ are met.

SOLUTION:

- *Random:* Independent random samples of 2309 U.S. adults and 1058 German adults. ✓
  - 10%: 2309 < 10% of all U.S. adults and 1058 < 10% of all German adults ✓
- Large Counts? $n_U\hat{p}_U = 2309(0.26) = 600.34 \rightarrow 600$, $n_U(1-\hat{p}_U) = 2309(0.74) = 1708.66 \rightarrow 1709$, $n_G\hat{p}_G = 1058(0.22) = 232.76 \rightarrow 233$, $n_G(1-\hat{p}_G) = 1058(0.78) = 825.24 \rightarrow 825$ are all ≥ 10. ✓

Be sure to mention *independent* random samples from the populations of interest when checking the Random condition.

We round these values to the nearest whole number because they represent the actual numbers of successes and failures in the two samples.

FOR PRACTICE, TRY EXERCISE 61

If the conditions are met, we can use our familiar formula to calculate a confidence interval for $p_1 - p_2$:

$$\begin{aligned}&\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})\\ &= (\hat{p}_1 - \hat{p}_2) \pm z^* \cdot (\text{standard deviation of statistic})\end{aligned}$$

In Section 7.2, you also learned that the standard deviation of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

when we have two types of independence:

- Independent samples, so we can add the variances of $\hat{p}_1$ and $\hat{p}_2$. This is why we reminded you to mention *independent* random samples when checking the Random condition in the preceding example.
- Independent observations within each sample. When sampling without replacement, the actual value of the standard deviation is smaller than the formula suggests. However, if the 10% condition is met for both samples, the given formula is approximately correct.

Because we don't know the values of the parameters p_1 and p_2, we replace them in the standard deviation formula with the sample proportions. The result is the *standard error* of $\hat{p}_1 - \hat{p}_2$:

$$SE_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

The formula sheet provided on the AP® Statistics exam uses the notation $s_{\hat{p}_1-\hat{p}_2}$ rather than $SE_{\hat{p}_1-\hat{p}_2}$ for the standard error of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$.

This value estimates how much the difference in sample proportions will typically vary from the difference in the true proportions if we repeat the random sampling or random assignment many times.

When the Large Counts condition is met, we find the critical value z^* for the given confidence level using Table A or technology. Our confidence interval for $p_1 - p_2$ is therefore

$$\text{statistic} \pm (\text{critical value}) \cdot (\text{standard error of statistic})$$
$$= (\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

This is often called a *two-sample z interval for a difference between two proportions.*

TWO-SAMPLE *z* INTERVAL FOR A DIFFERENCE BETWEEN TWO PROPORTIONS

When the conditions are met, a C% confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

where z^* is the critical value for the standard Normal curve with C% of the area between $-z^*$ and z^*.

Let's return to the brand names example. Recall that Harris Interactive took independent random samples of 2309 U.S. adults and 1058 German adults, and found that $\hat{p}_U = 0.26$ and $\hat{p}_G = 0.22$. We already confirmed that the conditions are met. A 95% confidence interval for $p_U - p_G$ is

$$(0.26 - 0.22) \pm 1.96\sqrt{\frac{0.26(1-0.26)}{2309} + \frac{0.22(1-0.22)}{1058}}$$
$$= 0.04 \pm 0.03$$
$$= (0.01, 0.07)$$

Interpretation: We are 95% confident that the interval from 0.01 to 0.07 captures $p_U - p_G$ = the difference in the true proportions of all U.S. adults and all German adults who think brand names are important when buying clothes.

The interpretation of a confidence interval for a difference in proportions is the same as for a single proportion. Interpreting the confidence *level* is the same as well: If we were to repeat the sampling process many times and compute a 95% confidence interval each time, about 95% of those intervals would capture the difference in the true proportion of all U.S. adults and all German adults who think brand names are important when buying clothes.

Note that the confidence interval does not include 0 (no difference) as a plausible value for $p_U - p_G$, so we have convincing evidence of a difference between the population proportions. In fact, it is believable that $p_U - p_G$ has any value between 0.01 and 0.07. We can restate this in context as follows: The interval suggests that the importance of brand names when buying clothes for U.S. adults is between 1 and 7 percentage points higher than for German adults.

It would *not* be correct to say that the importance of brand names when buying clothes for U.S. adults is between 1 and 7 *percent* higher than for German adults. To see why, suppose that $p_U = 0.25$ and $p_G = 0.20$. The difference $p_U - p_G = 0.25 - 0.20 = 0.05$, or 5 percentage points. But the proportion of U.S. adults who think brand names are important when buying clothes is $0.05/0.20 = 0.25$, or 25% higher than the corresponding proportion of German adults.

The researchers in the preceding example selected independent random samples from the two populations they wanted to compare. In practice, it's common to take one random sample that includes individuals from both populations of interest and then to separate the chosen individuals into two groups. For instance, a polling company may randomly select 1000 U.S. adults, then separate the Republicans from the Democrats to estimate the difference in the proportion of all people in each party who favor the death penalty. The two-sample z procedures for comparing proportions are still valid in such situations, provided that the two groups can be viewed as independent samples from their respective populations of interest.

INFERENCE FOR EXPERIMENTS So far, we have focused on doing inference using data that were produced by random sampling. However, many important statistical results come from randomized comparative experiments. Fortunately, the formula for calculating a confidence interval for a difference between two proportions is the same whether there are two independent random samples or two randomly assigned groups in an experiment. However, there are differences in the way we define parameters and check conditions.

In an experiment to compare treatments for prostate cancer, 731 men with localized prostate cancer were randomly assigned either to have surgery or to be observed only. After 20 years, $\hat{p}_S = 141/364 = 0.387$ of the men assigned to surgery were still alive and $\hat{p}_O = 122/367 = 0.332$ of the men assigned to observation were still alive.[22] The parameters in this setting are:

p_S = the true proportion of men like the ones in the experiment who would survive 20 years when getting surgery

p_O = the true proportion of men like the ones in the experiment who would survive 20 years when only being observed

AP® EXAM TIP

Many students lose credit when defining parameters in an experiment by describing the sample proportion rather than the true proportion. For example, "the true proportion of the men who *had* surgery and survived 20 years" describes $\hat{p}_S$ not p_S.

Most experiments on people use recruited volunteers as subjects. When subjects are not randomly selected, researchers cannot generalize the results of an experiment to some larger populations of interest. But researchers can draw cause-and-effect conclusions that apply to people like those who took part in the experiment. This same logic applies to experiments on animals or things.

In addition to the difference in the Random condition, there is a change to the 10% condition when analyzing an experiment. Unless the experimental units are randomly selected without replacement from some population, we don't have to check it. Fortunately, there is no change to the Large Counts condition.

Putting It All Together: Two-Sample z Interval for $p_1 - p_2$

The following example shows how to construct and interpret a confidence interval for a difference in proportions. As usual with inference problems, we follow the four-step process.

EXAMPLE

Treating lower back pain
Confidence interval for $p_1 - p_2$

Syda Productions/Shutterstock

PROBLEM: Patients with lower back pain are often given nonsteroidal anti-inflammatory drugs (NSAIDs) like naproxen to help ease their pain. Researchers wondered if taking Valium along with the naproxen would affect pain relief. To find out, they recruited 112 patients with severe lower back pain and randomly assigned them to one of two treatments: naproxen and Valium or naproxen and placebo. After 1 week, 39 of the 57 subjects who took naproxen and Valium reported reduced lower back pain, compared with 43 of the 55 subjects in the naproxen and placebo group.[23]

(a) Construct and interpret a 99% confidence interval for the difference in the proportion of patients like these who would report reduced lower back pain after taking naproxen and Valium versus after taking naproxen and placebo for a week.

(b) Based on the confidence interval in part (a), what conclusion would you make about whether taking Valium along with naproxen affects pain relief? Justify your answer.

SOLUTION:

(a) STATE: 99% CI for $p_1 - p_2$, where p_1 = true proportion of patients like these who would report reduced lower back pain after taking naproxen and Valium for a week and p_2 = true proportion of patients like these who would report reduced lower back pain after taking naproxen and placebo for a week.

> Be sure to indicate the order of subtraction when defining the parameter.

PLAN: Two-sample *z* interval for $p_1 - p_2$

- Random: Randomly assigned patients to take naproxen and Valium or naproxen and placebo. ✓
- Large Counts: 39, 57 − 39 = 18, 43, and 55 − 43 = 12 are all ≥ 10. ✓

> We don't have to check the 10% condition because researchers did not sample patients without replacement from a larger population.

DO: $\hat{p}_1 = \dfrac{39}{57} = 0.684$, $\hat{p}_2 = \dfrac{43}{55} = 0.782$

> $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

$$(0.684 - 0.782) \pm 2.576\sqrt{\frac{0.684(0.316)}{57} + \frac{0.782(0.218)}{55}}$$

$$= -0.098 \pm 0.214$$

$$= (-0.312, 0.116)$$

> Refer to the Technology Corner that follows the example. The calculator's 2-PropZInt gives (−0.3114, 0.1162).

CONCLUDE: We are 99% confident that the interval from −0.312 to 0.116 captures $p_1 - p_2$ = the difference in the true proportions of patients like these who would report reduced pain after taking naproxen and Valium versus after taking naproxen and a placebo for a week.

> The interval suggests that the true proportion of patients like these who would report reduced pain after taking naproxen and Valium is between 31.2 percentage points lower and 11.6 percentage points higher than after taking naproxen and placebo.

(b) Because the interval includes 0 as a plausible value for $p_1 - p_2$, we don't have convincing evidence that taking Valium along with naproxen affects pain relief for patients like these.

FOR PRACTICE, TRY EXERCISE 65

We could have subtracted the proportions in the opposite order in part (a) of the example. The resulting 99% confidence interval for $p_2 - p_1$ is

$$(0.782 - 0.684) \pm 2.576\sqrt{\frac{0.782(0.218)}{55} + \frac{0.684(0.316)}{57}}$$
$$= 0.098 \pm 0.214$$
$$= (-0.116, 0.312)$$

Notice that the endpoints of the interval have the same values but opposite signs to the ones in the example. This interval suggests that the true proportion of patients like these who would report reduced pain after taking naproxen and placebo is between 11.6 percentage points lower and 31.2 percentage points higher than after taking naproxen and Valium. That's equivalent to our interpretation of the confidence interval for $p_1 - p_2$ in part (a) of the example.

The fact that 0 is included in a confidence interval for $p_1 - p_2$ means that we don't have convincing evidence of a difference between the true proportions. Keep in mind that 0 is just one of many plausible values for $p_1 - p_2$ based on the sample data. **Never suggest that you believe the difference between the true proportions is 0 just because 0 is in the interval!**

You can use technology to perform the calculations in the "Do" step. Remember that this comes with potential benefits and risks on the AP® Statistics exam.

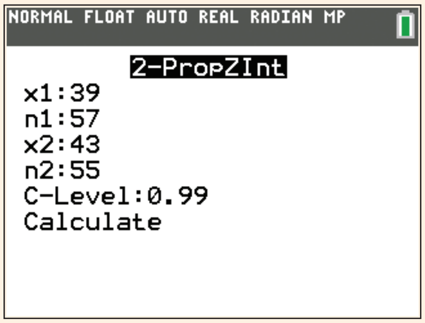
19. Technology Corner

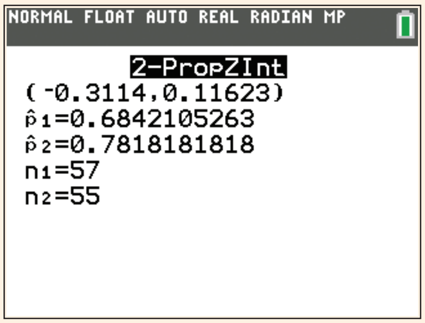
CONSTRUCTING A CONFIDENCE INTERVAL FOR A DIFFERENCE IN PROPORTIONS

TI-Nspire and other technology instructions are on the book's website at highschool.bfwpub.com/updatedtps6e.

The TI-83/84 can be used to construct a confidence interval for $p_1 - p_2$. We'll demonstrate using the preceding example. Of $n_1 = 57$ subjects who took naproxen and Valium, $X_1 = 39$ reported reduced lower back pain after a week. Of $n_2 = 55$ subjects who took naproxen and placebo, $X_2 = 43$ reported reduced lower back pain after a week. To construct a confidence interval:

- Press STAT, then choose TESTS and 2-PropZInt.
- When the 2-PropZInt screen appears, enter the values shown. Note that the values of x_1, n_1, x_2, and n_2 must be integers or you will get a domain error.
- Highlight "Calculate" and press ENTER.





AP® EXAM TIP

The formula for the two-sample z interval for $p_1 - p_2$ often leads to calculation errors by students. As a result, your teacher may recommend using the calculator's 2-PropZInt feature to compute the confidence interval on the AP® Statistics exam. Be sure to name the procedure (two-sample z interval for $p_1 - p_2$) in the "Plan" step and give the interval $(-0.311, 0.116)$ in the "Do" step.

CHECK YOUR UNDERSTANDING

A Pew Research Center poll asked independent random samples of working women and men how much they value job security. Of the 806 women, 709 said job security was very or extremely important, compared with 802 of the 944 men surveyed. Construct and interpret a 95% confidence interval for the difference in the proportion of all working women and men who consider job security very or extremely important.

Section 8.3 Summary

- Confidence intervals to estimate the difference between the proportions p_1 and p_2 of successes for two populations or treatments are based on the difference $\hat{p}_1 - \hat{p}_2$ between the sample proportions.
- When constructing a confidence interval for a difference in population proportions, we must check for independence in the data collection process and that the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal. The required conditions are:
  - **Random:** The data come from two independent random samples or from two groups in a randomized experiment.
    - **10%:** When sampling without replacement, $n_1 < 0.10N_1$ and $n_2 < 0.10N_2$.
  - **Large Counts:** The counts of "successes" and "failures" in each sample or group—$n_1\hat{p}_1$, $n_1(1-\hat{p}_1)$, $n_2\hat{p}_2$, $n_2(1-\hat{p}_2)$—are all at least 10.
- When conditions are met, a C% confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

  where z^* is the standard Normal critical value with C% of its area between $-z^*$ and z^*. This is called a **two-sample z interval for $p_1 - p_2$.**
- Be sure to follow the four-step process whenever you construct and interpret a confidence interval for the difference between two proportions.
- You can use a confidence interval for a difference in proportions to determine if a claimed value is plausible. For example, if 0 is included in a confidence interval for $p_1 - p_2$, it is plausible that there is no difference between the population proportions.

8.3 Technology Corner

TI-Nspire and other technology instructions are on the book's website at highschool.bfwpub.com/updatedtps6e.

19. Constructing a confidence interval for a difference in proportions Page 572

Section 8.3 Exercises

61. **Don't drink the water!** The movie *A Civil Action* (1998) tells the story of a major legal battle that took place in the small town of Woburn, Massachusetts. A town well that supplied water to east Woburn residents was contaminated by industrial chemicals. During the period that residents drank water from this well, 16 of 414 babies born had birth defects. On the west side of Woburn, 3 of 228 babies born during the same time period had birth defects. Let p_1 = the true proportion of all babies born with birth defects in west Woburn and p_2 = the true proportion of all babies born with birth defects in east Woburn. Check if the conditions for calculating a confidence interval for $p_1 - p_2$ are met. pg 568

62. **Broken crackers** We don't like to find broken crackers when we open the package. How can makers reduce breaking? One idea is to microwave the crackers for 30 seconds right after baking them. Randomly assign 65 newly baked crackers to the microwave and another 65 to a control group that is not microwaved. After 1 day, none of the microwave group were broken and 16 of the control group were broken.[24] Let p_1 = the true proportion of crackers like these that would break if baked in the microwave and p_2 = the true proportion of crackers like these that would break if not microwaved. Check if the conditions for calculating a confidence interval for $p_1 - p_2$ are met.

63. **Cockroaches** The pesticide diazinon is commonly used to treat infestations of the German cockroach, *Blattella germanica*. A study investigated the persistence of this pesticide on various types of surfaces. Researchers applied a 0.5% emulsion of diazinon to glass and plasterboard. After 14 days, they randomly assigned 72 cockroaches to two groups of 36, placed one group on each surface, and recorded the number that died within 48 hours. On glass, 18 cockroaches died, while on plasterboard, 25 died. If p_1 and p_2 are the true proportions of cockroaches like these that would die within 48 hours on glass treated with diazinon and on plasterboard treated with diazinon, respectively, check if the conditions for calculating a confidence interval for $p_1 - p_2$ are met.

64. **Digital video disks** A company that records and sells rewritable DVDs wants to compare the reliability of DVD fabricating machines produced by two different manufacturers. They randomly select 500 DVDs produced by each fabricator and find that 484 of the disks produced by the first machine are acceptable and 480 of the disks produced by the second machine are acceptable. If p_1 and p_2 are the proportions of acceptable DVDs produced by the first and second machines, respectively, check if the conditions for calculating a confidence interval for $p_1 - p_2$ are met.

65. **Young adults living at home** A surprising number of young adults (ages 19 to 25) still live in their parents' homes. The National Institutes of Health surveyed independent random samples of 2253 men and 2629 women in this age group.[25] The survey found that 986 of the men and 923 of the women lived with their parents. pg 571

(a) Construct and interpret a 99% confidence interval for the difference in the true proportions of men and women aged 19 to 25 who live in their parents' homes.

(b) Does your interval from part (a) give convincing evidence of a difference between the population proportions? Justify your answer.

66. **Where's Egypt?** In a Pew Research poll, 287 out of 522 randomly selected U.S. men were able to identify Egypt when it was highlighted on a map of the Middle East. When 520 randomly selected U.S. women were asked, 233 were able to do so.

(a) Construct and interpret a 95% confidence interval for the difference in the true proportions of U.S. men and U.S. women who can identify Egypt on a map.

(b) Based on your interval, is there convincing evidence of a difference in the true proportions of U.S. men and women who can identify Egypt on a map? Justify your answer.

67. **More young adults** Interpret the confidence level for the interval in Exercise 65.

68. **More about Egypt** Interpret the confidence level for the interval in Exercise 66.

69. **Response bias** Does the appearance of the interviewer influence how people respond to a survey question? Ken (white, with blond hair) and Hassan (darker, with Middle Eastern features) conducted an experiment to address this question. They took turns (in a random order) walking up to people on the main street of a small town, identifying themselves as students from a local high school, and asking them, "Do you support President Obama's decision to launch airstrikes in Iraq?" Of the 50 people Hassan spoke to, 11 said "Yes," while 21 of the 44 people Ken spoke to said "Yes." Construct and interpret a 90% confidence interval for the difference in the proportions of people like these who would say they support President Obama's decision when asked by Hassan versus when asked by Ken.

70. **Quit smoking** Nicotine patches are often used to help smokers quit. Does giving medicine to fight depression help? A randomized double-blind experiment assigned 244 smokers to receive nicotine patches and another 245 to receive both a patch and the antidepressant drug bupropion. After a year, 40 subjects in the nicotine patch group had abstained from smoking, as had 87 in the patch-plus-drug group. Construct and interpret a 99% confidence interval for the difference in the true proportion of smokers like these who would abstain when using bupropion and a nicotine patch and the proportion who would abstain when using only a patch.

71. **Ban junk food!** A CBS News poll asked 606 randomly selected women and 442 randomly selected men, "Do you think putting a special tax on junk food would encourage more people to lose weight?" 170 of the women and 102 of the men said "Yes."[26] A 99% confidence interval for the difference (Women – Men) in the true proportion of people in each population who would say "Yes" is −0.020 to 0.120.

(a) Does the confidence interval provide convincing evidence that the two population proportions are different? Explain your answer.

(b) Does the confidence interval provide convincing evidence that the two population proportions are equal? Explain your answer.

72. **Artificial trees?** An association of Christmas tree growers in Indiana wants to know if there is a difference in preference for natural trees between urban and rural households. So the association sponsored a survey of Indiana households that had a Christmas tree last year to find out. In a random sample of 160 rural households, 64 had a natural tree. In a separate random sample of 261 urban households, 89 had a natural tree. A 95% confidence interval for the difference (Rural – Urban) in the true proportion of households in each population that had a natural tree is −0.036 to 0.154.

(a) Does the confidence interval provide convincing evidence that the two population proportions are different? Explain your answer.

(b) Does the confidence interval provide convincing evidence that the two population proportions are equal? Explain your answer.

Multiple Choice: *Select the best answer for Exercises 73–75.*

73. Earlier in this section, you read about an experiment comparing surgery and observation as treatments for men with prostate cancer. After 20 years, $\hat{p}_S = 141/364 = 0.387$ of the men who were assigned to surgery were still alive and $\hat{p}_O = 122/367 = 0.332$ of the men who were assigned to observation were still alive. Which of the following is the 95% confidence interval for $p_S - p_O$?

(a) $(141-122) \pm 1.96\sqrt{\dfrac{141\cdot 223}{364}+\dfrac{122\cdot 245}{367}}$

(b) $(141-122) \pm 1.96\left(\sqrt{\dfrac{141\cdot 223}{364}}+\sqrt{\dfrac{122\cdot 245}{367}}\right)$

(c) $(0.387-0.332) \pm 1.96\sqrt{\dfrac{0.387\cdot 0.613}{364}+\dfrac{0.332\cdot 0.668}{367}}$

(d) $(0.387-0.332) \pm 1.96\left(\sqrt{\dfrac{0.387\cdot 0.613}{364}}+\sqrt{\dfrac{0.332\cdot 0.668}{367}}\right)$

(e) $(0.387-0.332) \pm 1.96\sqrt{\dfrac{0.387\cdot 0.613}{364}-\dfrac{0.332\cdot 0.668}{367}}$

74. When constructing a confidence interval for a difference between two population proportions, why is it important to check that the number of successes and the number of failures in each sample is at least 10?

(a) So we can generalize the results to the populations from which the samples were selected.

(b) So we can assume that the two samples are independent.

(c) So we can assume that the observations within each sample are independent.

(d) So we can assume the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal.

(e) So we can assume that population 1 and population 2 are approximately Normal.

75. To estimate the difference in the proportion of students at high school A and high school B who drive themselves to school, a district administrator selected a random sample of 100 students from each school. At school A, 23 of the students said they drive themselves; at school B, 29 of the students said they drive themselves. A 90% confidence interval for $p_A - p_B$ is -0.16 to 0.04. Based on this interval, which conclusion is best?

(a) Because -0.06 is in the interval, there is convincing evidence of a difference in the population proportions.

(b) Because 0 is in the interval, there is convincing evidence of a difference in the population proportions.

(c) Because -0.06 is in the interval, there is not convincing evidence of a difference in the population proportions.

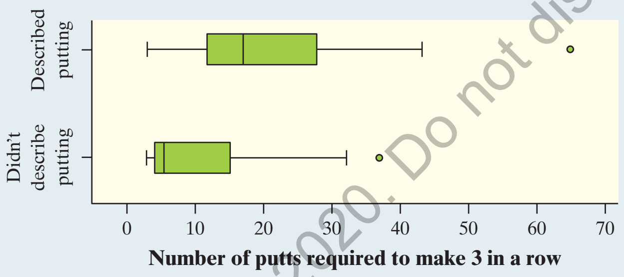
(d) Because 0 is in the interval, there is not convincing evidence of a difference in the population proportions.

(e) Because most of the interval is negative, there is convincing evidence that a greater proportion of students at high school B drive themselves to school.

Review and Recycle

Exercises 76 and 77 refer to the following setting. Athletes often comment that they try not to "overthink it" when competing in their sport. Is it possible to "overthink it"? Or is this just another cliché that athletes use in interviews? To investigate, researchers put some golfers to the test.[27] In the experiment, researchers recruited 40 experienced golfers and allowed them some time to practice their putting. After practicing, they randomly assigned the golfers to one of two groups. Golfers in one group had to write a detailed description of their putting technique (which could lead to "overthinking it"). Golfers in the other group had to do an unrelated verbal task for the same amount of time. After completing their tasks, each golfer was asked to attempt putts from a fixed distance until he or she made 3 putts in a row.

76. **Don't overthink it!** (1.3, 4.2) The boxplots summarize the results of this experiment.



(a) Compare these distributions.

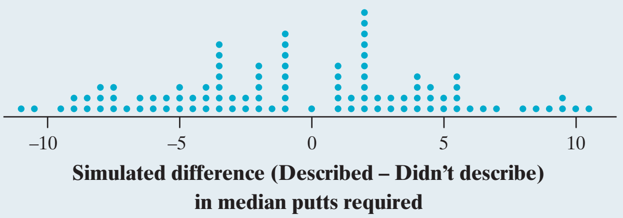
(b) Why might it be better to compare medians rather than means?

(c) What was the purpose of randomly assigning the 40 golfers to the two treatments?

77. **More overthinking it** (4.3) The difference in the medians for the two groups was $17 - 5.5 = 11.5$ putts.

(a) Explain how this difference gives some evidence that it is harder for golfers like these to make putts after being asked to describe their putting technique.

(b) To determine if a difference in medians of 11.5 or more could happen simply due to the chance variation in random assignment, 100 trials of a simulation were conducted assuming that the treatment a subject receives does not affect the number of putts required. Each dot in the dotplot represents the difference in medians for one trial. Based on the results of the simulation, is there convincing evidence that it is harder for golfers like these to make putts after being asked to describe their putting technique? Explain your answer.



Chapter 8 Wrap-Up

FRAPPY! FREE RESPONSE AP® PROBLEM, YAY!

The following problem is modeled after actual AP® Statistics exam free response questions. Your task is to generate a complete, concise response in 15 minutes.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Members at a popular fitness club currently pay a \$40 per month membership fee. The owner of the club wants to raise the fee to \$50 but is concerned that some members will leave the gym if the fee increases. To investigate, the owner plans to survey a random sample of the club members and construct a 95% confidence interval for the proportion of all members who would quit if the fee was raised to \$50.

(a) Explain the meaning of "95% confidence" in the context of the study.

(b) After the owner conducted the survey, he calculated the confidence interval to be 0.18 ± 0.075. Interpret this interval in the context of the study.

(c) According to the club's accountant, the fee increase will be worthwhile if fewer than 20% of the members quit. According to the interval from part (b), can the owner be confident that the fee increase will be worthwhile? Explain.

(d) One of the conditions for calculating the confidence interval in part (b) is that $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$. Explain why it is necessary to check this condition.

After you finish, you can view two example solutions on the book's website (highschool.bfwpub.com/updatedtps6e). Determine whether you think each solution is "complete," "substantial," "developing," or "minimal." If the solution is not complete, what improvements would you suggest to the student who wrote it? Finally, your teacher will provide you with a scoring rubric. Score your response and note what, if anything, you would do differently to improve your own score.

Chapter 8 Review

Section 8.1: Confidence Intervals: The Basics

In this section, you learned that a point estimate is the single best guess for the value of a population parameter. You also learned that a confidence interval provides an interval of plausible values for a parameter based on sample data. To interpret a confidence interval, say, "We are C% confident that the interval from ___ to ___ captures the [parameter in context]," where C is the confidence level of the interval.

The confidence level C describes the percentage of confidence intervals that we expect to capture the value of the parameter in repeated sampling. To interpret a C% confidence level, say, "If we took many samples of the same size from the same population and used them to construct C% confidence intervals, about C% of those intervals would capture the [parameter in context]."

Confidence intervals are formed by including a margin of error on either side of the point estimate. The size of the margin of error is determined by several factors, including the confidence level C and the sample size n. Increasing the sample size n makes the standard deviation of our statistic smaller, decreasing the margin of error. Increasing the confidence level C makes the margin of error larger, to ensure that the capture rate of the interval increases to C%. Remember that the margin of error only accounts for sampling variability—it does not account for any bias in the data collection process.

Section 8.2: Estimating a Population Proportion

In this section, you learned how to construct and interpret confidence intervals for a population proportion. Several important conditions must be met for this type of confidence interval to be valid. First, the data used to calculate the interval must come from a random sample from the population of interest (the Random condition). When the sample is taken without replacement from the population, the sample size should be less than 10% of the population size (the 10% condition). Finally, the observed number of successes $n\hat{p}$ and observed number of failures $n(1-\hat{p})$ must both be at least 10 (the Large Counts condition).

The formula for calculating a confidence interval for a population proportion is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $\hat{p}$ is the sample proportion, z^* is the critical value, and n is the sample size. The value of z^* is based on the confidence level C. To find z^*, use Table A or technology to determine the values of z^* and $-z^*$ that capture the middle $C\%$ of the standard Normal distribution.

The four-step process (State, Plan, Do, Conclude) is perfectly suited for problems that ask you to construct and interpret a confidence interval. You should *state* the parameter you are estimating and the confidence level, *plan* your work by naming the type of interval you will use and checking the appropriate conditions, *do* the calculations, and make a *conclusion* in the context of the problem. You can use technology for the Do step, but make sure that you identify the procedure you are using and type in the values correctly.

Finally, an important part of planning a study is determining the size of the sample to be selected. The necessary sample size is based on the confidence level, the proportion of successes, and the desired margin of error. To calculate the minimum sample size, solve the following inequality for n, where $\hat{p}$ is a guessed value for the sample proportion:

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

If you do not have an approximate value of $\hat{p}$ from a previous study or a pilot study use $\hat{p} = 0.5$ to determine the sample size that will yield a value less than or equal to the desired margin of error.

Section 8.3: Estimating a Difference in Proportions

In this section, you learned how to construct confidence intervals for a difference between two proportions. To verify independence in data collection and that the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal, we check three conditions. The Random condition says that the data must be from two independent random samples or two groups in a randomized experiment. The 10% condition says that each sample size should be less than 10% of the corresponding population size when sampling without replacement. The Large Counts condition says that the number of successes and the number of failures from each sample/group should be at least 10. That is, $n_1\hat{p}_1, n_1(1-\hat{p}_1), n_2\hat{p}_2, n_2(1-\hat{p}_2)$ are all ≥ 10.

A confidence interval for a difference between two proportions provides an interval of plausible values for the difference in the true proportions. The formula is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

The logic of confidence intervals, including how to interpret the confidence interval and the confidence level, is the same as when estimating a single population proportion. Likewise, you can use a confidence interval for a difference in proportions to evaluate claims about the population proportions. For example, if 0 is not included in a confidence interval for $p_1 - p_2$, there is convincing evidence that the population proportions are different.

Comparing confidence intervals for proportions

	Confidence interval for p	**Confidence interval for $p_1 - p_2$**
Name (TI-83/84)	One-sample z interval for p (1-PropZInt)	Two-sample z interval for $p_1 - p_2$ (2-PropZInt)
Conditions	• **Random:** The data come from a random sample from the population of interest. ∘ **10%:** When sampling without replacement, $n < 0.10N$. • **Large Counts:** Both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10. That is, the number of successes and the number of failures in the sample are both at least 10.	• **Random:** The data come from two independent random samples or from two groups in a randomized experiment. ∘ **10%:** When sampling without replacement, $n_1 < 0.10N_1$ and $n_2 < 0.10N_2$. • **Large Counts:** The counts of "successes" and "failures" in each sample or group—$n_1\hat{p}_1, n_1(1-\hat{p}_1), n_2\hat{p}_2, n_2(1-\hat{p}_2)$—are all at least 10.
Formula	$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

What Did You Learn?

Learning Target	Section	Related Example on Page(s)	Relevant Chapter Review Exercise(s)
Identify an appropriate point estimator and calculate the value of a point estimate.	8.1	538	R8.1
Interpret a confidence interval in context.	8.1	541	R8.4, R8.6
Determine the point estimate and margin of error from a confidence interval.	8.1	541	R8.2
Use a confidence interval to make a decision about the value of a parameter.	8.1	541	R8.2, R8.6
Interpret a confidence level in context.	8.1	543	R8.2
Describe how the sample size and confidence level affect the margin of error.	8.1	547	R8.3
Explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval.	8.1	547	R8.4
State and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.	8.2	555	R8.4
Determine the critical value for calculating a *C*% confidence interval for a population proportion using a table or technology.	8.2	558	R8.4
Construct and interpret a confidence interval for a population proportion.	8.2	558, 559	R8.4
Determine the sample size required to obtain a *C*% confidence interval for a population proportion with a specified margin of error.	8.2	562	R8.5
Determine whether the conditions are met for constructing a confidence interval about a difference between two proportions.	8.3	568	R8.6
Construct and interpret a confidence interval for a difference between two proportions.	8.3	571	R8.6

Chapter 8 Review Exercises

These exercises are designed to help you review the important ideas and methods of the chapter.

R8.1 We love football! A Gallup poll conducted telephone interviews with a random sample of adults aged 18 and older. Data were obtained for 1000 people. Of these, 370 said that football is their favorite sport to watch on television.

(a) Define the parameter p in this setting.

(b) What point estimator will you use to estimate p? What is the value of the point estimate?

(c) Do you believe that the value of the point estimate is equal to the value of p? Explain your answer.

R8.2 Sports fans Are you a sports fan? That's the question the Gallup polling organization asked a random sample of 1527 U.S. adults.[28] Gallup reported that a 95% confidence interval for the proportion of all U.S. adults who are sports fans is 0.565 to 0.615.

(a) Calculate the point estimate and the margin of error.

(b) Interpret the confidence level.

(c) Based on the interval, is there convincing evidence that a majority of U.S. adults are sports fans? Explain your answer.

R8.3 It's about *ME* Explain how each of the following would affect the margin of error of a confidence interval, if all other things remained the same.

(a) Increasing the confidence level

(b) Quadrupling the sample size

R8.4 Running red lights A random digit dialing telephone survey of 880 drivers asked, "Recalling the last 10 traffic lights you drove through, how many of them were red when you entered the intersections?" Of the 880 respondents, 171 admitted that at least one light had been red.[29]

(a) Construct and interpret a 95% confidence interval for the population proportion.

(b) Nonresponse is a practical problem for this survey—only 21.6% of calls that reached a live person were completed. Another practical problem is that people may not give truthful answers. What is the likely direction of the bias: Do you think more or fewer than 171 of the 880 respondents really ran a red light? Why? Are these sources of bias included in the margin of error?

R8.5 Do you go to church? The Gallup Poll plans to ask a random sample of adults whether they attended a religious service in the past 7 days. How large a sample would be required to obtain a margin of error of at most 0.01 in a 99% confidence interval for the population proportion who would say that they attended a religious service?

R8.6 Facebook As part of the Pew Internet and American Life Project, researchers conducted two surveys. The first survey asked a random sample of 1060 U.S. teens about their use of social media. A second survey posed similar questions to a random sample of 2003 U.S. adults. In these two studies, 71.0% of teens and 58.0% of adults used Facebook.[30] Let p_T = the true proportion of all U.S. teens who use Facebook and p_A = the true proportion of all U.S. adults who use Facebook.

(a) Calculate and interpret a 99% confidence interval for the difference in the true proportions of U.S. teens and adults who use Facebook.

(b) Based on the confidence interval from part (a), is there convincing evidence of a difference in the population proportions? Explain your answer.

Chapter 8 AP® Statistics Practice Test

Section I: Multiple Choice *Select the best answer for each question.*

T8.1 The Gallup Poll interviews 1600 people. Of these, 18% say that they jog regularly. The news report adds: "The poll had a margin of error of plus or minus 3 percentage points at a 95% confidence level." You can safely conclude that

(a) 95% of all Gallup Poll samples like this one give answers within ±3% of the true population value.

(b) the percent of the population who jog is certain to be between 15% and 21%.

(c) 95% of the population jog between 15% and 21% of the time.

(d) we can be 95% confident that the sample proportion is captured by the confidence interval.

(e) if Gallup took many samples, 95% of them would find that 18% of the people in the sample jog.

T8.2 A confidence interval for a difference in proportions is −0.077 to 0.013. What are the point estimate and the margin of error for this interval?

(a) −0.032, 0.045

(b) −0.032, 0.090

(c) −0.032, 0.180

(d) −0.045, 0.032

(e) −0.045, 0.090

T8.3 In a random sample of 100 students from a large high school, 37 regularly bring a reusable water bottle from home. Which of the following gives the correct value and interpretation of the standard error of the sample proportion?

(a) In samples of size 100 from this school, the sample proportion of students who bring a reusable water bottle from home will be at most 0.095 from the true proportion.

(b) In samples of size 100 from this school, the sample proportion of students who bring a reusable water bottle from home will be at most 0.048 from the true proportion.

(c) In samples of size 100 from this school, the sample proportion of students who bring a reusable water bottle from home typically varies by about 0.095 from the true proportion.

(d) In samples of size 100 from this school, the sample proportion of students who bring a reusable water bottle from home typically varies by about 0.048 from the true proportion.

(e) There is not enough information to calculate the standard error.

T8.4 Many television viewers express doubts about the validity of certain commercials. In an attempt to answer their critics, Timex Group USA wishes to estimate the true proportion p of all consumers who believe what is shown in Timex television commercials. Which of the following is the smallest number of consumers that Timex can survey to guarantee a margin of error of 0.05 or less at a 99% confidence level?

(a) 550
(b) 600
(c) 650
(d) 700
(e) 750

T8.5 Which of the following is the critical value for calculating a 94% confidence interval for a population proportion?

(a) 1.555
(b) 1.645
(c) 1.881
(d) 1.960
(e) 2.576

T8.6 A radio talk show host with a large audience is interested in the proportion p of adults in his listening area who think the drinking age should be lowered to 18. To find this out, he poses the following question to his listeners: "Do you think that the drinking age should be reduced to 18 in light of the fact that 18-year-olds are eligible for military service?" He asks listeners to go to his website and vote "Yes" if they agree the drinking age should be lowered and "No" if not. Of the 100 people who voted, 70 answered "Yes." Which of the following conditions are violated?

I. Random II. 10% III. Large Counts

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I, II, and III

T8.7 A marketing assistant for a technology firm plans to randomly select 1000 customers to estimate the proportion who are satisfied with the firm's performance. Based on the results of the survey, the assistant will construct a 95% confidence interval for the proportion of all customers who are satisfied. The marketing manager, however, says that the firm can afford to survey only 250 customers. How will this decrease in sample size affect the margin of error?

(a) The margin of error will be about 4 times larger.
(b) The margin of error will be about 2 times larger.
(c) The margin of error will be about the same size.
(d) The margin of error will be about half as large.
(e) The margin of error will be about one-fourth as large.

T8.8 Thirty-five people from a random sample of 125 workers from Company A admitted to using sick leave when they weren't really ill. Seventeen employees from a random sample of 68 workers from Company B admitted that they had used sick leave when they weren't ill. Which of the following is a 95% confidence interval for the difference in the proportions of workers at the two companies who would admit to using sick leave when they weren't ill?

(a) $0.03 \pm \sqrt{\frac{(0.28)(0.72)}{125} + \frac{(0.25)(0.75)}{68}}$

(b) $0.03 \pm 1.96\sqrt{\frac{(0.28)(0.72)}{125} + \frac{(0.25)(0.75)}{68}}$

(c) $0.03 \pm 1.96\sqrt{\frac{(0.28)(0.72)}{125} - \frac{(0.25)(0.75)}{68}}$

(d) $57 \pm 1.96\sqrt{\frac{(0.28)(0.72)}{125} + \frac{(0.25)(0.75)}{68}}$

(e) $57 \pm 1.96\sqrt{\frac{(0.28)(0.72)}{125} - \frac{(0.25)(0.75)}{68}}$

T8.9 A telephone poll of an SRS of 1234 adults found that 62% are generally satisfied with their lives. The announced margin of error for the poll was 3%. Does the margin of error account for the fact that some adults do not have telephones?

(a) Yes; the margin of error accounts for all sources of error in the poll.
(b) Yes; taking an SRS eliminates any possible bias in estimating the population proportion.
(c) Yes; the margin of error accounts for undercoverage but not nonresponse.
(d) No; the margin of error accounts for nonresponse but not undercoverage.
(e) No; the margin of error only accounts for sampling variability.

T8.10 At a baseball game, 42 of 65 randomly selected people own an iPod. At a rock concert occurring at the same time across town, 34 of 52 randomly selected people own an iPod. A researcher wants to test the claim that the proportion of iPod owners at the two venues is different. A 90% confidence interval for the difference (Game − Concert) in population proportions is $(-0.154, 0.138)$. Which of the following gives the correct outcome of the researcher's test of the claim?

(a) Because the interval includes 0, the researcher can conclude that the proportion of iPod owners at the two venues is the same.

(b) Because the center of the interval is −0.008, the researcher can conclude that a higher proportion of people at the rock concert own iPods than at the baseball game.

(c) Because the interval includes 0, the researcher cannot conclude that the proportion of iPod owners at the two venues is different.

(d) Because the interval includes −0.008, the researcher cannot conclude that the proportion of iPod owners at the two venues is different.

(e) Because the interval includes more negative than positive values, the researcher can conclude that a higher proportion of people at the rock concert own iPods than at the baseball game.

Section II: Free Response *Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

T8.11 The U.S. Forest Service is considering additional restrictions on the number of vehicles allowed to enter Yellowstone National Park. To assess public reaction, the service asks a random sample of 150 visitors if they favor the proposal. Of these, 89 say "Yes."

(a) Construct and interpret a 99% confidence interval for the proportion of all visitors to Yellowstone who favor the restrictions.

(b) Based on your work in part (a), is there convincing evidence that more than half of all visitors to Yellowstone National Park favor the proposal? Justify your answer.

T8.12 For some people, mistletoe is a symbol of romance. Mesquite trees, however, have no love for the parasitic plant that attaches itself and steals nutrients from the tree. To estimate the proportion of mesquite trees in a desert park that are infested with mistletoe, a random sample of mesquite trees was randomly selected. After inspecting the trees in the sample, the park supervisor calculated a 95% confidence interval of 0.2247 to 0.5753.

(a) Interpret the confidence level.

(b) Calculate the sample size used to create this interval.

T8.13 Do "props" make a difference when researchers interact with their subjects? Emily and Madi asked 100 people if they thought buying coffee at Starbucks was a waste of money.[31] Half of the subjects were asked while Emily and Madi were holding cups from Starbucks, and the other half of the subjects were asked when the girls were empty handed. The choice of holding or not holding the cups was determined at random for each subject. When holding the cups, 19 of 50 subjects agreed that buying coffee at Starbucks was a waste of money. When they weren't holding the cups, 23 of 50 subjects said it was a waste of money. Calculate and interpret a 90% confidence interval for the difference in the proportion of people like the ones in this experiment who would say that buying coffee from Starbucks is a waste of money when asked by interviewers holding or not holding a cup from Starbucks.